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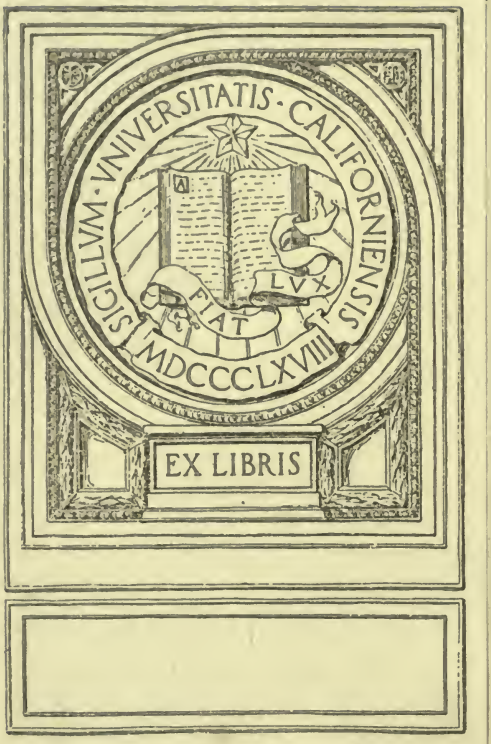
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EDITORIAL NOTE

THE DIRECTLY-USEFUL TECHNICAL SERIES requires a few words by way of introduction. Technical books of the past have arranged themselves largely under two sections: the Theoretical and the Practical. Theoretical books have been written more for the training of college students than for the supply of information to men in practice, and have been greatly filled with descriptions of an academic character. Practical books have often sought the other extreme, omitting the scientific basis upon which all good practice is built, whether discernible or not. The present series is intended to occupy a midway position. The information, the problems, and the exercises are to be of a directly-useful character, but must at the same time be wedded to that proper amount of scientific explanation which alone will satisfy the inquiring mind. We shall thus appeal to all technical people throughout the land, either students or those in actual practice.

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AUTHOR'S PREFACE

THIS work is intended as a text-book for the architect, surveyor, and student, and for practical men engaged in the various branches of the building industries. The examples throughout are specially selected with a view to their practical application, and are chosen in such a way as to simplify some of the difficulties which arise in actual workshop practice. The book is not intended as a primer or elementary work for beginners. Hence, the usual instructions as to the use of the various drawing tools have been omitted. It is assumed that the student is a practical man with some knowledge of the more elementary principles of geometry, and the simpler elementary problems, particularly in plane geometry, have been omitted. The space so gained allows of the inclusion of a larger number of more advanced practical examples than would otherwise have been possible. The work should prove of much use to technical students who are studying practical geometry in connection with building construction, carpentry and joinery, and other similar subjects. It is hoped that the problems are sufficiently self-explanatory to appeal not only to those students who have the advantage of studying under the direction of a competent teacher, but to those, also, who are so far removed from such advantages, and who have to rely upon private study. Such students will probably find the exercises at the end of the book a useful means of testing their knowledge of the subject matter of each chapter, and the inclusion of many examples from the questions set at the Board of Education "Lower" Examination in Geometry and Graphics should materially assist those who are studying for these examinations. A certain part of the Board of Education Syllabus in Building Construction is also covered by the various chapters of this volume.

The Author wishes to render his tribute to the late Mr. W. J. Lineham, B.Sc., M.I.C.E., to whose inspiration and guidance this book largely owed its inception. The Author's gratitude is also due to Mr. John L. Bale for his very painstaking assistance, criticism, and help most freely rendered in the compilation of this book.

Care has been taken to avoid errors, but the Author will be grateful for any notification of mistakes which may have crept into the work.

J. E. PAYNTER.

GOLDSMITH'S COLLEGE,
NEW CROSS, LONDON, S.E.14.
February, 1921.

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(NOTE.—*To assist the student in the systematic study of his geometry, examples, covering the principles of the various chapters, have been set at the end of the book*—see p. 371.)

PRACTICAL GEOMETRY FOR BUILDERS

INTRODUCTION

THE Science of Geometry is of very great importance to those engaged in the various branches of the building trades. A thorough knowledge of the practical applications of geometrical principles will go far to correct the *rule of thumb* knowledge which often leads to mistakes. The joiner in his more difficult "setting out," such as "circle on circle" work and staircase work, must of necessity be familiar with these principles. The carpenter who aspires to be something more than a mere journeyman must know how these geometrical principles are applied to roof construction, centreing for arches and groins, raking mouldings, etc.; and much the same may be said of the bricklayer and the plumber. The older school of craftsmen were sometimes wont to decry the work of the studious technical youth, and assert that such matters could not be learnt from books. There is a certain amount of truth underlying these statements. The *Art* of any craft, such as carpentry or joinery, can only be learnt in one way, by actually handling the tools and executing the various examples of handicraft. No amount of study will ever help a man to make a mortise and tenon joint. This can only be learnt by *doing*. The same is true of the *Art* of any craft. There is, however, a *Science* of carpentry and joinery, much of which can be learnt from well-written text-books and from practical teachers. There is nothing to be said for the man who knows all about the *Science* of his particular trade, but little or nothing about the *Art* of it. There is everything to be said for the man who can handle the tools and execute work in the most craftsmanlike manner, and who, in addition, knows sufficient of the Science underlying his work to be able to correct and amplify the deductions which he has made as a result of his own experience. Given two men, both equally good craftsmen, the one possessing no knowledge of any of the various Sciences

underlying his work; the other possessed of such knowledge; there can be no doubt which man is likely to be of most value to his employer, and most likely to be placed in a position of greater responsibility—and higher pay.

The **Science of Geometry**, as its name suggests, is of great antiquity. The name, **Geometry**, is derived from two Greek "roots," Ge = the Earth, and Metron = a measure, and it is clear that in the earliest period of which we have any record of the use of Geometry, it was used for "**Earth-measurement**," or *Land-surveying*, as we should now term it. According to Greek historians Geometry originated in Egypt as early as 600 B.C., and was used for the measurement of land, or for readjusting the boundaries of the various provinces which had been inundated by the overflow of the Nile. Thales, who lived in Egypt from 639 B.C. to 548 B.C., taught his fellow-countrymen how to calculate the height of an obelisk by measuring the length of its shadow. Pythagoras, Archimedes, and Euclid, carried on the study of certain branches of the Science, and much of our more practical work in Geometry to-day is based upon the principles which they established in a somewhat abstract manner.

CHAPTER I

SCALES

Choice of Suitable Scale.—Most of the work for which the building student has to prepare drawings is of such large dimensions that it would be impossible, or at least impracticable, to make **full size** drawings of it. Hence a suitable **scale** has to be chosen. The choice of scale is largely a matter of judgment. For instance, the drawings of a certain job as prepared by the architect may be drawn to a scale of $\frac{1}{8}$ in. = 1 ft., but the general foreman of the job, in preparing working drawings of any details of constructional woodwork or ironwork, may probably draw them to a scale of $\frac{1}{2}$ in. = 1 ft. and so on. **Plain scales:** For such purposes a plain scale, as illustrated in Fig. 1, is used. Fig. 1 (*a*) represents a plain scale of 1 in. to 1 ft. That is to say, each 1 in. on the scale represents an actual measurement of 12 ins., *i.e.* the drawing is $\frac{1}{12}$ full size. This fraction ($\frac{1}{12}$) is spoken of as the *Representative Fraction.* On a scale of $\frac{1}{8}$ in. = 1 ft. each inch represents 8 ft. or 96 ins. Hence the drawing made with such a scale is $\frac{1}{96}$ of the actual size of the work. The representative fraction of such a scale is $\frac{1}{96}$. Scales are usually bought ready for use, but students should be familiar with the method of preparing a scale to fulfil any special requirement in case the need should arise. The preparation of any plain scale involves very little except accurate division of a line into a given number of equal parts. **Decimal scales:** The method of doing this is sufficiently explained in Fig. 1 (*b*), which is a decimal scale. The construction line OB is drawn, and on this line ten *equal* units *of any length* are marked off with dividers. The last point 10 is joined up to A, and parallels to this line BA are drawn from the points 9, 8, 7, etc., on the line OB. In this way a series of so called "similar" triangles are formed, with the result that the line OA is divided in the same ratio as OB, *i.e.* into ten equal parts. From such a scale it is possible to take off measurements in inches and *tenths* of an inch, as shown on the diagram. It

will be observed, however, that it is only possible to read off accurately to the *first* decimal place in such a measurement as 2·57 ins. To read off such a measurement from a plain scale would mean that the 2·5 ins. was taken off *accurately*, and the 7 in the second decimal place had to be *guessed* at; or at best, approximately estimated. Thus arises the need of a decimal scale diagonally divided, as shown in Fig. 1 (*c*). **Diagonal scales:** Compare Fig. 1 (*c*) with Fig. 1 (*b*). Note that the 1 in. on the extreme left-hand side of scale (*i.e.* OA) as before, has been

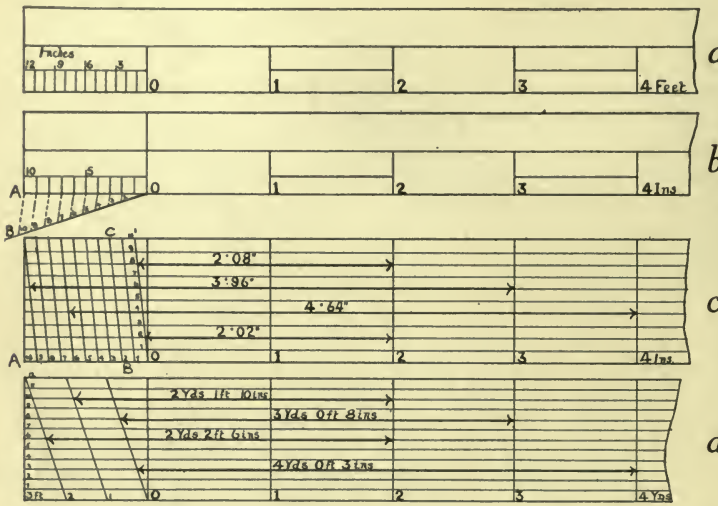


FIG. 1.—Scales.

a. Plain scale. *b*. Decimal scale.
c. Decimal scale diagonally divided. *d*. Diagonal scale of yards, feet, and inches.

divided into ten equal parts, but the top edge of the scale has been similarly divided into ten equal parts, each ·1 in. long, and diagonal lines, as O–10′ drawn. The scale has also been divided into ten equal divisions in the width, and fine lines drawn parallel to the edge of the scale. Refer again to the diagonal lines O, 10′, etc. Note that each diagonal line slopes in such a way that its top end, say C, is $\frac{1}{10}$ in. (·1 in.) nearer the left than its bottom end B. Hence, if in crossing the whole scale a diagonal line travels $\frac{1}{10}$ in. to the left, it is clear that by the time it reaches the parallel No. 3 it will be $\frac{3}{10}$ of $\frac{1}{10}$ in. nearer the left than it was at the bottom edge of the scale. Now $\frac{3}{10}$ of $\frac{1}{10}$ in. = $\frac{3}{100}$ in. or ·03 in. Hence, by using such a scale, we are able to read off

measurements correct, not only in tenths of an inch, but in *hundredths* also. Thus, in the examples indicated in Fig. 1 (*c*) we can take account of the $\frac{2}{100}$ in. in 2·02 ins, the $\frac{4}{100}$ in. in 4·64 ins., the $\frac{6}{100}$ in. in 3·96 ins, and the $\frac{8}{100}$ in. in 2·08 ins.

Scale to read Yards, Feet, etc.—Fig. 1 (*d*) illustrates still further the use of a diagonal scale. In this scale each inch on the scale represents 1 yd. of actual measurement, *i.e.* a scale of 1 in. to 1 yd. By subdividing the extreme left-hand unit of the scale into three equal parts we are able to read off *yards and feet* from the scale. In order to read off *inches* accurately on such a

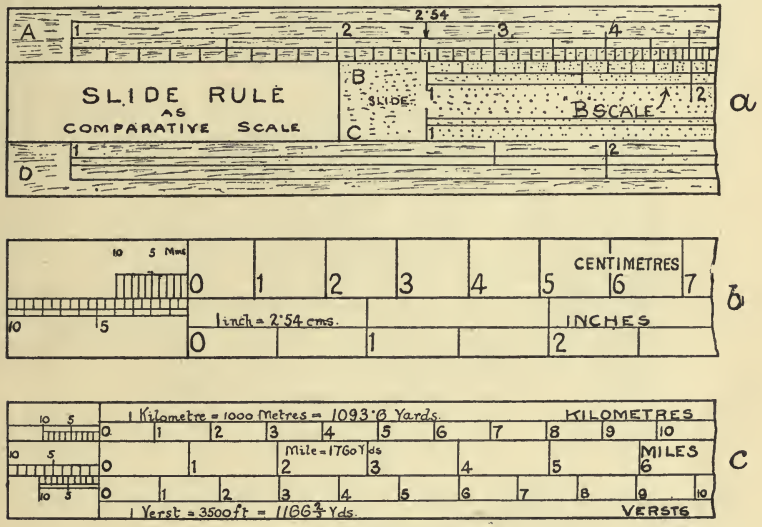


FIG. 2.—Comparative scales.

a. Slide rule as comparative scale.
b. Comparative scale of English and metric lengths.
c. Scale showing comparison between English, French, and Russian road distances.

scale, the width of the scale is divided into twelve equal parts, and parallel lines drawn along the whole length of the scale.

Diagonal lines are then drawn from points 1 and 2 at the left-hand end of the scale. Note that each of these diagonals slopes 1 ft. (scale reading) to the left by the time it has crossed the twelve parallels of the scale. Hence by the time it has reached the third parallel it has sloped $\frac{3}{12}$ of 1 ft. to the left, *i.e.* it is 3 ins. to the left (scale reading) of the point representing 1 ft. on the bottom edge of scale. Thus we can read off the 3 ins. accurately in the dimension 4 yds. 0 ft. 3 ins. as shown. The diagram

indicates with sufficient clearness the method of reading off any other scalar measurement.

Comparative Scales.—These are really plain scales arranged in such a way (usually one on each edge of a scale) as to show at a glance the relative number of units of one scale contained in any given number of units of another scale. A *comparative scale* showing the relative values of English lengths and metrical lengths is a very useful one, in view of the fact that English firms of builders may be called upon to execute continental work dimensioned according to the *metric* system. Fig 2 (*b*) shows a comparative scale of centimetres and inches ; and Fig. 2 (*c*) a comparative scale which would be useful for larger survey work. In this case the larger metrical unit—a kilometre—is employed, and a

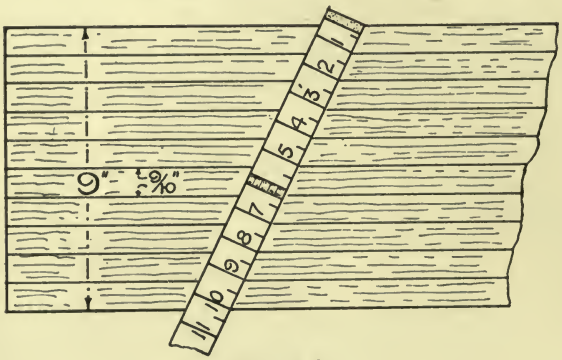


FIG. 2 *d*.

comparative scale is prepared showing the number of kilometres in any given number of English miles. This scale also shows a comparison between Russian versts and English miles.

Slide Rule used as a Comparative Scale.—In this connection it is interesting to note that the slide rule may be set in such a way as to serve as a comparative scale for English and metrical measurements. Reference to the metric tables on page 9, shows that a metre—the French standard of length—is 39·37 ins. Hence 1 in. is equal to 2·54 centimetres. If the left-hand 1 of the B scale of the slide rule is set to the 2·54 mark on the A scale, as shown in Fig 2 (*a*) then, taking the figures on the B scale as representing English measurements, the equivalent values in metrical lengths may be read off on the A scale as shown in Fig. 2 (*b*). Note that the 2 on the B scale almost coincides with 5 on the A scale, *i.e.* 2 ins. = 5 centimetres

(approx.). Most builders look upon the slide rule as essentially an engineer's device for shortening arithmetical calculations, but often it is equally useful to architects and builders. Its use is explained fully in other works in this series.*

Division of Board into Equal Widths.—Refer to Fig. 16. Note method of division of a straight line into any number of equal parts. Fig. 8 shows how this method is employed when it is required to mark off a board quickly, and without calculation, into any given number of equal parts. The ordinary 2 ft. rule is so arranged that one end coincides with one edge of the board to be divided into ten equal widths, while the 10 in. mark on the rule is made to coincide with the other edge of the board. Dots are

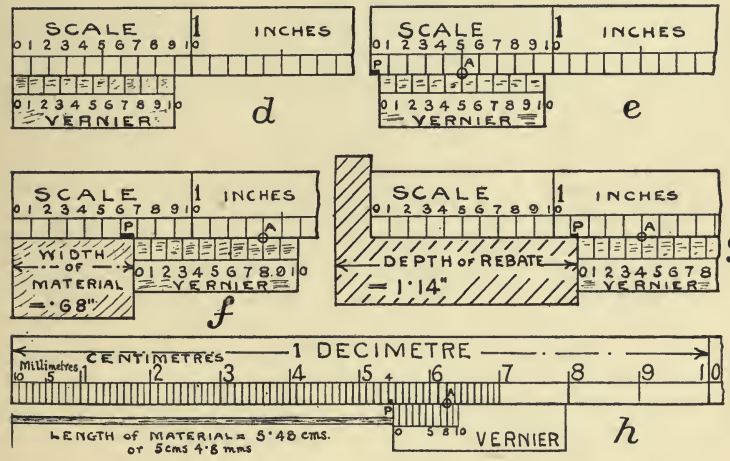


FIG. 3.—Verniers.

d. Vernier attachment to ordinary rule. *e*. Vernier used for measuring a distance P. *f*, *g*, *h*. Practical illustrations of the use of a vernier.

marked on the board opposite each inch mark on the rule and lines are then drawn though these dots, as shown.

Vernier Scales.—A **vernier** (Fig. 3) is a *graduated attachment* so made and arranged as *to slide along an ordinary scale.* By using such an attachment it is possible to *measure more minutely* than would be possible with an ordinary scale. Fig. 3 (*d*) represents an ordinary scale of inches decimally divided, *i.e.* each inch divided into tenths. A *vernier* for such a scale is constructed as

* *Arithmetic for Engineers*, by C. B. Clapham, 2nd Edition, 7*s.* 6*d.* net. *Mathematics for Engineers*, by W. N. Rose, Part I., 2nd Edition, 10*s.* 6*d.* net.

follows. The total length of the graduated space 0–10 on the vernier is made exactly equal to $\frac{9}{10}$ in. on the scale (Fig. 3 (*d*)). This length ($\frac{9}{10}$ in.) is then divided carefully into ten equal parts, as shown. Hence, if the 0 on the vernier is placed opposite the 0 on the scale, as shown in (*d*), the 10 *on the vernier* will *coincide exactly* with the 9 (*i.e.* $\frac{9}{10}$ in.) *on the scale*. Thus, each division, 0–1 ; 1–2, etc., on the vernier is *exactly* $\frac{9}{10}$ of any division, 0–1 ; 1–2, etc., on the scale. Suppose the vernier to slide along the edge of the scale (Fig. 3 (*e*)) until the *fifth division* on the vernier happens to coincide exactly with *some* point of division on the scale, as shown at A. Then the space P, between the end of the scale and the end of the vernier is known to be exactly $\frac{5}{10}$ of $\frac{1}{10}$ in., *i.e.* $\frac{5}{10} \times \frac{1}{10} = \frac{5}{100}$ in. or ·05 in.

The use of the vernier is more clearly illustrated in (*f*) Fig. 3. Here the width of a piece of material has to be measured correctly to the *second* decimal place. The ordinary scale of tenths would only give an approximate reading, correct, that is, to the *first* decimal place. By sliding the vernier up until its 0 end is close against the edge of the piece to be measured, as shown in (*f*), an accurate measurement can be read off in the following manner. It is evident that the width of the material lies somewhere *between* $\frac{6}{10}$ in. and $\frac{7}{10}$ in. The vernier tells us how much *greater* than $\frac{6}{10}$ in. this width really is. Looking along the vernier we notice that its *eighth division* coincides exactly with a division on the scale of inches at A. This tells us that the space P is $\frac{8}{10}$ of $\frac{1}{10}$ in. or $\frac{8}{10} \times \frac{1}{10} = \frac{8}{100} =$ ·08 in. We have already noted that the width of the material is greater than $\frac{6}{10}$ in. and less than $\frac{7}{10}$ in. Thus, if we add the $\frac{6}{10}$ in. or ·6 in. (*as read off from the scale of inches*) to the $\frac{8}{100}$ in. or ·08 in. (*as read off from the vernier*) we have a total of ·6 in. + ·08 in. = ·68 in. This is the exact width of the material. In (*g*) Fig. 3 the exact depth of a rebate in a piece of timber is measured in a similar manner. From the scale of inches we note that the rebate is *slightly over* $1\frac{1}{10}$ in. In order to ascertain exactly *how much* the rebate exceeds $1\frac{1}{10}$ in. we use the vernier as shown. We note that division 4 on the vernier coincides with a mark (A) on the scale. This tells us that the space P is $\frac{4}{10}$ of $\frac{1}{10}$ in., *i.e.* $\frac{4}{10} \times \frac{1}{10} = \frac{4}{100}$ in. or ·04 in. Adding this ·04 in. to the 1·1 in. or $1\frac{1}{10}$ in. already read off from the top scale, we arrive at the total depth of the rebate, *i.e.* 1·1 in. + ·04 in. = 1·14 in.

N.B.—In reading off the dimensions at A in Fig. 3 (*f* and *g*) we take no notice as to *which* division *on the scale* of inches happens to coincide with a division on the vernier. We note only the number of the particular division *on the vernier* which coincides with *any* division on the scale.

Verniers may be used in a similar manner on any scale, but a specially graduated vernier must be made to suit each scale. Thus, for metrical measurements a metre rod (39·37 ins.) might be graduated in centimetres (·39 in.) and millimetres (·039 in.). The vernier for such a scale would have a total graduated length equal to $\frac{9}{10}$ of a centimetre, *i.e.* 9 millimetres, as shown at *h*, Fig. 3. Thus each division on the vernier would be ·9 of a millimetre. By sliding the vernier along the scale an exact measurement in centimetres, millimetres, and tenths of a millimetre could be read off. The *plain scale* would give the measurement correct in *centimetres and millimetres*, and the *vernier* would supply the further information as to how much the measurement exceeded, say, 15·4 centimetres (15 centimetres 4 millimetres), and how much less than 15·5 centimetres. Suppose the eighth division on the vernier coincided with some mark on the scale. This would tell us that the measurement we were reading off was $\frac{8}{10}$ of a millimetre greater than 15·4 centimetres, *i.e.* 15·48 centimetres, or 15 cms. 4·8 mms.

METRIC SYSTEM.

Handy Conversion Rules.

Standard of Linear Measurement:—**Metre** = 39·37 inches, approx. 39⅜ ins.
Fractional parts or *multiples* of the metre are obtained by shifting the decimal point to *left* or *right* of its position in 39·37; thus:—

1 *Decimetre*	= $\frac{1}{10}$ of a metre = 3·93 ins.	Fractional parts of metre.
1 *Centimetre*	= $\frac{1}{10}$,, decimetre = ·39 in.	
1 *Millimetre*	= $\frac{1}{10}$,, centimetre = ·03 in.	
1 **Dekametre**	= 10 metres = 393·7 ins.	Multiples of metre.
1 **Hectametre**	= 10 dekametres = 3937· ins.	
1 **Kilometre**	= 10 hectametres = 1093⅓ yds.	

Useful Memoranda.

1 foot = 30 centimetres (approx.).
1 millimetre = $\frac{1}{25}$th of an inch (approx.).
1 centimetre = $\frac{13}{32}$,, ,, ,,

To convert	Multiply by
Inches to centimetres	2·54
Centimetres to inches	0·3937
Feet to metres	0·3048
Metres to feet	3·281
Yards to metres	0·9144
Metres to yards	1·094
Miles to kilometres	1·609
Kilometres to miles	0·6214

CHAPTER II

ANGLES AND ANGLE MEASUREMENT

A **Plane Angle** may be defined as the inclination of two straight lines to one another; assuming that the lines *meet together*, that they are in the *same plane*, and are not in the *same straight line*. In measuring angles, as in linear measurement, some **unit** must be chosen as a standard, and the unit usually employed is the **degree**. The advantage of using such a unit is that it is *invariable*, as may thus be demonstrated. The hands of a clock at 12 o'clock are exactly in line with each other. From 12 o'clock until after 1 o'clock the minute hand is moving around the face until it comes again to the 12. Suppose the hour hand remains stationary (*i.e.* at 12 o'clock) while the minute hand makes one complete revolution of the face. Then the minute hand may be looked upon as "*generating*" angles which gradually increase in size until the minute hand has described a complete circle. Suppose the circular dial to be marked off into 360 equal parts. Then in describing a complete revolution of the dial the minute hand travels over the whole 360 spaces. Call these spaces "**degrees.**" Assume that the minute hand, starting as before from 12, has just passed through one of these 360 spaces, then the angular space passed through is $\frac{1}{360}$ of the whole circular dial, or 1 *degree*. Thus at 3 o'clock, if the clock was working, the hands would have 90 degrees between them, *i.e.* $\frac{1}{4}$ of the complete 360 equal angular spaces (see Fig. 4).

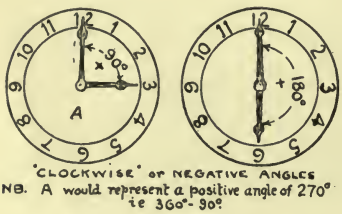


FIG. 4.—Plane angles.

Thus we might take either the $\frac{1}{360}$ part of the whole circle or $\frac{90}{360}$ as our unit of measurement. In either case we should have a unit that was *definite* and *invariable*. Degrees are indicated thus, 27°. Degrees are further subdivided into *minutes* and

seconds, $\frac{1}{60}$ of a degree = 1 minute; $\frac{1}{60}$ of a minute = 1 second. 35° 24′ 13″ represents 35 degrees, 24 minutes, 13 seconds.

Another unit of angular measure is the **radian.** This form of measurement was formerly described as the "*circular measure*" of an angle. This is illustrated in (*a*) Fig. 5. A straight line, LM, is drawn, and with radius LM the arc MON described. Keeping the compasses set to the same radius (LM) the chord MN is stepped off and the straight line LN drawn. These two lines include an angular space of 60°. Note that in this case it is the *chord* MN which is equal to the radius of the arc. Hence the distance around the arc MON must be greater than MN, and therefore greater than the radius of the arc.

In (*b*) Fig. 5 the *radius* LM of the arc is the same as in (*a*), but the angular distance between M and N is *less* than in (*a*).

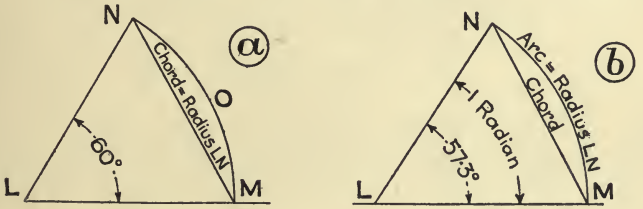


FIG. 5.—"Radian measure" of an angle.

In this case the distance around the arc MN has been made *equal to the radius* LN. Hence the chord MN is *shorter* than in (*a*). (Test this with a pair of dividers.) The angular space (57·3°) between LM and LN (*b*) is spoken of as a **radian,** and may be looked upon as another *unit of measurement of angular space.* Thus we state the magnitude of an angle either in (1) *degrees,* or (2) *radians.* The second method is known as the *circular measure* of an angle.

Scales of Chords.—On many boxwood scales and protractors is a graduated scale, marked CHO (chords). This scale may be used as a further means of measuring the magnitude of any given angle, or of setting out an angle of a given number of degrees. The use of such a scale will be best understood by reference to Fig. 6. This illustrates the method of constructing a scale of chords. On a sheet of Bristol board or stiff cardboard describe a semicircle of any radius OP. Graduate the semicircle into thirty-six equal spaces. This can be done most expeditiously by using a protractor and marking off intervals of 5° around the semicircle. Number these divisions 0, 5, 10, etc., as shown. With centre O and radius O5, O10, etc., describe

arcs of circles as shown. Number the points at which these arcs meet the diameter of the semicircle as at P, R, etc. Then the graduated line OPR may be looked upon as a scale of chords similar to that usually found engraved on scales.

To use a Scale of Chords.

PROBLEM (Fig. 7).—To set out an angle of 80° by means of a scale of chords. Draw a line MK as shown. Measure O60 on the scale

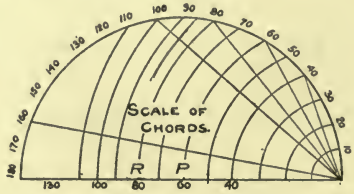


FIG. 6.—Method of constructing a scale of chords.

(Fig. 7). Describe a semicircle MQL with this as radius. Measure OR (marked 80) on scale of chords. With centre M (Fig. 8) and radius OR, describe the arc QN. Join KQ. Then MK and KQ will include an angle of 80°.

N.B.—The first step is always that of *measuring the* O60 *length* given on scale of chords, *i.e.* OP.

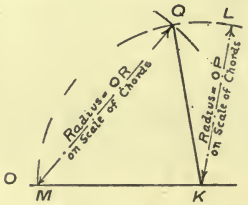


FIG. 7.—Method of using a scale of chords.

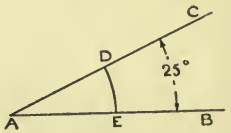


FIG. 8.—Measuring an angle by means of a scale of chords.

PROBLEM (Fig. 9).—**Given an angle, to measure its magnitude by means of a Scale of Chords.**—Draw any two lines AB and AC, including an acute angle (Fig. 8), *i.e.* one known to be less than 90°. Measure OP (*i e.* O60) on scale of chords. With this as radius and centre A describe the arc DE. Measure DE with dividers. Apply this measurement to scale of chords, measuring from O, and noting which division and number on scale of chords is opposite the other point of the dividers, in this case 25. Then the magnitude of the given angle is found to be 25°. In order to obtain accurate readings, it follows that the semicircle (scale of chords) must be divided into 180

equal parts. In the diagram (Fig. 6) only intervals of 10° have been marked, in order to avoid confusion.

Use of 2-ft. Rule for setting out Angles.—The ordinary 2-ft. rule may be used as a rough-and-ready means of setting out angles as shown in Fig. 9. The *distance* D for any angle A may be found by reference to the following table and the rule opened to the required distance. The rule may then be used in the same way as a bevel and is sufficiently accurate for ordinary bench work. Thus suppose that two strips of moulding are to be mitred together in such a way that the angle at the mitred corner is 40°. Referring to the table of distances which follows, the correct *distance* to which the rule should be set for a 40° angle is found to be 8·2 ins. or $8\frac{2}{10}$ ins. This gives the correct distance apart D of the ends of the rule so that the **angle** A may be 40°.

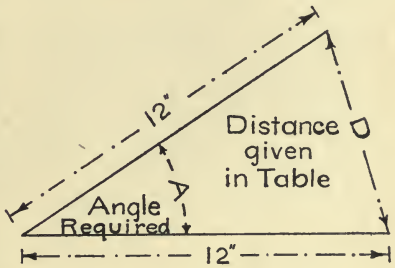


FIG. 9.

ANGLES AND DISTANCES.

ANGLES AND DISTANCES FOR SETTING THE TWO-FOOT RULE.

Angle.	Distance.	Angle.	Distance.	Angle.	Distance.
Degrees.	Inches.	Degrees.	Inches.	Degrees.	Inches.
5	1·05	29	6·01	45	9·18
10	2·09	30	6·21	46	9·38
15	3·13	31	6·41	48	9·76
16	3·34	32	6·62	50	10·14
17	3·55	33	6·82	52	10·52
18	3·75	34	7·02	54	10·9
19	3·96	35	7·22	56	11·27
20	4·17	36	7·42	58	11·64
21	4·37	37	7·61	60	12·0
22	4·58	38	7·81	62	12·36
23	4·78	39	8·01	64	12·72
24	4·99	40	8·2	66	13·07
25	5·19	41	8·4	68	13·42
26	5·4	42	8·6	70	13·77
27	5·6	43	8·8	72	14·11
28	5·81	44	8·99	74	14·44

The correct distance for the angles between 74° and 90° may be found by calculation.

Theorem (Fig. 10*a*). *If one straight line AB* (A_1B, A_2B) *meets another straight line CD, the sum of the angles on either side of AB is* 190° *or two right angles.*

Test for Accuracy of Try Square (Fig. 10*b*).—The principle enunciated in the foregoing paragraph is involved in the method of testing a try square. The square is first placed in position *a*, and a line is drawn along the blade. The square is then reversed and placed in position *b*. If the outer edge of the blade coincides exactly with the line drawn when the square was in position *a*, the square is known to be true, *i.e.* to contain an angle of 90°. Fig. 10*b* illustrates how it would appear if the square was not true, *i.e.* if the angle between stock and blade was less (or greater) than 90°.

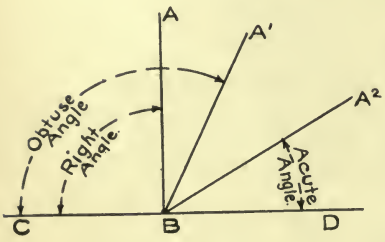


FIG. 10*a*.

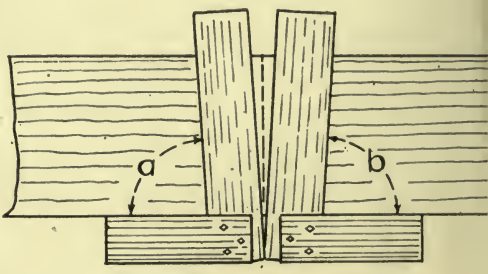


FIG. 10*b*.—Testing a Try Square.

Trigonometrical Ratios of Angles.—A **ratio** is a *comparison* of two (or more) quantities or lengths. Thus, of two men A and B: if A earns £200 per annum and B earns £350 per annum, A's salary is $\frac{200}{350}$ or $\frac{20}{35}$ or $\frac{4}{7}$ of B's salary. Here, in comparing A's income with B's, we express the *comparison* in the form of a *fraction*. We could also express it thus: A's salary as compared with B's salary is as 200 compared with 350. In order to shorten this statement we employ a sign :, thus A : B as 200 : 350. This is read as, A is to B as 200 is to 350. When we have two pairs of terms compared we further shorten the statement thus, A : B :: 200 : 350. Such comparisons are called "**ratios.**"

Trigonometrical Ratios are *comparisons* of the lengths of the respective *sides of a right-angled triangle.*

EXPERIMENT.—Draw any two lines AB and AC (Fig. 11) including an angle *a*. Make AC 10 units long, say 1¼ ins. long, *i.e.* unit ⅛ in. Make BC perpendicular to AC. We now have a right-angled triangle

ABC right-angled at C, in which AC is the base; BC is the perpendicular; and AB is the hypotenuse. Measure BC. It measures, say, 7 units of $\frac{1}{8}$ in. Then, comparing the perpendicular BC with the base AC we find that BC : AC :: 7 : 10. That is, BC is $\frac{7}{10}$ of AC, or the ratio of perpendicular : base is 7 : 10. Measure AB (say 12 units), then ratio of base : hypotenuse is 10 : 12. Similarly ratio of perpendicular : hypotenuse is 7 : 12.

Mark off AC_1 10 units long, using a larger unit (say $\frac{1}{4}$ in.); measure B_1C_1 and AB_1 as before. Note that there is the same ratio between any two sides in this triangle as in the triangle ABC.

Mark off AC_2 10 units long (unit, say, $\frac{3}{8}$ in.); complete the triangle AB_2C_2, measure and compare sides as before. Ratios of sides are the same as in the triangles ABC, AB_1C_1. From this we deduce the following :—

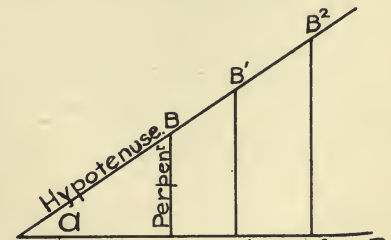


FIG. 11.

The ratios of the sides of any right-angled triangle having an angle α are constant, no matter how large the triangle may be.

The ratio **perpendicular : hypotenuse**, *i.e.* BC : AB or $\frac{BC}{AB}$ is called the **sine** ratio of the angle α.

The ratio **base : hypotenuse**, *i.e.* AC : AB or $\frac{AC}{AB}$ is called the **cosine** ratio of the angle α.

The ratio **perpendicular : base**, *i.e.* BC : AC or $\frac{BC}{AC}$ is called the **tangent** ratio of the angle α.

These are the three simple trigonometrical ratios. There are three others, not so frequently used, viz. **cotangent**, **secant**, and **cosecant**, of which more will be said later.

These six ratios are abbreviated as follows: **sin**, **cos**, **tan**; **cot**, **sec**, and **cosec**.

Trigonometrical Tables. Method of Use.—The ratios of the sides of right-angled triangles containing any angle from 0° up to 90° have been calculated and tabulated (see tables, p. 25), and these tables are used for reference in dealing with any given angle.

Part of such a table is given here to illustrate the method of procedure.

ANGLE.									
Degrees.	Radians.	Chord.	Sine.	Tangent.	Cotan.	Cosine.			
1	·0175	·017	·0175	·0175	57·2900	·9998	1·414	1·5708	90
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
45	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45
			Cosine.	Cotan.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								ANGLE.	

Method of using Trigonometrical Tables.—To find the sine value of a certain angle, say 5°. Look down the column headed "degrees" at left-hand side of table until 5 is reached. Follow this line across horizontally until ·0872 is reached. This is the *sine ratio* for a 5° angle; *i.e.* for every 5° angle the ratio between the perpendicular : hypotenuse is ·0872; or in other words, the perpendicular is $\frac{87}{100}$ of the length of the hypotenuse.

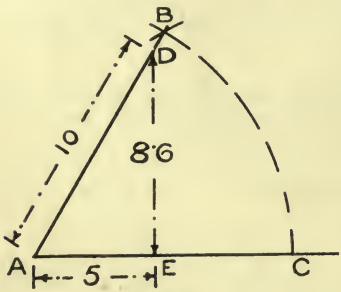


FIG. 12.

To find the cosine value of an angle greater than 45° (say 50°). Look at the column having degrees at its foot, at right-hand side of table. Find 50°. Follow this line across to the *left* horizontally until the column marked *cosine* at the *lower* end is reached. This gives the cosine value of an angle of 50° as ·6428. That is, the ratio between the base : perpendicular for every 50° angle is ·6428; or in other words, the base is $\frac{64}{100}$ or $\frac{642}{1000}$ or $\frac{6428}{10000}$ of the length of the hypotenuse.

PROBLEM (Fig. 12).—*Given an angle, to find its sine and cosine ratios.*—Make an angle of 60° BAC accurately with compasses, by making the *chord* BC = *radius* AC. Make AD (the hypotenuse) 10 units long (any length of unit may be chosen) and from D draw DE at right angles to AC. Measure DE and AE. AE is 5 units long; DE is slightly over 8½ units long, say 8·6. Thus, for a 60° angle, we find by experiment that the sine ratio, *i.e.* perpendicular : hypotenuse, is 8·6 : 10, and the cosine ratio, *i.e.* base : hypotenuse, is 5 : 10. These ratios could

be expressed otherwise thus: sin 60° = ·86, and cos 60° = ·5. Refer to Table of Ratios, page 25, and compare the ratios found experimentally with those given in the table. Here the ratios are given as sin 60° = ·8660 and cos 60° = ·5000. This shows that the estimate of ·86 was slightly under the actual value *if worked out to four places of decimals*. Instead of the perpendicular being $\frac{86}{100}$ of the length of hypotenuse it is really $\frac{866}{1000}$, while the cosine ratio as given in table shows that, even if worked out correctly to four decimal places, the ratio of base : hypotenuse would still be ·5, *i.e.* base is $\frac{1}{2}$ of hypotenuse.

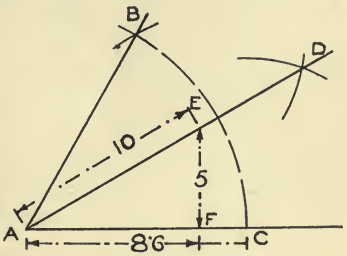


FIG. 13.

PROBLEM.—*To find sine and cosine ratios for* 30° *angle.*—Construct an angle of 60°, Fig. 13, as in last problem. Bisect the angle and draw AD the bisector. Then the angle DAC is a 30° angle. Make AE (hypotenuse) 10 units long and proceed as in the last case. Measure the perpendicular (EF) and the base AF. We discover that the sine ratio of 30° is ·5, and the cosine ratio is ·86. Comparing with last problem we note that—

sin 60° = cos 30° = ·866
and cos 60° = sin 30° = ·5 } Compare this with tables.

From this we deduce the following:—**The sine ratio of a given angle is the cosine ratio of the complement of the given angle,** and *vice versâ*.

Complementary and Supplementary Angles.—The complement of an angle is an angle large enough to make the given angle up to 90°. See Fig. 14. Thus 90° − 30° = 60°; hence 60° is the complement of 30°, and 30° is the complement (not compliment) of 60°.

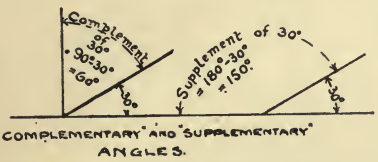


FIG. 14.—Complement and supplement of an angle.

The supplement of an angle is an angle large enough to make the given angle up to two right angles, *i.e.* 180°. Thus the supplement of 30° is 180° − 30° = 150°.

PROBLEM.—To find the tangent ratio of any given angle. Construct angles of 60° and 30°, Fig. 15, as in the preceding problems. Make AE (base) 10 units long and erect a perpendicular DE. Measure the length of DE; 17 units. Then the ratio of perpendicular : base or tangent ratio for 60° is $\frac{17}{10}$ or 1·7. Compare with tables. Our estimate of 1·7 is slightly less than the correct ratio, which is 1·7321, *i.e.* instead

of the perpendicular being $1\frac{7}{10}$ as long as the base, it is really $1\frac{7321}{10000}$. Referring again to Fig. 25, make AC 10 units of any length and draw CG at right angles to AC. Measure this perpendicular CG. It measures 5·7 units, *i.e.* the tangent ratio is estimated as being $\frac{57}{100}$. Refer to tables. Tangent ratio of 30° is really ·5774, *i.e.* perpendicular is $\frac{5774}{10000}$ of length of base, or slightly greater than that found by experiment.

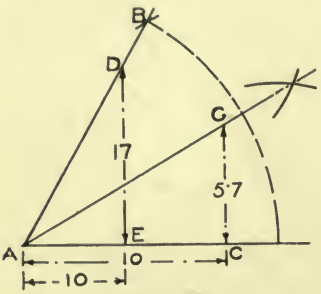


FIG. 15.

PROBLEM.—Consider now the converse of the foregoing problems. *To construct an angle, the sine ratio of which is given as* ·4. Draw any line AB, Fig. 16, and mark off 10 units long. This is to be the *hypotenuse*. The *perpendicular* for the angle required must be ·4 of the length of hypotenuse. Hence, it is necessary to construct a right-angled triangle, using the line AB as hypotenuse, making it right angled at C, and having BC 4 units long. *The angle in a semicircle is always* 90° (*vide* Chap. VI., Segments of Circles). Hence, describe a semicircle on AB, and with radius 4 units and centre B, mark off the point C. Join BC and AC. Then the angle BAC is the angle required. Measure this angle with your protractor. It should measure 24°. Refer to tables, p. 25. Here the sine value of 24° is given as ·4067. Thus, strictly speaking, the sine ratio for 24° should be perpendicular :

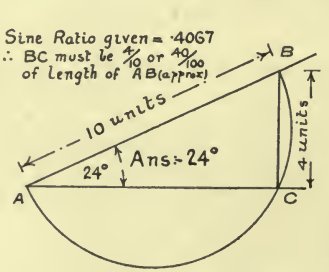


FIG. 16.—Setting out an angle when the sine ratio is known.

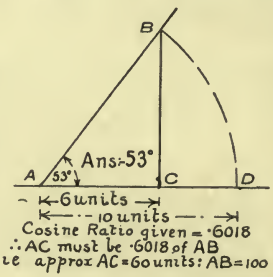


FIG. 17.—Setting out an angle when the cosine ratio is known.

hypotenuse : : 406 : 1000; the ratio 4 : 10 makes the perpendicular slightly shorter than it should be. Does this make the angle BAC less or greater than 24°?

PROBLEM.—*Given the cosine ratio of an angle:* ·6018, *to construct the angle.* Here base : hypotenuse : : 6018 : 10,000, or approximately 60 : 100. Hence, draw any line AC, Fig. 17, 6 units long; this is to be the *base* of right-angled triangle. Produce AC to P so that AP is 10 units

long. At C draw CB at right angles to AC. With centre A and radius AP draw the arc PB cutting CB at B. Join AB. Then BAC is the required angle, in which base : hypotenuse : : 6 : 10, or as 60 : 100. Measure the angle. It measures approximately 53°. Refer to tables, p. 25. Find 53° in column at right-hand side of table. Follow this line horizontally to the left until column marked *cosine* at its foot is reached. Here the cosine of 53° is given as ·6018. We have made the base AC 6 units long. Is the angle BAC really 53°? If not, is it less or greater than 53°?

PROBLEM.—*Given the tangent ratio of an angle, say* ·7, *to construct the angle.* Draw AC, Fig. 18, 10 units long. Make CB at right angles to AC and 7 units long. Draw AB. Then BAC is the required angle. Measure it. It measures approximately 35°. Refer to tables. Here tangent ratio of 35° angle is given as ·7002. We have neglected the figure 2 in the fourth decimal place. What effect will this have? Will the angle BAC, Fig. 28, be greater or less than 35°?

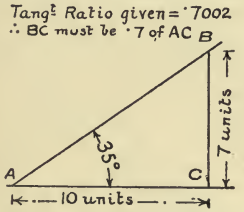


FIG. 18.—Setting out an angle when the tangent ratio is known.

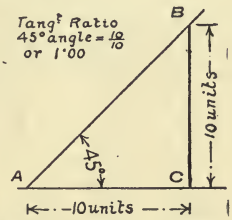


FIG. 19.—Use of tangent ratio.

PROBLEM.—Draw two lines AC, CB, Fig. 19, at right angles, and of the same length, *i.e.* 10 : 10 or 1 : 1. Draw AB. The angle BAC is found to be 45°. Refer to tables. Note that tangent 45° is there given as 1·0000, *i.e.* the ratio of perpendicular : base is unity.

Application of Trigonometry to Calculations of Height and Distance.—The "*functions*" or properties of angles, as illustrated in the foregoing examples, are of very great importance, and are frequently used by the practical man for *calculating the height* of an object, the top of which is *inaccessible*, or for *calculating the distance from some accessible point to another*, on the same level, which is *inaccessible.*

Angles of Elevation and Depression. Use of Sextant and Theodolite.—Suppose an observer at the top of a hill O (Fig. 20) measures, by some means, the angle between a level line OT and a line OY which just meets the top of another hill Y. This angle is the "**Angle of Depression**" of the hill Y as observed from the station point O. For accurate survey work it

is sometimes necessary thus to measure the "*angular distance*" between two straight lines, *i.e.* the angle between two straight lines which meet at the eye of the observer. For this purpose **sextants** and **theodolites** are used. These are so accurately made that it is possible to read off the angular distance between two points in degrees and minutes, and by the aid of a *vernier* attachment, the angle can be read off in *fractions of a minute*. For rough purposes, however, some simple home-made device may be used. For instance, a box may be held in a horizontal position at the level of the observer's eye, see Fig. 21 (*a*). The lid can then be opened until the observer can just sight the distant object in line with the lid. The angle between the box and lid, if measured, gives roughly the *angle of elevation* of the object.

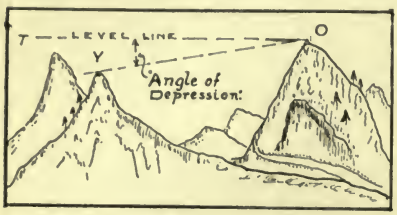


FIG. 20.—Illustration of meaning of the term angle of depression in survey work.

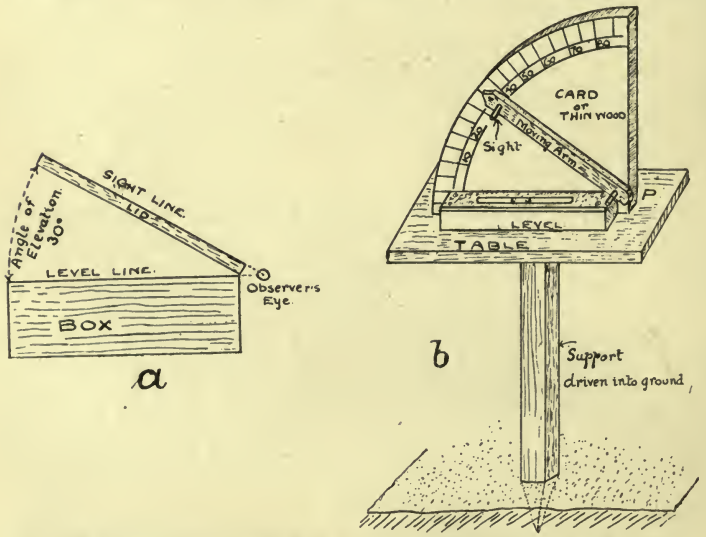


FIG. 21.—Simple method of estimating angles in survey work.

For more accurate calculations a simple device such as that shown in Fig. 21 (*b*) may be used. This consists of a simple table supported upon a stake driven into the ground. On the table is a simple arrangement consisting of a *level*, for levelling the top

of the table, a *card graduated in degrees* and a *moving arm*, pivoted at P; into which two short pieces of wire are fixed. These are the **sights** and are used in the same manner as the *sights* on a rifle. The observer moves the arm into a position such that the line from his eye, passing along the *sights* P and Q meets the object, *e.g.* the top of a cliff or mountain, the height of which is to be calculated.

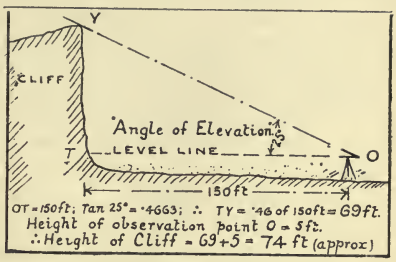

Y
CLIFF
Angle of Elevation
T
LEVEL LINE
O
150ft
OT = 150ft; Tan 25° = ·4663; ∴ TY = ·46 of 150ft = 69ft.
Height of observation point O = 5ft.
∴ Height of Cliff = 69 + 5 = 74 ft (approx)

FIG. 22.—Calculating the height of a cliff by use of table.

PROBLEM. — *An observer, stationed at* O, Fig. 22, 150 *feet from the bottom of a cliff, finds that the angle of elevation of the top of the cliff is* 25°. *What is the height of the cliff?* Referring to the trigonometrical tables in his pocket-book, he finds that tangent 25° is ·4663. From this he is able to calculate the height of the cliff TY in the manner shown.

PROBLEM.—*An observer stationed at the top of a building,* A, *Fig.* 23, *wishes to know the height of a monument,* B, *some distance away on the level of the street. He finds that the angle of depression of the top of the monument is* 18°, *and that of the bottom is* 43°. *The point at which he is stationed is* 120 *feet above the level of the street. What is the height of the monument?* He finds from tables that tangent 43° = ·9325. Using

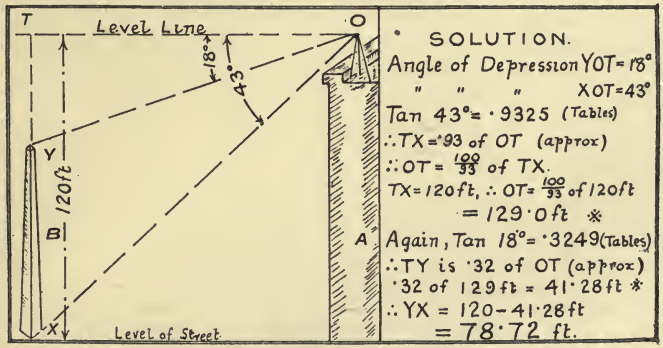


FIG. 23.—Practical illustration of "angle of depression."

this ratio, and knowing the height from the ground to T, *i.e.* the perpendicular TX, he is able to *calculate* the length of OT, *i.e.* the base. Referring again to the tables, he finds that tangent 18° = ·3249. Knowing the length of the *base* OT he is able to calculate the length of the *perpendicular* TY, as shown in Fig. 24. *Subtracting* the length of

TY from the length of TX, the remainder, *i.e.* XY, gives him the height of the monument, *i.e.* 78·72 feet.

PROBLEM.—*Find the length of two guy ropes*, OA *and* OB, *stretched from the top of a post* O, *Fig.* 24, *on one bank of a river, to the top and bottom, respectively, of a bank on the other side of the river.* The angles of depression, TOA and TOB, are first observed at O, and are found to be 32° and 48° respectively. The height from B to the level line, *i.e.* the perpendicular BT, is known. Note that the length of the guy rope OB is the hypotenuse of the right-angled triangle OTB. Hence, the sine ratio of 48° is found, from tables, = ·7431. Knowing TB and

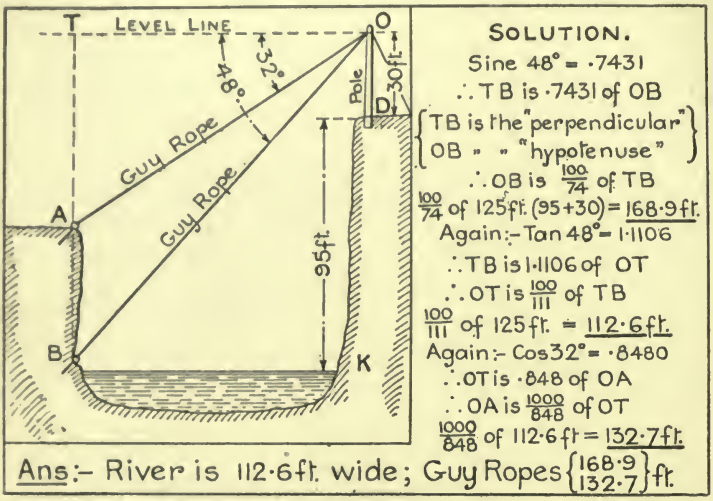


FIG. 24.—Finding length of guy ropes by means of trigonometrical ratios.

the ratio which exists between TB and OB, *i.e.* ·7431, the length of OB can be found as shown (168·9 feet).

Tangent 48° (tables) is 1·1106. Making use of this ratio, *i.e.* perpendicular : base, the length of the base, OT, in the right-angled triangle OTB is found (**112·6** feet). Note that in the triangle TOA we now know the length of the base OT and the angle TOA, 32°. We require the length of the hypotenuse OA. From the tables we find that cos 32° = ·8480. This gives the ratio between OA and OT. Knowing OT to be 112·6 feet, we are able to find the length OA of the other guy rope (**132·7** feet). Note also, that in finding the length of OT we have found the width of the river.

PROBLEM.—*The angles of elevation of a high building* A, *Fig.* 25, *are taken at two different points*, O *and* P; *the latter* 100 *feet nearer the building than* O. *The angles are respectively* 30° *and* 45°. *A wall* B *which intervenes, makes it impracticable to measure the distance of the observer from the building. Calculate the height of the building.*

Calling the angles of elevation a and b respectively, and the distance apart, OP, of the two observation points d, then the height CT may be found by using the formula $CT = d\left(\frac{\sin a \times \sin b}{\sin (b - a)}\right)$ as shown on the diagram.

Note that in this and other similar calculations, the *height of the observer's eye above the ground must be added* to the height of building as found by calculation. The reason for this is obvious upon reference to the various diagrams.

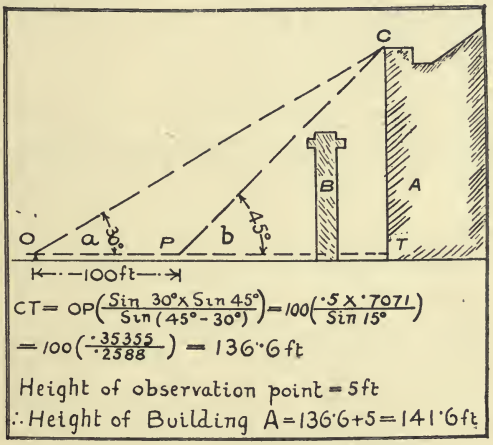


FIG. 25.—Estimating height of building.

PROBLEM.—*A person stationed at* P, *Fig.* 26, *on one side of a river, wishes to find the width of the river.* He observes a tree Q on the other side of the river, as nearly as possible in a line at right angles to the direction of the flow of the river. He measures off some definite distance PR, say 250 feet, *parallel to the river*, and therefore, approximately *at right angles to* PQ. He reads off the angular distance between RP and RQ as 46°. From the tables he finds tangent 46° = 1·0355. He knows the *length of base* PR, and is able to make use of this ratio to calculate *length of perpendicular* PQ, *i.e.* the *width of the river*, as shown.

PROBLEM.—The second case shown in Fig. 26 (right side) illustrates a more accurate method of calculating the width of a river than that shown in last problem. Observations are made at two points P and R of the angular distance between the base line PR and PQ and RQ respectively. Knowing that the angle PRQ is 46°, we know that angle RQS must be 90° − 46° = 44°, *i.e.* the *complement* of 46°. Similarly, the angle PQS must be 90° − 57° = 33°. These two angles, *i.e.* 33° and 44°, give the ratio of the two parts of the base line PS : RS as 33 : 77 or 3 : 7. Knowing this (the *base*) and the tangent ratio of 57°, it is possible to find QS the perpendicular, as shown in Fig. 26.

The foregoing examples have dealt principally with the three ratios: sine, cosine, and tangent. It should, however, be noted that the three remaining ratios, viz. cotangent, secant, and cosecant, are at times equally useful. Referring again to Fig. 21, these three ratios would be expressed as follows: —

(1) Ratio $\frac{\text{hypotenuse}}{\text{perpendicular}}$, *i.e.* $\frac{AB}{BC}$, is called the **cosecant** of the given angle A.

Compare this ratio with the *line ratio* which is $\frac{\text{perpendicular}}{\text{hypotenuse}}$ or $\frac{BC}{AB}$.

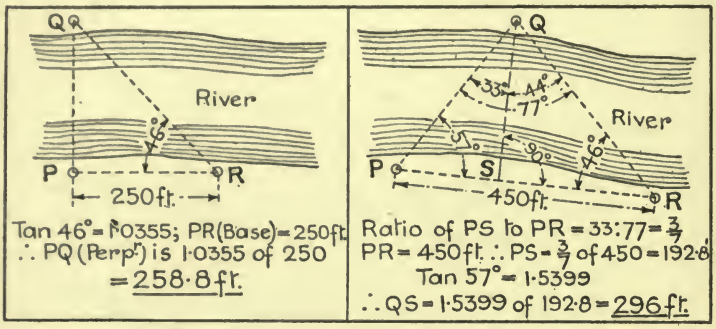


FIG. 26.—Estimating the width of a river.

(2) Ratio $\frac{\text{hypotenuse}}{\text{base}}$, *i.e.* $\frac{AB}{AC}$, is called the **secant** of the angle A.

Compare this ratio with the *cosine ratio*, which is $\frac{\text{base}}{\text{hypotenuse}}$ or $\frac{AC}{AB}$

(3) Ratio $\frac{\text{base}}{\text{perpendicular}}$, *i.e.* $\frac{AC}{BC}$, is called the **cotangent** of the given angle A.

Compare this ratio with the *tangent ratio*, which is $\frac{\text{perpendicular}}{\text{base}}$ or $\frac{BC}{AC}$.

It has already been observed that the sine ratio of a certain angle (say 40°) is the cosine ratio of the complement of that angle 40°, *i.e.* 90° − 40° = 50°.

Similarly, it may be noted that *the tangent ratio of an angle is equal to the cotangent of the complementary angle.*

Table of Trigonometrical Ratios

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians.								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine	Co-tangent	Tangent	Sine	Chord	Radians	Degrees
								Angle	

CHAPTER III

CONSTRUCTION OF TRIANGLES

A **triangle** has *three sides* and *three angles*. These are spoken of as the *elements* of the triangle. Fig. 27 illustrates various kinds of triangle. Before it is possible to construct any triangle, it is necessary to know *at least three* of the *six elements* of the triangle. For instance, if we are given the respective *lengths* of the *three sides* of a triangle we know *three* of the *six* elements and are able to construct the triangle; thus suppose in a triangle

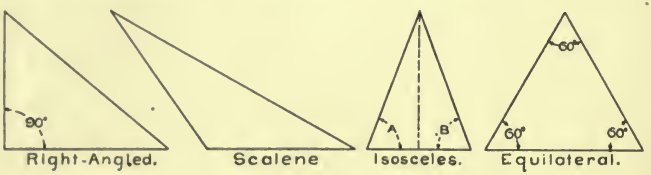


FIG. 27.—Triangles.

ABC, Fig. 28, of which AC is the base, the lengths of sides are as follows: AC = $3\frac{1}{2}$ ins., AB = $2\frac{3}{4}$ ins., BC = $1\frac{3}{4}$ ins. Then, making the base AC = $3\frac{1}{2}$ ins., and with radii respectively $2\frac{3}{4}$ ins. and $1\frac{3}{4}$ ins., and centres A and C respectively, describe arcs intersecting at B. This point is $2\frac{3}{4}$ ins. from A and $1\frac{3}{4}$ ins. from C. Joining AB and BC, the resulting triangle ABC is the one required.

Perimeter of a Triangle.—The *perimeter* of a triangle or any other rectilinear figure is the *sum of its sides*. In the triangle shown in Fig. 28, the perimeter is $3\frac{1}{2}$ ins. + $2\frac{3}{4}$ ins. + $1\frac{3}{4}$ ins. = 8 inches.

Altitude of a Triangle.—The *altitude* of a triangle is the *shortest distance* between its base and a line drawn through the vertex parallel to the base.

The sum of the angles in any triangle is always 180°, *or two right angles.*

In the various problems which follow, the data given for the construction of the triangle always includes *three* of the six elements of the triangle.

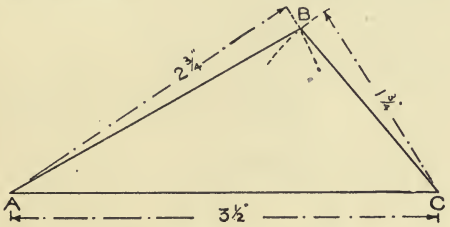


FIG. 28.—Construction of triangle.

Cutting Triangular Wooden Template.—*A triangular template, Fig.* 29, *is to be cut from thin wood to fulfil the following conditions : vertical angle* 40° ; *one base angle* 60° ; *altitude* 2·5 ins.

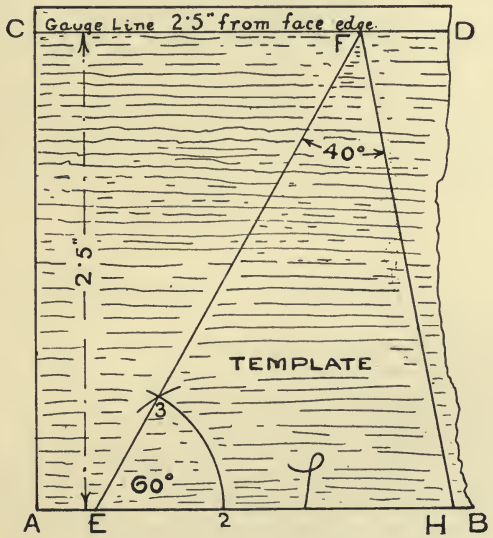


FIG. 29.—Method of setting out triangular wooden template.

A gauge line CD, 2·5 ins. from the face edge AB, is a locus or path along which the vertex must fall. With centre E and any

semicircle at J. From J draw a vertical projector to meet the height line at L. Join DL, EL. Then DLE is the plan of the field.

N.B.—This solution is based upon the method of constructing an *ellipse* by making use of the *major and minor auxiliary circles*. The two semicircles drawn, radii CF and CA respectively, are *half* the major and minor auxiliary circles of an ellipse which has AC for its semi-major axis and CF for its semi-minor axis, and L is a point through which the ellipse would pass. The student should refer to Chapter IX., where this construction is fully explained. The solution employed in the foregoing problem will then be more easily understood.

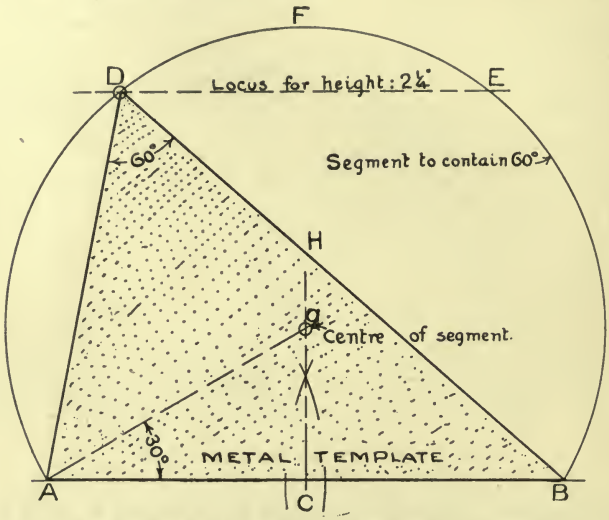


FIG. 32.—Setting out a triangular metal template to given data.

Shaping a Triangular Metal Plate.—*A triangular piece of sheet metal* ADB, *Fig.* 32, *is to be shaped as follows, to make a template. Base* AB 3 *ins. Vertical angle at* D 60°. *Vertical distance of* D *measured square with* AB = 2¼ *ins.*

Make AB = 3 ins. and bisect it at C. Draw the bisector CH. At A set out an angle of 30°, *i.e.* the *complement* of the *vertical angle* required in the triangle (90° − 60° = 30°). The intersection of this 30° line and the bisector CH gives the centre for describing a *segment of a circle* AFB *to contain* 60°. Then D, which is on the *locus line* for the height of the triangle and on the segment AFB, is the *vertex* of the required triangle. Join AD, BD, and ADB is the shape of the template.

Medians of a Triangle.—*If any triangle* DEF, *Fig.* 33 (*a*), *has its sides bisected as at* AB *and* C, *and lines are drawn from each corner of the triangle to the middle point of the opposite side, as* FA, DC, *and* EB, *these lines are called the* **medians** *of the triangle* DEF.

The *medians* of any triangle have a common point of intersection, such that each of the three medians is cut at this point into parts in the ratio 1 : 2 or $\frac{1}{3} : \frac{2}{3}$. This property forms the basis of the solution of the problem which follows.

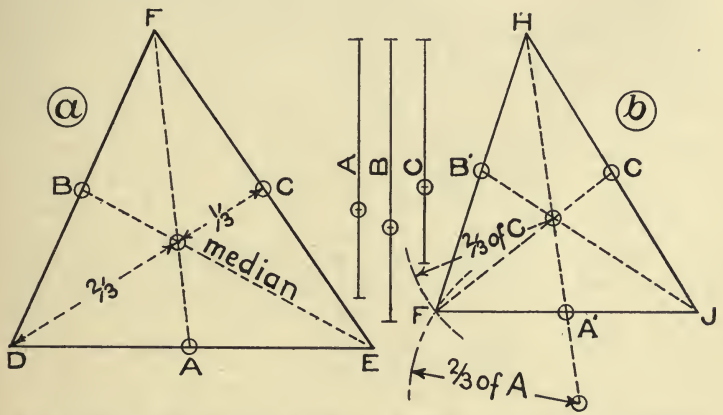


FIG. 33.—(*a*) "Medians" of a triangle. (*b*) Method of constructing triangle when "medians" are known.

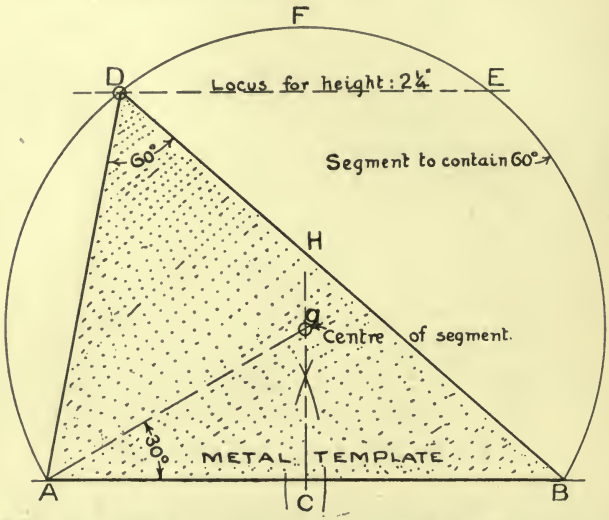
Method of Construction of Triangle when its Medians are given.—*Given the three medians* A, B, *and* C *of a triangle, Fig.* 33 (*b*), *to construct the triangle.*

Draw a line HA′, making it equal to the *longest* median B. Divide HA into three equal parts in order to find the point D, $\frac{2}{3}$ of the length of the whole median, measured from H. Produce HA′ to E, making A′E = A′D, *i.e.* $\frac{1}{3}$ of HA′. With E as centre and radius $\frac{2}{3}$ of the given median A describe an arc. With D as centre and radius $\frac{2}{3}$ of the given radian C describe an arc. These two arcs intersect at F which is one corner of the triangle. From F draw FC through D making DC = $\frac{1}{2}$ of FD. Through C draw HJ to meet FA′, produced, at J. From J draw JB′ through D, making B′D = $\frac{1}{2}$ of DJ. Draw HF through B′. Then FHJ is the required triangle.

Triangulation in Survey Work. Plotting.—The *plotting* of triangles to given data is an important part of survey work. When a survey on a large scale is made, such as the **Ordnance**

semicircle at J. From J draw a vertical projector to meet the height line at L. Join DL, EL. Then DLE is the plan of the field.

N.B.—This solution is based upon the method of constructing an *ellipse* by making use of the *major and minor auxiliary circles.* The two semicircles drawn, radii CF and CA respectively, are *half* the major and minor auxiliary circles of an ellipse which has AC for its semi-major axis and CF for its semi-minor axis, and L is a point through which the ellipse would pass. The student should refer to Chapter IX., where this construction is fully explained. The solution employed in the foregoing problem will then be more easily understood.



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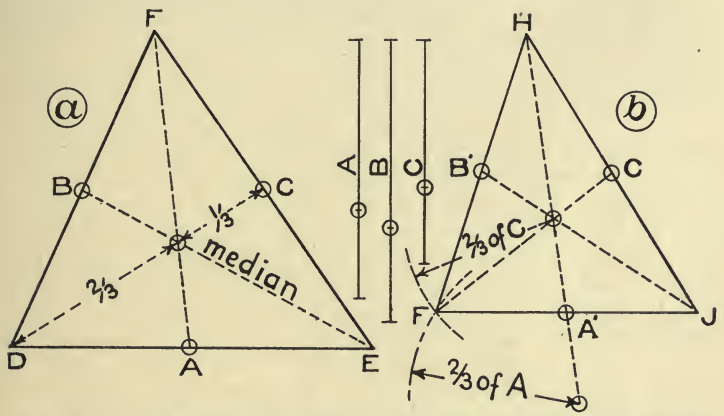
The *medians* of any triangle have a common point of intersection, such that each of the three medians is cut at this point into parts in the ratio 1 : 2 or $\frac{1}{3} : \frac{2}{3}$. This property forms the basis of the solution of the problem which follows.



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Triangulation in Survey Work. Plotting.—The *plotting* of triangles to given data is an important part of survey work. When a survey on a large scale is made, such as the **Ordnance**

Survey of England, the surveyor chooses some *level* plain as a starting point, say Salisbury Plain. On this plain a *base line* is very carefully *laid down* and measured (see Fig. 34). From each end of this base line, AB, observations are taken and distances measured in order to determine the position of other prominent points on the landscape with relation to the base line already laid down. By this means the exact position of D (a tower) and C (a tree) are located and the *distances* AC and BC and the *angles* CAB and CBA carefully noted. Similarly, the distances AD and BD and the angles DAB and DBA are carefully observed and noted. The lines joining A to D and B to C are now used as new base lines, from each end of which measurements are taken and observations made with a view to locating the position of E, and so on. So that, working outwards in every direction from the original base line AB, a complete survey, made up of an *elaborate system of* **triangulation**, is eventually made of the whole tract of country. This work is all *field* work. The results of these observations are handed on to others in the office, who *plot* or draw to scale the network of triangles from the data supplied them. As a result of this elaborate system we eventually have an extremely accurate and reliable map of any district in the whole of England.

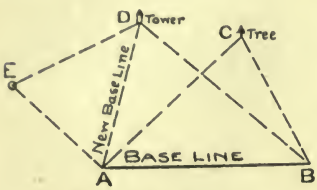

FIG. 34.—Triangulation in survey work.

Plane Tabling in Survey Work.—The **plane table** is one of the simplest and most readily applicable of all surveying instruments. Its use is closely associated with the method of *triangulation* to which reference has already been made. In the more elaborate survey work carried out in the Ordnance Survey, however, costly instruments for levelling and sighting were an absolute necessity. The *plane table*, on the other hand, is so simple and inexpensive that it can be made and used by those with little or no surveying experience. In its least elaborate form, it may be simply a drawing board mounted on a tripod and provided with a ruler (see Fig. 35). Supposing that a

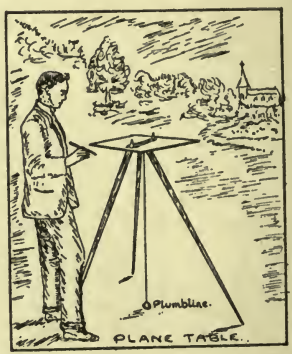

FIG. 35.—Tripod and drawing board used as "plane table" in survey work.

simple survey of a small estate is required: a *base-line* is measured and marked off on some suitable part of the estate by means of a stretched string. The plane table is then placed *at one end* of this line, the tripod being arranged so that the plumb bob falls *exactly over the base-line.* A sheet of drawing paper is pinned to the board which serves the purpose of a plane table, and a point A is marked on this paper vertically above the end A of the base line already "laid out" on the ground (see Fig. 36). From the point A on the paper a line is drawn in line with the base line. For this purpose the ruler is used for *sighting*, and it is an advantage to have a somewhat heavy ruler provided with vertical wire *sights* (see Fig. 35). On the line drawn on the paper from A the position of B, the other end of the base line, is marked off to scale (*b*).

Thus, supposing the base line on the ground to be 50 yds., and a scale of $\frac{1}{2}$ in. = 1 yd. is chosen for the plan, the base line on the paper would be 25 ins.

Sighting lines are then taken along the ruler in order to determine the position of various objects, *e.g.* gate, elm tree, stile, etc., Fig. 35. Lines are then drawn from the point A on the plan accurately in line with each of these objects (Fig. 36, inset sketch, top left-hand corner). Each line is marked in some suitable way to indicate with which of these various objects it was drawn in line. The board is then moved to the other end of the 50 yards base line marked by the string on the ground, and so placed that the ruled line on the paper lies *exactly in the same direction* as the base-line, and the point B of the plan exactly above the end B of the base-line. A second series of lines towards the same objects is then drawn on the paper by means of

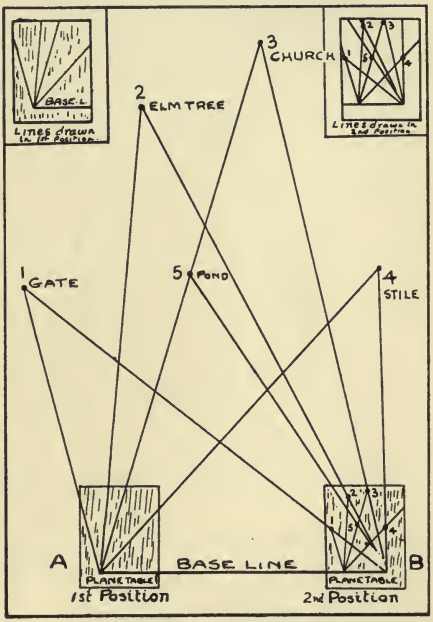

FIG. 36.—Use of plane table for plotting position of points on plan.

the *sights* on the ruler, and the position of each object marked at the *intersection* of its *two sight lines* (Fig. 36, inset sketch, top right-hand corner). Having obtained the position of the main objects on the estate in this manner, the positions of other objects of less importance may be fixed with reference to the various triangles already obtained. The diagram as shown at A should be compared with the inset sketch, top left-hand corner; and that at B with inset sketch, top right-hand corner.

A little practice with the plane table will give the student much useful information with regard to triangulation, and will

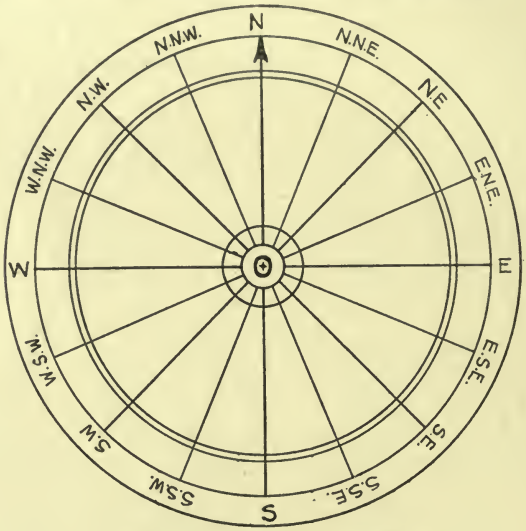

FIG. 37.—Points of the compass.

make the elaborate work of the Ordnance Survey more easily understood.

Plotting Positions of Points.—The positions of points or fixed objects on the landscape are measured and denoted, first, by specifying the distance of each point or object, say B, C, D, etc., from some fixed object A, which is used as a starting point; second, by specifying the magnitude of the angle which the line drawn from A to B or A to C, etc., makes with a line drawn in some fixed direction, such as a due east line.

Points of the Compass.—Fig. 37 illustrates the marking of the most important points of the compass. The positions of the four *cardinal* points, viz. *north*, *south*, *east*, and *west*, are

familiar to all. The points intermediate between the respective cardinal points are sometimes misread. The point exactly *midway* between any two cardinal points, say N. and E., is denoted by the two letters N.E. The direction of this line with reference to a due east line may be stated as 45° *north of east*, written thus: 45° N.E. The point of the compass midway between N. and N.E., since it is *more north* than east, is denoted by two N's and one E, thus: N.N.E. This is described as *north-north-east.* A line drawn from O, Fig. 37, towards N.N.E. would be spoken of as a line $67\frac{1}{2}°$ north of east. The same line could be denoted by "$22\frac{1}{2}°$ east of north." In the same way the point midway between E. and N.E., seeing that it is *more east* than north, is denoted by E.N.E., *i.e. east-north-east.* A line drawn from O towards E.N.E. would be stated as a $22\frac{1}{2}°$ *N.E. line, i.e.* north of east, and so on. The process of fixing the position of objects or points relatively to each other in this manner is described as **plotting.**

The following problems are examples of this process.

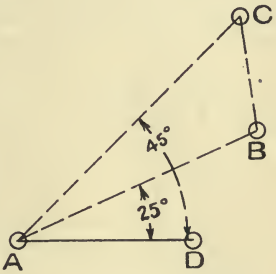

FIG. 38.—Plotting position of points.

Plotting Points with reference to a Base Line.—A surveyor stationed at A, Fig. 38, wishes to fix the position of two objects B and C with reference to a base line AD which he has already "*laid down.*" By means of a sextant he finds that the angle DAB between the base-line and the line AB is 25°, and the angle DAC between the base-line and the line AC is 45°. Measuring the distances from A to B and C respectively, he finds AB = 15 chains and AC = 18 chains: *How far are the two objects* A *and* C *apart?*

Draw the line AD, and at A set out the lines AB and AC, making, respectively, angles of 25° and 45° with the base line AD. Make AB and AC to scale to represent 15 and 18 chains respectively. The scale should not be too small, say, $\frac{1}{8}$ in. = 1 chain. Measure off the length BC to the same scale.

Ans. BC = 6·5 chains.

Positive and Negative Angles—Reference Line and Angular Displacement.—The data from which to plot certain points and lines are not always given quite in the same form as that of the last problem. The direction of a line may be stated by specifying the **angular displacement** between it

and a *reference line* (Fig. 39). Suppose the rod OP, turning on O as a centre, moves towards R until it is in the position OR. In doing this it is moving in the *opposite direction* to that of the hands of a clock. This is spoken of as an "*anti-clockwise*" movement, and the angle ROP so formed is termed a **positive angle**, in this case 30°. This would be denoted as a + 30° angle. In mathematics, however, it is usual to omit the plus sign, unless it comes between two terms. Hence, a "30° angle" would mean a + 30° angle, *i.e.* the *plus sign is understood.*

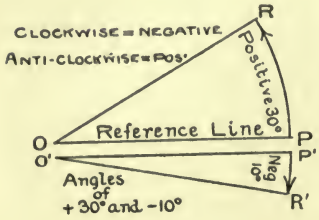

FIG. 39.

Referring again to Fig. 39, suppose the rod O′P′ to move in the *same direction* as the hands of a clock, *i.e.* "*clockwise*," turning on O′ as centre until it had arrived at the position indicated by O′R′; then the rod O′P′ is said to have moved through a **negative angle** of 10°. This would be written thus — 10°. The line OP or O′P′, from which the magnitude of the angle is measured, is called the **reference line**, and is usually taken as *running due east and west*, unless otherwise stated.

Plotting Results of Traverse Survey.—*In a traverse survey lengths and directions are measured by using a chain and a magnetic compass.* A, B, *and* C *being three trees, the following measurements are made:—*

AB = 8·35 chains in direction E. 27° N., *i.e.* $8{\cdot}35_{27°}$ chains.

BC = 6·24 chains in direction E. 110° N., *i.e.* $6{\cdot}24_{110°}$ chains.

Plot AB *and* BC *to a scale of* 1 *cm. to* 1 *chain. Measure and calculate the distance* AC. *How much is* C *to the east of* A, *and how much to the north of* A? *What is the direction of* AC *relative to* AE? (B.E. 1905.)

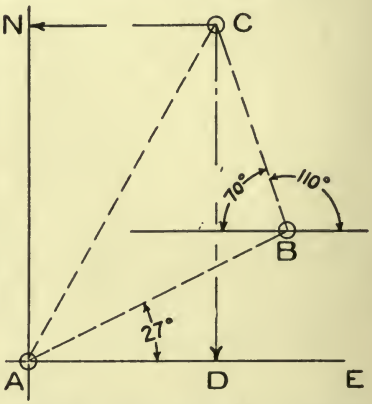

FIG. 40.—Plotting position of points in traverse survey.

Draw a reference line AE, Fig. 40, running due east and west. Note that the *direction* of AB is given as 27° north of east, *i.e.* AB is to make a *positive angle* of 27° with the reference line. At A set out the direction of AB as a line, AB making

27° with AE, and make it 8·35 centimetres long. This fixes the position of B relative to A. Through B draw a line *parallel to the reference line* AB. The direction of BC is given as 110° north of east, *i.e.* BC is to make a *positive angle* of 110° with the *new reference line* drawn through B. Draw BC, making 110° with the new reference line, and make it 6·24 centimetres long. This fixes the position of C relative to A and B. Join AC and measure it (11 chains). Measure the angle between AC and AE (61°). Draw a due north line AN, and draw the co-ordinates CN and CD parallel to AN and AE respectively. Measure AD (5·4) and AN (9·6).

Ans.—(*a*) AC = 11 chains; (*b*) C is 5·4 chains to the east and 9·6 chains to the north of A; (*c*) direction of AC = 61° north of east. This (*c*) might be stated thus, AC = $11_{61°}$ chains.

The following are the field notes of the survey of a triangular plot of ground ABC. *"Bearing" means the direction from one station to the next succeeding station, and N.* 31½ *W. indicates the direction* 31½° *west of north.*

Stations.	Bearings.	Distances.
A	N. 31½° W.	10 chains
B	N. 62¾ E.	8·25 chains
C	——	——

Plot the survey to a scale of 1 *cm. to* 1 *chain. Find and measure the bearing and distance from* C *to* A. *What is the enclosed area in square chains?* (B.E. 1908.)

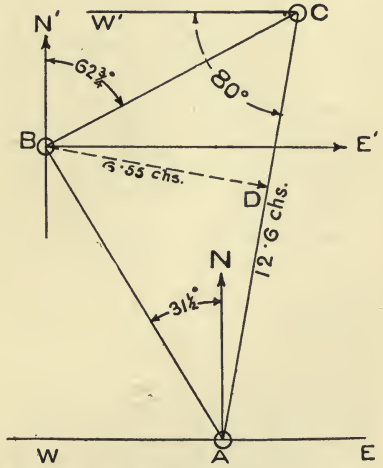

FIG. 41.—Plotting survey of a triangular field.

Draw the reference line WE, Fig. 41, and at any point draw a due north line AN. At A set out the **bearing** *direction* of AB, *i.e.* 31½° to the west of AN, *i.e.* 31½ west of north, and make A = 10 chains (10 cm.). Through B draw a *due north line* BN', and at B set out the *bearing* or direction BC, as given, *i.e.* 62¾° to the east of north. Make BC = 8·25 chains (8·25 cm.). Join AC and measure it to scale (12·65 chains). Draw CW' and measure the angle ACW' (80°). Then CA is a distance of 12·65 chains in a direction 80° south of west. The table given in the question above could then be completed as follows :—

Stations.	Bearings.	Distances.
C	S. 80° W.	12·6 chains.

The position of AC relative to WE could be stated alternatively thus, AC = $12{\cdot}6_{80°}$ chains.

Draw BD at right angles to AC and measure it to scale (6·55 chains). Then the area of the triangle ABC is found by taking *one-half the product of the base* AC *into the height* BD of the triangle, *i.e.* $\frac{12{\cdot}6 \times 6{\cdot}55}{2} = 82{\cdot}65 = 41{\cdot}32$ chains.

Areas of Triangles.—The area of a triangular piece of material PRS (Fig. 42) is found by the formula $\frac{\text{base} \times \text{height}}{2}$ or *half the product of the base into the height.* In the given

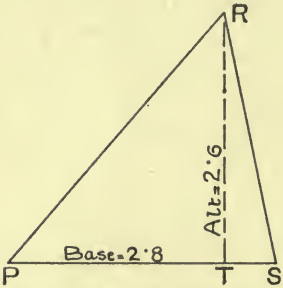

FIG. 42.—Method of finding area of triangular plot of ground.

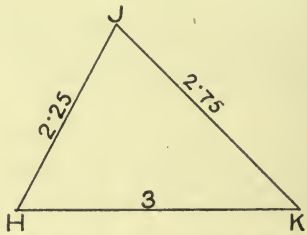

FIG. 43.—Determination of area of triangle when lengths of sides only are given.

triangle the *base* PS = 2·8 units, and the *altitude* or height RT = 2·6 units. Then the area = $\frac{2{\cdot}8 \times 2{\cdot}6}{2} = \frac{7{\cdot}28}{2} = 3{\cdot}64$ square units. If only the *lengths of the sides* are given (see Fig. 43), the area may be found without reference to the height of the triangle as follows:—

2·25 + 2·75 + 3 = 8 feet = perimeter or sum of sides.
$\frac{8}{2}$ = 4 feet = half sum of sides.
4 − 2·25 = 1·75
4 − 2·75 = 1·25
4 − 3·0 = 1·0
} each side subtracted separately from half sum of sides.
1·75 × 1·25 × 1·0 = 2·1875 = product of remainders.
2·1875 × 4 = 8·75 = product of remainders × $\frac{1}{2}$ sum of sides.

Then $\sqrt{8{\cdot}75}$, *i.e.* the square root of 8·75, is the required area.

Ans.—2·95 square feet.

Draw a triangle exactly the same size as HJK *and verify this result by using the formula* $\frac{base \times height}{2}$.

Division of the Triangle into Various Areas.—*A triangular plot of ground* TUV, *Fig.* 44, *is to be shared equally among three men. The two fences required are to be placed parallel to one side of the field,* TV.

Suppose the *height* of the triangle were divided into three equal parts, as shown at 1 and 2, Fig. 44. Would these positions satisfy the requirements? If not, suppose *one side* of the field, say T′U′, Fig. 44, was divided into three equal parts and fences erected in the positions indicated by A and B. Would *these* positions satisfy the requirements? It is evident that some method is required which will *increase* the size of the top space (above A) at the expense of the other two spaces. Fig. 44

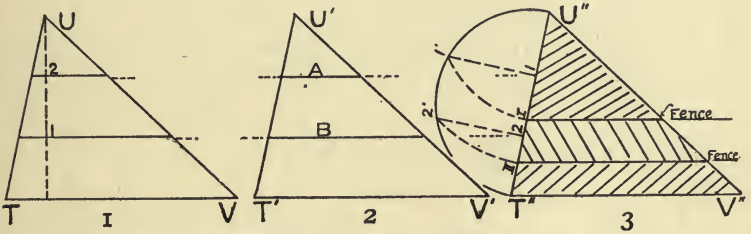

FIG. 44.—Dividing a triangular plot of ground TUV into three equal areas by means of fences parallel to TV.

shows a method for securing this. Divide the *side* T″U″ into three equal parts (1, 2), as suggested in 2. On T″U″ describe a semicircle, and at 1 and 2 erect perpendiculars from T″U″ to meet the semicircle in 1′ and 2′. With centre U″ and radii U″1′ and U″2′ respectively, describe arcs to cut T″U″ in I and II. Then I and II are the positions at which the fences should be erected parallel to T″V″, as shown.

A triangular field ACB (Fig. 45) *is left to be shared equally between* A *and* B, *who live at opposite corners of the field, on condition that the fence must be erected at* **right angles** *to the fence* AB *which runs from* A's *to* B's *end of the field.*

A and B each offer suggestions as to where the fence should be placed. A suggests that the fairest way is to divide the base AB into two equal parts, as at *a*, and erect the fence at *aa′*. B suggests that the vertex, C, of the triangle should be used, and the fence erected at C*b*. A mutual friend, to whom they take their difficulty, says that what is required is the **geometric**

mean between the distance A*a* suggested by A, and the distance B*b* suggested by B. The method of doing this is shown at (*b*), Fig. 45. B's suggestion (B*b*) and A's suggestion (A*a*′) are laid

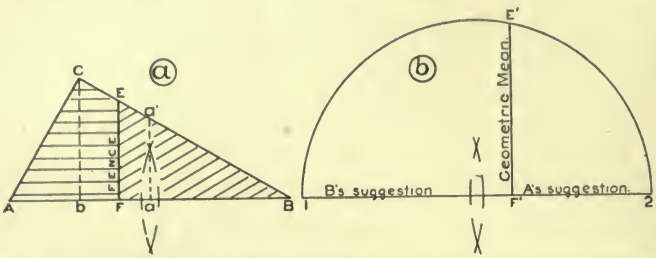

FIG. 45.—Triangular space ACB divided into two equal areas by fence EF.

end to end at F′ in (*b*), to make a straight line. A semicircle 1E′2 is described on this line, and a line E′F′ erected at F′, the point at which A's suggestion and B's suggestion meet, to meet the semicircle at E′. Then E′F′ is the length of fence required for dividing the field into two equal areas. This fence is shown in position at EF in (*a*).

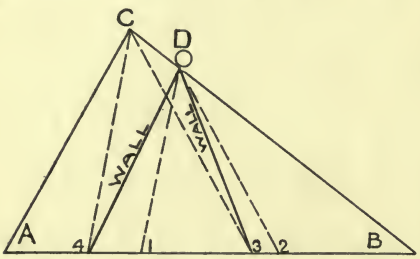

FIG. 46.—Dividing a plot of ground ACD into three equal areas by walls D4 and D3.

A triangular plot of ground, ACB, Fig. 46, *belongs to a man whose house is situated at* D. *He wishes to build two walls in such a position that they shall divide the plot of ground into three equal areas, but stipulates that each wall must terminate at his house*, D. *The position of the walls is found as follows:—*

Draw a plan of the field ACB to scale. Mark the position of the house, D. Divide the base, AB, into as many parts as the plot of ground is to be divided into areas, *i.e.* three, as at 1 and 2. Join D1 and D2. From C, the vertex of the triangle, draw C3 parallel to D2 and C4 parallel to D1. Join D4 and D3. These two lines give the position of the two walls which divide the ground into three equal areas, ACD4; 4D3; and 3DB.

Triangles on the same base (or equal bases) and between the same parallels are equal in area. See Fig. 47.

A landlord whose house is situated at H, Fig. 48, *has let a triangular plot*, RPS, *to a tenant. Finding that his grounds at*

H *are overlooked from his tenant's grounds at* P, *he asks him to plot out a new piece of ground equal in area to* RPS, *but in such a way as not to have any part of the plot anywhere near* H.

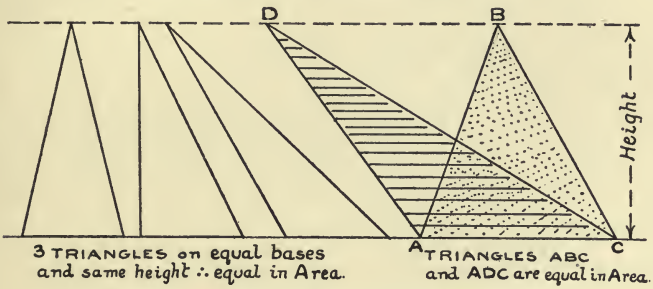

FIG. 47.—Equality of area of triangles on same base, etc.

In Fig. 48, if a line PP′ is drawn parallel to the base RS, and a point P′ immediately over the centre of the base is chosen for the vertex of the new triangle, RP′S will then represent a new plot of ground equal in area to the original plot, and having the same base RS.

The Right-angled Triangle.—Some of the properties of the right-angled triangle have already been considered (Chapter II). There are, however, a few other practical illustrations of these properties which deserve attention. The following is one of the most important:—

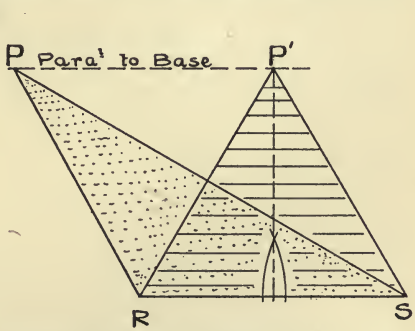

FIG. 48.—Plotting a triangular piece of ground RP′S equal in area to RPS.

In any right-angled triangle, the square constructed on the hypotenuse is equal in area to the sum of the squares on the two sides which contain the right angle.

This principle is illustrated in Fig. 49. In this case the lengths of the *sides* which contain the *right angle* are 8 units and 6 units respectively. In the *squares* constructed on these sides there are 64 square units and 36 square units (8×8 and 6×6, or 8^2 and 6^2). If a square is constructed on the *hypotenuse* it will contain 100 square units of the same size as those in the squares on the sides: $100 = 64 + 36$. Hence the *length of the*

hypotenuse is equal to the *square root* of 100, written thus, $\sqrt{100} = 10$.

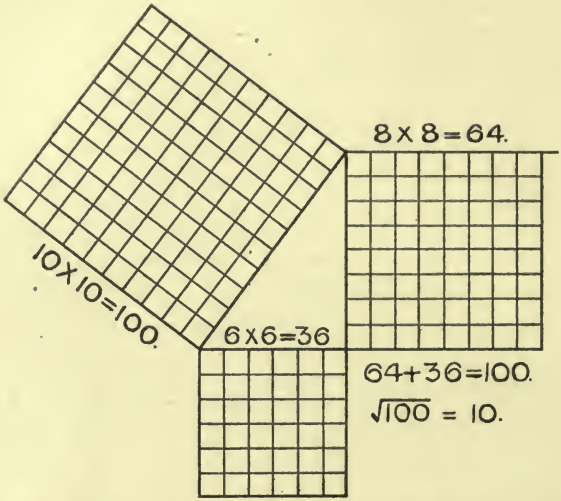

FIG. 49.—Squares on sides and hypotenuse of right-angled triangle.

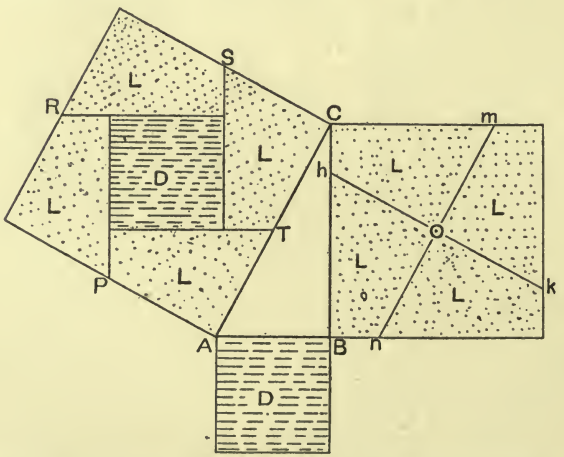

FIG. 50.—Practical proof of property of right-angled triangles.

Experimental Proof of Property of Right-angled Triangle.—Fig. 50 illustrates a simple method of proving, experimentally, the truth of this principle. Draw any right-angled

triangle ABC and construct squares on the two sides and on the hypotenuse as shown. Bisect the sides of the square on the hypotenuse at P, R, S, and T. From R and T draw lines parallel to AB, and from P and S draw lines parallel to BC. Shade the four outer areas light as indicated at L, etc. Shade the inner area dark, as at D. Cut out the four lightly shaded pieces marked L, and fit them together. They fit exactly into and fill the square on the longest side BC, as shown, the four pieces having their joints along the *hk* lines and *mn*. The lines *hk* and *mn* are drawn through O, the centre of the square, and parallel respectively to SC and AP. The small square piece, D, which remains will just fit into the square on the shortest side AB.

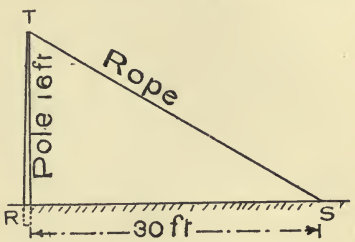

FIG. 51.—Method of finding length of rope.

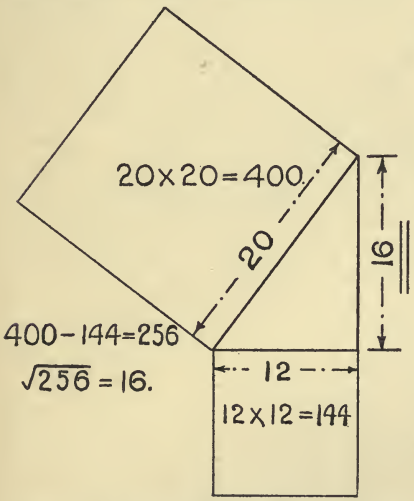

FIG. 52.—Method of finding length of side of right-angled triangle.

Pole and Rope as Example of Right-angled Triangle.

PROBLEM.—*A pole* RT, *Fig.* 51, *stands* 16 *ft. above the ground. Find the length of a rope* TS *stretched tightly from the top of the pole to a point* S *on the ground* 30 *ft. away from the bottom of the pole.*

SRT is a right-angled triangle, hence the square on the hypotenuse TS is equal to the sum of the squares on RS and RT respectively, *i.e.* $RS^2 + RT^2 = TS^2$.

$RS^2 = RS \times RS$
$= 16 \times 16 = 256$
$RT^2 = RT \times RT$
$= 30 \times 30 = 900$
$\therefore RS^2 + RT^2 = 256 + 900$
$= 1156$

$\therefore TS^2 = 1156$
$\therefore TS = \sqrt{1156}$ (*i.e.* the square root of 1156) $= 34$
$\therefore$ Length of rope TS = 34 ft.

Method of finding Length of Side when One Side and

Hypotenuse are given.—In the foregoing example, the *sides of* the right-angled triangle were known and the length of the *hypotenuse* had to be determined. In certain cases, however, the length of the *hypotenuse* and *one side* are known and the length of the *other side* of the triangle has to be determined.

PROBLEM.—*The hypoteneuse of a right-angled triangle, Fig.* 52, *is* 20 *ft. long, and one side of the triangle is* 12 *ft. long. Find the length of the other side.* From the foregoing examples we know that the square on the hypoteneuse is equal to the *sum* of the squares on the sides. Hence, if the square on one side, *i.e.* 12^2 or 144, is *subtracted* from the square on the hypotenuse, *i.e.* 20^2 or 400, the *remainder*, 256, is the *square on the remaining side.*

∴ $\sqrt{256}$, or **16 ft.**, is the length of the **other side** of the triangle.

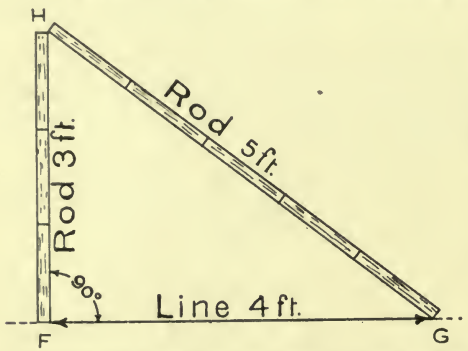

FIG. 53.—Link method and application to right angles.

Link Method of setting out Larger Right Angles.—It is sometimes necessary, in builder's work, to set out a right angle of large dimensions on the ground in order to mark a line for excavating or for commencing the brickwork. As a guide in such cases it is interesting to note that by selecting three lengths which bear a certain *ratio* to each other, a right-angled triangle can be constructed. The most useful ratio is that shown in Fig. 53, the ratio or comparison of the lengths being 3 : 4 : 5. In order to make use of this principle, a straight line FG is laid down and marked off to 4 ft. Two rods, 5 ft. and 3 ft. respectively, are then used, the end of the 5 ft. rod held at G, and the end of the 3 ft. rod held at F. The point H, at which the other ends of the rods meet when held closely together, is the third corner of a right-angled triangle of which the two points F and G are the other two corners. Any length of rods may be used, provided that the ratio of the lengths is the same. Thus the rods might be 15 feet and 25 feet and the base line 20 feet, since 15 : 20 : 25 is the same ratio as 3 : 4 : 5.

The 3 : 4 : 5 ratio is, however, one of the most convenient because the three numbers forming the ratio are *whole* numbers, and the lengths of the three sides can be conveniently marked off in feet or in links by means of a surveyor's chain.

CHAPTER IV

PLANE RECTILINEAR FIGURES

DEF. *A* **plane rectilinear figure** *is one in which all the sides are straight lines, all of which lie wholly in the same plane.*

The surface of a table, Fig. 54, may be taken as an illustration of a **plane surface.** Suppose a piece of wire ABCDE to be bent in such a way that the parts AE and ED rest *upon the surface* of the table, while the other three parts AB, BC, and CD of the wire are *above the surface* of the table. The wire, so bent,

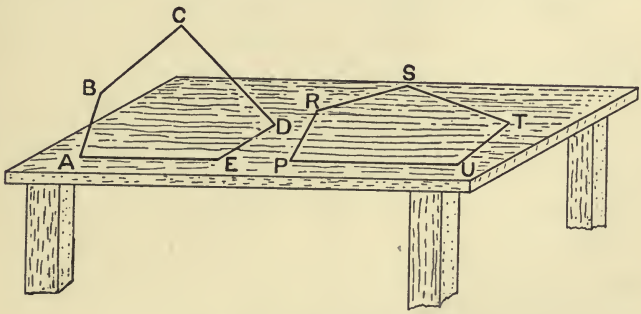

FIG. 54.—Plane rectilinear figures.

would make a *rectilinear* figure, because all its sides are *straight*, but it would not be a *plane* rectilinear figure, because *all* its sides *do not rest wholly upon the plane surface* of the table.

PRSTU, Fig. 54, represents a piece of wire bent in such a way that each part PR, RS, etc., *rests wholly upon the plane surface* of the table. This wire may be taken as an illustration of a **plane rectilinear figure.**

Plane rectilinear figures which have *three* sides are called *triangles.*

Quadrilaterals.—Any plane rectilinear figure which has **four** sides is called a **quadrilateral** (Fig. 55). If the *opposite sides* of a quadrilateral are *parallel*, it is called a **parallelogram**. Thus the **square, rectangle, rhombus,** and **rhomboid,** (*a*), (*b*), (*c*), and (*d*), Fig. 55, are **parallelograms,** but the **trapezium** is not a parallelogram because it *has only one pair of parallel sides.*

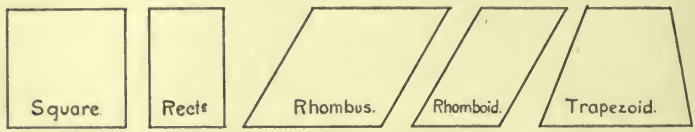

FIG. 55.

Square and Rectangle. Rule for Area.—These figures are too well known to need any explanation as to their construction. **Area of square = side²**. Thus a square with a 25·5 yds. length of side has an area of 25·5 × 25·5 = 650·25 sq. yds. **Area of rectangle** = length × breadth. Thus a rectangle with long side 16 ft. and short side 5 ft. has an area of 16 × 5 = 80 sq. ft.

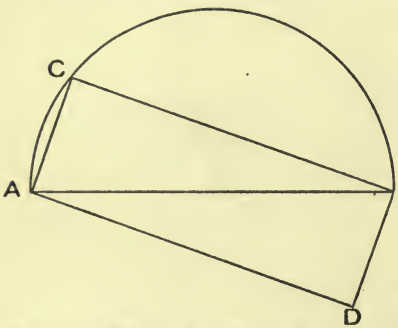

FIG. 56.—Method of constructing a rectangle when the length of diagonal and one side are known.

PROBLEM.—To construct a rectangle, the *diagonal* of which shall be $2\frac{1}{4}$ ins., and *one side* $\frac{3}{4}$ in. (Fig. 56).

Draw AB $2\frac{1}{4}$ ins., and on it describe a semicircle ACB. *The angle in a semicircle is a right angle.* Hence if any point such as C is taken, and lines CA and CB are drawn to A and B respectively, the angle ACB will be a *right angle.* In order to fulfil the conditions of the problem, AC must be $\frac{3}{4}$ in. long. With A as centre, and $\frac{3}{4}$ radius, cut the semicircle at C, and draw AC, BC. The remainder of the rectangle may be completed symmetrically about the diagonal AB. Note that D should fall on the circle of which ACB is the semicircle.

Geometric Mean or Mean Proportional.

PROBLEM.—*A rectangular space* 16 *ft.* × 4 *ft. is covered with floor boards. Find the size of the* **largest square space** *which could be floored with the boards taken up from the given rectangular floor, assuming that there was no waste in taking up and re-laying the boards.*

This problem involves the use of what is known as the **geometric mean** or **mean proportional** between two lines. The term *mean* implies some number or quantity *intermediate between two extremes.*

Thus if a man had two planks, one of which was 20 ft. long and the other 15 ft. 6 in., the *average* or *mean* lengths of the planks is $\frac{20 + 15\frac{1}{2}}{2}$ $= \frac{35\frac{1}{2}}{2} = 17\frac{3}{4}$ ft. or 17 ft. 9 ins. In such a case 17 ft. 9 ins. is a **mean** between two **extremes,** *i.e.* 20 ft. and $15\frac{1}{2}$ ft. Such a mean or average is called the **arithmetic mean** of two or more quantities, and is determined by *adding together the various quantities and dividing by the number of quantities there are in question.*

The **geometric mean** or **mean proportional,** is quite another kind of *mean.* Suppose two terms are arranged thus, 4 : ? : 16. The

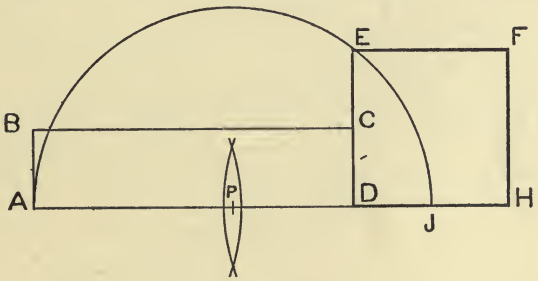

FIG. 57.—Geometric mean or mean proportional.

product of the two *extremes,* 4 and 16, is 64. There must be some number *intermediate between* 4 and 16 which, if *squared,* would give the same result as 4 × 16. Thus $\sqrt{64}$ or 8 is the number which should be filled in to take the place of the query mark in the above statement, since 8^2 or 8 × 8 = 4 × 16. In the same way, it follows that in the following statements: (*a*) 3 : ? : 27; (*b*) 4·9 : ? : 10; the number which, squared, will be equal to the product of the two extremes 3 and 27 (81) is 9. Similarly in case (*b*), the number required in place of the query mark is 7, since $7^2 = 4{\cdot}9 \times 10$. ∴ 9 is the *geometric mean* between 3 and 27, and 7 between 4·9 and 10 respectively.

In the foregoing cases, the geometric mean was found by arithmetical calculation. It is easy, however, to find it graphically, *i.e.* by means of lines, as follows.

Construct a rectangle ABCD (Fig. 57) to scale of $\frac{1}{8}$ in. to 1 ft. to represent the rectangular floor, 4 ft. × 16 ft. With centre D and radius CD, describe the arc CJ. Then AD + DJ = sum of a long and short side of the rectangle. Bisect AJ, and with P as centre, describe a semicircle on AJ. At D, the junction of the long and short, erect a perpendicular to meet the semicircle. Measure DE. It measures 8

units on the same scale on which AD measures 16 and AB measures 4. Thus DE, found graphically, is the geometric mean between AD and AB, or CD. Hence, the square DEFH, represents, to scale, the largest square floor which could be covered with the material taken up from the rectangular floor ABCD.

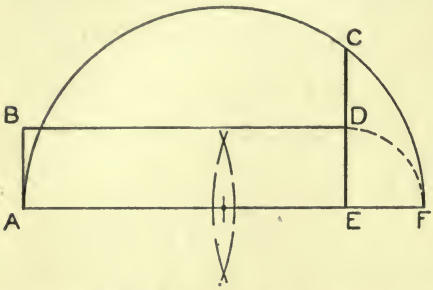

FIG. 58.—Method of finding side of square when area is known.

Given the Area of a Square to find Length of Side.

PROBLEM. — *Determine, graphically, the* **length of side** *of a square field which has an area of* 400 *sq. yds.*

Draw, to scale, say $\frac{1}{2}$ in. = 10 yds., the rectangle ABDE, Fig. 58, making AE to scale = 40 yds., and AB = 10 yds. Find the geometric mean, EC, between the long and short sides of this rectangle. Then CE, to scale, measures the length of side of the field, which contains 400 sq. yds.

Length of CE in the diagram = 1 in. This represents 20 ft. to a scale of $\frac{1}{2}$ in. = 10 ft. 20^2 or $20 \times 20 = 400$. Thus $\sqrt{400}$ or 20 ft. is the required length of side of square.

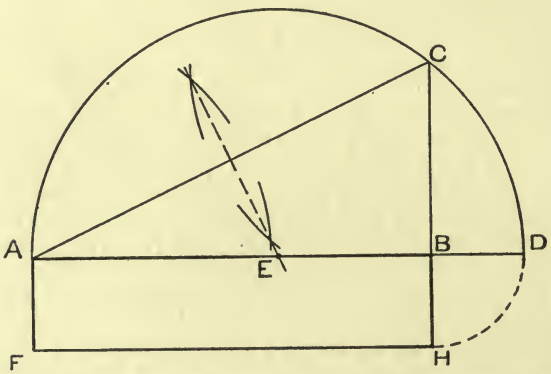

FIG. 59.—Construction of rectangle when long side and area are known.

Given the Longest Side of a Rectangle and its Area to find Length of Short Side.

PROBLEM.—A gardener wishes to turf a rectangular lawn ABHF, Fig. 59. The *long* side of his lawn has to be as long as AB. The length of short side is immaterial, but he has to take up the turves from a *square* plot CPRB and make these serve for the *rectangular* lawn.

Find the *greatest possible length* which he can have for the *short side* of his plot.

Refer to Fig. 57 (*geometric mean*). In that case the *long* side and the *short* side of the rectangle were known, and the "*mean*" between the long and short sides had to be found by construction. This case is the converse of that in Fig. 57. Here, the "*long*" side is known and the "*mean*" between the long and short sides is known, but the "*short*" side has to be found by construction.

Join AC and bisect it. The bisector meets AB in E. This is the centre for describing a semicircle to pass through A + C. This semicircle cuts AB (produced) at D. Then for a *long* side equal in length to AB and a *mean* of length equal to BC, it follows that BD is the required length for the *short* side. In other words, BD used as a short side for the plot with AB as the long side would require all the turves covering the area of the square plot CPRB.

The square and rectangle are too well known to need any explanation as to their construction. Note, however, that the **square** and **rhombus** *a* and *c*, Fig. 55, resemble each other in that the *diagonals of each bisect each other at right angles.* Comparing the rectangle and the rhomboid *b* and *d*, Fig. 55, it will be seen that the diagonals of each bisect each other, but not at right angles, as in the square and rhombus.

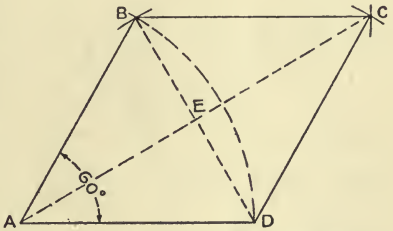

FIG. 60.—Setting out a rhombus-shaped piece of wood.

Rhombus and Rhomboid. — *A rhombus-shaped piece of wood,* ABCD, *Fig.* 60, *has an* **edge** 8 ins. long, *and the* **angle** *at one corner* 60°. *Construct the figure* $\frac{1}{4}$ *full size.*

Set out AD to scale to represent 8 ins. With centre A and radius AD describe the arc DB. With centre D and radius DA describe an arc cutting DB at B. Draw AB. Then the angle DAB is 60°. With centres D and B and radius DB, describe arcs intersecting at C. Then ABCD is the required figure.

Area of Rhombus.—The diagonals AC of the rhombus, Fig. 60, divides the figure into *two equal triangles*, ABC and ADC. The area of the triangle ABC is half the product of the base AC and the height BE, *i.e.* $\frac{AC \times BE}{2}$. The area of the remaining triangle ADC is found in the same way, *i.e.* $\frac{AC \times DE}{2}$. Then, since BE = DE, the *area of the whole*

rhombus may be found by taking *half the product of the diagonals* AC and BD, *i.e.* $\frac{AC \times BD}{2}$.

A **parallelogram**, such as the **rhomboid**, Fig. 61, can be constructed if we know—

(*a*) Two sides and the included angle.
(*b*) „ „ „ angle between its diagonals.
(*c*) „ diagonals „ „ „ them.
(*d*) A diagonal, the angle opposite this diagonal, and the length on one side.

The following problems illustrate the method of using either of the data given above.

PROBLEM.—The plan of a field ABCD, Fig. 61, is a parallelogram, the *long side* of which is 250 yds., the *short side* 125 yds., and the *angle between the sides* 50°. Draw the plan of the field to a scale of 1 in. = 100 yds., and calculate its area.

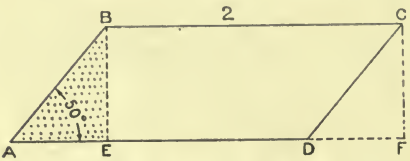

FIG. 61.—Setting out parallelogram as the plan of a field.

Set out AD to scale to represent the long side of the field. At A make angle EAB = 50° (using a protractor). Make AB to scale = 125 yds. With centre D and radius AB, and with centre B and radius AD, describe arcs intersecting at C. Draw BC and DC. The ABCD is the required plan.

Area of Parallelogram.—The **area** of any **parallelogram**, such as ABCD, Fig. 61, is *the product of any side, e.g.* AD, *and the perpendicular distance between it and the side parallel to it, i.e.* BC. Thus the area of ABCD = AD × BE.

Part of the parallelogram (the triangle ABE) is shaded in order to prove this rule. In the figure the triangles ABE and DCF are equal, DF being equal to AE. Hence the rectangle BEFC is equal to a rectangle on AD, since EF = AD. *Take away* the triangle DCF from the rectangle BEFC and *add on* the equal triangle ABE. Then the rhomboid ABCD = the rectangle BEFC, *i.e.* areas of ABCD = AD × BE.

Plan of Field.

PROBLEM.—The diagonal PS of a field is 400 yds., Fig. 62. The *length of one side*, PR, of the field is 200 yds., and the *angular distance* between P and S as observed from R is 110°. Draw a plan of the field and calculate its area.

Make PS to scale = 400 yds. On PS describe a segment of a circle to contain 110° (see Chapter VI, "*Segments of circles to contain*

any given angles"). Bisect PS and draw BC at right angles to it. This is one *locus* of the centre of the segment. Set out PC at 20° with PS to intersect BC at C (110° − 90° = 20°). Then PC is another *locus* of the centre of the segment, and C, which is *common* to the two loci, is the centre of the segment required. With centre C and radius CP or CS, describe the arc PRS. With centre P and radius (to scale) 200 yds. draw an arc to intersect the arc RRS at R. Join PR and RS. As the figure is symmetrical about the diagonal PS, the remaining sides PT and ST can be drawn parallel to RS and PR respectively.

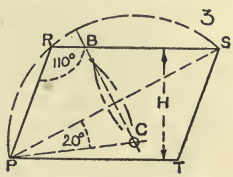

FIG. 62.—Setting out plan of field from given data.

The area is found as in last problem, *i.e.* PT × H. PT = 286·2 yds.; H = 186·2 yds. ∴ area = 286·2 × 186·2 = 53,290·44 sq. yds.

Plan of Quadrilateral Field.

PROBLEM.—The field notes of the survey of a quadrilateral-shaped field (DEFH, Fig. 63) give the following particulars: Diagonal DF = 250 yds.; DB = 80 yds.; BC = 100 yds., measured from D towards F. *Offsets* at B (left hand) and C (right hand) are *laid out* as follows: BE = 90 yds.; CH = 120 yds. Draw a plan of the field and calculate its area.

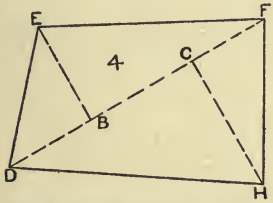

FIG. 63.—Plan of a quadrilateral field from given "field notes."

Make DF to scale = 250 yds., and at B and C set out BE and CH respectively at right angles to DF, making BE = 90 yds. and CH = 120 yds. to a suitable scale. Join DE, FE, DH, and FH. Then DEFH is the required plan.

Area of Quadrilateral.—The diagonal DF divides the field into *two unequal triangles*, DEF and DHF.

$$\text{Area of triangle DEF} = \frac{DE \times BE}{2} = \frac{250 \times 90}{2} = 11{,}250 \text{ sq. yds.}$$

$$\text{„ „ DHF} = \frac{DF \times CH}{2} = \frac{150 \times 120}{2} = 13{,}000$$

∴ „ quadrilateral = 11,250 + 15,000 = 26,250 sq. yds.

The area of any **quadrilateral** may be found in the same manner.

Polygons.—*Rectilinear figures of more than four sides are called* **polygons** (polygon = many sides). Special names are assigned to the various polygons in order to denote how many sides they have, as follows :—

Pentagon = 5 sides	Octagon = 8 sides
Hexagon = 6 „	Nonagon = 9 „
Heptagon = 7 „	Decagon = 10 „

Regular and Irregular Polygons.—A **regular** polygon is one in which all the *sides are equal in length*, and all the *angles equal.* An **irregular** polygon is one in which the sides and angles are not all equal. Thus (*a*), Fig. 64, is an irregular heptagon, while (*b*) is a regular heptagon.

To construct any Regular Polygon.—The method of construction shown in Fig. 65 will serve equally well for any *regular* polygon. The line OB is made equal to *twice the length of side* AB of the required polygon. Upon this line describe a semicircle. Divide the semicircle (using dividers) into *as many equal parts as the polygon is to have sides*—in this case 5. Join A to the

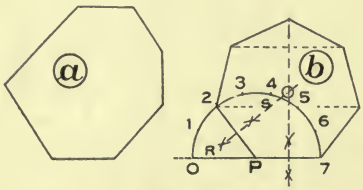

FIG. 64.—(*a*) Irregular polygon. (*b*) Regular polygon.

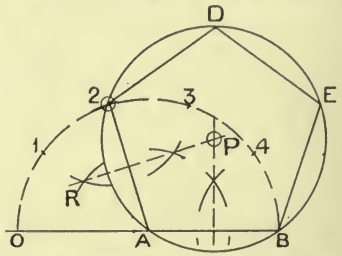

FIG. 65.—Method of construction for regular polygon.

point 2. Bisect A2 and draw RP in order to find the centre P of a circle to pass through 2, A and B. Draw this circle, *i.e.* the circumscribing circle of the polygon. All the remaining corners of the polygon will lie upon this circle. Hence it is only necessary to step off the length of side AB from the point 2 around this circle in order to fix the position of the other sides of the polygon. Proceed in the same manner for any other regular polygon. Thus for a heptagon the semicircle is divided into seven equal parts (see Fig. 64 (*b*)), and the point P joined to the second of these seven points of division. In such a case it is evident that the point numbered 2 would be nearer the point O than in Fig. 65, thus giving a greater angle between the first two sides of the heptagon than that between the first two sides of the pentagon in the given case, Fig. 65.

Correct Angle between any two Adjacent Sides of a Regular Polygon.—*All the internal angles of a regular polygon are equal, and if they be added together, their sum will equal twice*

as many right angles as the polygon has sides, less 4. Thus, in a pentagon, Fig. 65, there are five *internal* angles. Hence the correct angle between any two adjacent sides of such a figure is calculated as follows: $(5 \times 2) - 4 \times 90° = 540°$. This is the total of the five angles. Hence each angle is $\frac{540}{5} = 108°$.

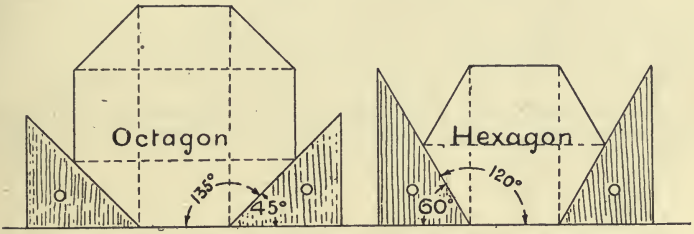

FIG. 66.—Use of 45° and 60° set square for setting out bolt heads and nuts.

The angle between any two adjacent sides in any regular polygon may be determined in the same way. Thus, for a hexagon, Fig. 66, the working would appear as follows: $(6 \times 2) - 4 \times 90° = 720°$. Thus $\frac{720}{6} = 120°$.

For an octagon, Fig. 66, $(8 \times 2) - 4 \times 90° = 1080°$; $\frac{1080}{8} = 135°$.

Practical Method of setting out Angles for Hexagonal and Octagonal Bolt Heads and Nuts.—In the foregoing calculations it has been shown that the correct angles between any two adjacent sides in a hexagon and octagon are respectively 120° and 135°. Hence, in practice, the 60° and 45° set squares are used mechanically to determine these angles, as shown in Fig. 66, the 60° being the *supplement* of 120°, *i.e.* 180° − 120°, and 45° the *supplement* of 135°, *i.e.* 180° − 135°. The dotted lines in the diagram illustrate the method of finding the remaining points of the polygon by symmetry.

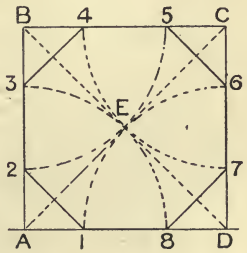

FIG. 67.—Largest octagonal bar possible out of square section rod.

Method of determining largest possible Octagonal Prism which can be made from any Square Prism.—ABCD, Fig. 67, represents the cross section of a piece of wood of *square* section. Find the size of the largest *octagonal* blind roller which can be made from this piece of wood.

Draw the two diagonals AC and BD. With A, B, C, and D respectively as centres, and radius = AE, describe arcs 3,8; 2,5; etc. Join points 1, 2; 3, 4; 5, 6; 7, 8. Then 1, 2, 3, 4, 5, 6, 7, 8

is the cross section of the largest piece of octagonal section which can be made from ABCD.

Thumb-gauge or Pencil-gauge for lining Square Rod ready for planing to Octagonal Shape.—When a rod of *square section* is to be planed to make one of *octagonal section*, Fig. 68, it is only necessary to draw the *two diagonals* of the square as shown. The notch cut out of the pencil gauge is then made equal in length to *half the length of a diagonal.* The lines so drawn are correct for chamfering off the corners of the square rod.

Area of Regular Polygon.—*The diagonals of any regular polygon, Fig.* 69, *divide the figure into as many equal triangles as*

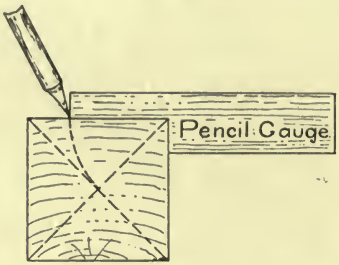

FIG. 68.—Pencil gauge for chamfering octagonal prism.

FIG. 69.—Method of finding the area of a regular polygon.

the polygon has sides. In the hexagon, Fig. 71, these triangles are **equilateral.** In the other regular polygons the triangles are **isosceles.** In any case, the area of the polygon is found thus :—

$$\text{Area of triangle POR, Fig. 69} = \frac{\text{PR} \times h}{2}$$

There are eight such triangles. Hence the area of the octagon is $\frac{\text{PR} \times h \times 8}{2}$ or $\text{PR} \times h \times 4$.

In the case of an **irregular** polygon the area of *each* of the triangles made by the intersecting diagonals must be found *separately.* The *total area* of these several triangles is the area of the polygon.

Relation between Diagonal and Side of Regular Pentagon.—The diagonal AB, Fig. 70, of the cross section of a pentagonal rod is given. Find the length of its side and construct the figure.

The construction here depends upon the *medial division* of a straight line. When a line AB, Fig. 70, is divided into two parts at a point E, such that *the rectangle on the whole line* AB *and the lesser segment* BE *is equal to the square on the greater*

segment AE, the line AB is said to be **divided medially** at E, *i.e.* AB × BE = AE².

The *medial division* of AB, the diagonal of the pentagon, is obtained as follows: At B make BC at right angles to AB and equal to half AB. Join AC. With C as centre, and radius CB describe an arc BD, cutting AC in D, and with centre A and radius AD describe an arc DE, cutting AB in E. Then AB is *medially divided* at E. The length AE is the *length of side* required for a pentagon which has a *diagonal* equal in length to AB. Hence, with A and B respectively as centres, and radius = AE, describe arcs to intersect at F. Join AF and BF. The remaining points H and J may now be determined either (*a*) by finding the centre of the circumscribing circle to pass through A, F, and B, and stepping the length of side AF around this circle. (*b*) All the diagonals of the pentagon are equal, ∴ with B and F as centres, and radius = AB, describe arcs to intersect at H, and with A and F as centres and radius = AB describe arcs to intersect at J. Join AH, HJ, and BJ. Then AFBJH is the required pentagon.

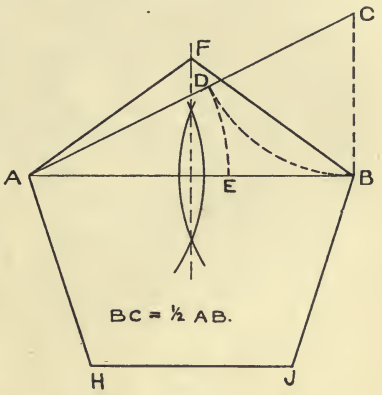

FIG. 70.—Diagonal of pentagon medially divided in order to find length of side.

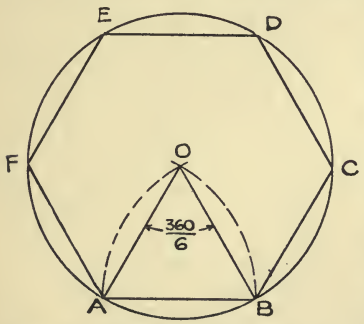

FIG. 71.—Making largest hexagonal bar possible from one of circular section.

Polygons inscribed in Circles.—A polygon is said to be inscribed in a circle when *all the corners of the polygon occur exactly upon the circle, i.e.* no corner falls *without* or *within* the circle. Thus the hexagon, Fig. 71, and the octagon, Fig. 72, are polygons inscribed in circles.

To make the largest Hexagonal Bar possible from a Cylindrical Rod.

PROBLEM.—Fig. 71. *Given the cross section of a* **round** *rod, to*

determine the size of the largest **hexagonal** *rod which can be made from it.*

The simplest and most direct method is to find the centre O of the circular rod, and set out at O an angle equal to $\frac{1}{6}$ of $360° = 60°$. This determines the position of the points A and B. The length of side AB will step around the circle six times exactly at F, E, D, etc. Join these points and ABCDEF is the required hexagonal cross section.

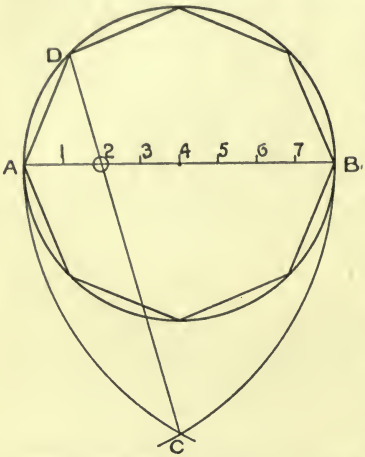

FIG. 72.—Making the largest octagonal shaft possible to fit a cylindrical pipe.

Method of Determining the Largest Octagon which can be Inscribed in a Given Circle.

PROBLEM. — *The circle* ADB, Fig. 72, *represents the cross section of a cylindrical pipe. Find the size of the largest shaft of* **octagonal** *section which will just fit within the* **cylindrical** *pipe.*

The method illustrated here is that discovered by *Renaldinus* in the seventeenth century.

Draw AB the diameter of the circle, and divide it into as many equal parts as the inscribed figure is to have sides (in this case eight). With A and B as centres and radius = AB (*i.e.* the diameter of the circle) describe arcs intersecting at C. From C draw a straight line to pass through the second point of division (2) of AB, and produce to meet the circle at D. Then AD is the length of side of the *largest regular octagon* which can possibly be *inscribed in the circle* ADB. Step AD around the circle (eight times) and the required octagonal cross section is determined.

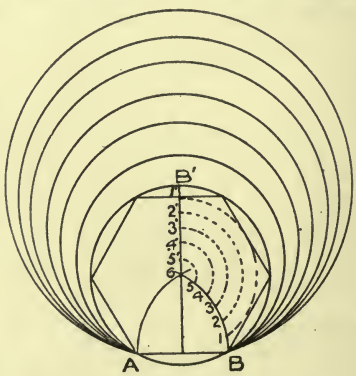

FIG. 73.—Alternative method of constructing various regular polygons.

PROBLEM (Fig. 73).—*Given a certain length of side* AB, *to determine the circles required respectively for inscribing a hexagon, octagon, etc., having* AB *as the length of side of the respective polygons.*

The method shown is only approximately correct, but serves well enough for all ordinary purposes.

With centres A and B and radius AB describe arcs intersecting

at 6. Divide the arc B6 into six equal parts. With centre 6 and radius 6,5, describe the arc 5,5′. In the same manner describe arcs 4,4′; 3,3′ etc. Then 6 is the centre of a circle large enough to allow of a hexagon of side AB being inscribed within it. Similarly 5′ is the centre of a circle for inscribing a heptagon, 4′ for an octagon, 3′ for a nonagon, 2′ for a decagon, and so on.

Strongest Beam to resist Tranverse Strain.

PROBLEM.—*Given the cross section of a log of wood*, ADBC, (*a*) Fig. 74, *to determine the dimensions of the* **strongest** *beam of* **rectangular** *section which can be cut from the given log.*

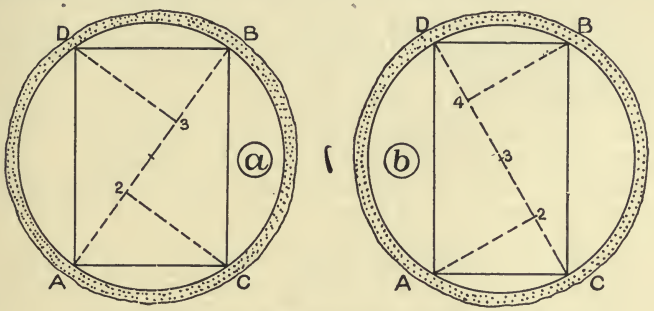

FIG. 74.—Rectangular wooden beams cut from round log.
(*a*) Strongest beam possible. (*b*) Stiffest beam possible.

Draw AB the diameter of the circle, and divide into three equal parts at 2 and 3. At these points set out 2C and 3D respectively at right angles to AB. Join AD, BD; AC, BC. Then ADBC is the required cross section. Note that this is a *rectangle* of a special character *inscribed within the circle* ADBC.

Stiffest Beam, considered with Regard to Deflection.

PROBLEM (*b*) (Fig. 75).—*The circle* ADBC *represents the cross section of a log of wood. Determine the cross section of a beam of* **rectangular** *section which shall give the* **minimum** *of deflection when loaded.*

Draw the diagonal DC in last problem, but divide into four equal parts, at 2, 3, and 4. At 2 and 4 set out A2 and B4, respectively, at right angles to DC. Join AD, DB, etc. Then ADBC is the required cross section. Note that by this method a *deeper* beam is obtained though of *smaller sectional area* than that obtained in the preceding case. In other words, the **maximum value** of *bd* is obtained.

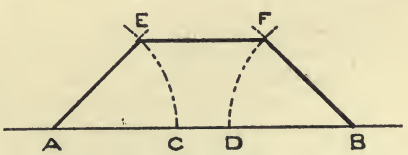

FIG. 75.—Setting out an octagonal bay window.

To set out an Octagonal Bay-window.—AB, Fig. 75, represents the plan of the opening in a wall for an octagonal bay. If this distance is divided into three parts as shown, at C and D, such that the ratio AC : CD is 12 : 5 and DB : CD is 12 : 5, then AC or DB is the length of side required for the octagon, *i.e.* the width of the side windows of bay. In other words, CD is $\frac{5}{12}$ of the length of AC or DB.

With A as centre and AC as radius describe the arc CE, and with B as centre and BD as radius describe the arc DF. With C as centre and radius equal to AC (the length of side of octagon) describe an arc at F intersecting the arc DF, and with D as centre and radius = AC or DB, describe an arc cutting the arc CE at E. Join AE, EF, FB, and thus complete the plan of the bay.

Conversely if the length of side of the octagon, *i.e.* width of the side windows of the bay, is known and the width of the opening in the main wall of the house has to be determined, multiply the length of side of octagon by $2\frac{5}{12}$, and the product is the width of opening required in main wall.

To set out a Hexagonal Bay-window.—Draw a line *ab*, Fig. 76, to represent the line of frontage of the house. Bisect *ad* at *e*, and with *e* as centre and radius *ea* describe the semicircle *abcd*. With *a* as centre, and radius *ae*, describe the arc *eb*, and with *d* as centre, and radius *de*, describe the arc *ec*. Join *ab*, *bc*, *cd*, and thus complete the plan of the bay.

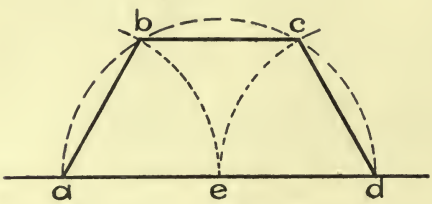

FIG. 76.—Setting out a hexagonal bay window.

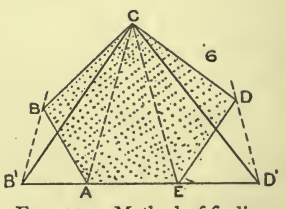

FIG. 77.—Method of finding triangle equal to given pentagon.

Equivalent Areas.

PROBLEM.—ABCDE, *Fig.* 77, *represents the plan of a floor, of an irregular* **pentagonal** *shape. Find the size and shape of the largest* **triangular** *shaped space that could be covered with the flooring taken up from the given floor, supposing there was no waste of material in taking up and relaying it.*

The construction in such cases depends upon the following important principle: *Triangles on the same base and between the same parallels are equal in area* (see Chapter III, Triangles). Join AC,

thus making a triangle ABC with AC as its base. Through B draw BB′ parallel to AC. Then, since AC and BB′ are *parallels*, all triangles which have AC as their *common base* and have their *vertexes on* BB′ are *equal.* Join B′C, making one such triangle. Then the two triangles ABC and AB′C *on the same base* AC and *between the same parallels* AC and BB′ are **equal in area.** Join CE, and through D draw DD′ parallel to CE. Join CD′. Then the two triangles CDE and CD′E, which are *on the same base* CE and *between the same parallels* CE and DD′, are **equal in area.** The triangle ACE *has not been altered in any way.* Hence it is evident that the *three triangles* B′CA, ACE, and ECD′ which go to make up the larger triangle B′CD′ are *equal in area to the original figure* ABCDE.

Square Equal in Area to Irregular Polygon.

PROBLEM.—*A tenant rents a piece of ground, the plan of which is an* **irregular polygon,** ABCDEF, Fig. 78 (*a*). *He asks his landlord to*

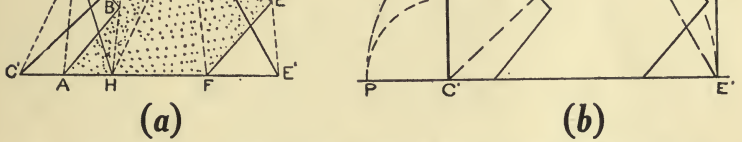

FIG. 78.—Method of finding triangle equal in area to given irregular polygon.

exchange it for a **square** *plot of the same area. Find the size of the square plot.*

The steps to be taken in this case are as follows: (*a*) Proceed as in last problem to find a *triangle* equal in area to the given figure ABCDEF. (*b*) Construct a *rectangle* equal in area to this *triangle.* (*c*) Find the *mean proportional* between the *long and short sides* of this rectangle. Join DF and draw EE′ parallel to it. Join DE′, thus making the triangle DE′F equal to the triangle DEF. The left-hand side presents a little more difficulty. In the last problem the projecting corners B and D of the polygon were *triangulated.* The first step here, however, is to deal with the space which has been cut into the polygon at B. Thus join AC, and through B draw BH parallel to AC. Join CH. Then the triangle AHC = triangle ABC. In place of the three sides AB, BC, and CD of the polygon we now have only two, viz. HC and CD. Join HD and through C draw CC′ parallel to DH, and join DC′. Then the triangle DHC′ is equal in area to the triangle DHC. The triangular patch HDF has not been interfered with; hence the triangle C′DE′ is equal in area to the original figure ABCDEF.

The remainder of the construction is shown on a separate diagram, (*b*), Fig. 78, to avoid confusion. Construct a rectangle using the base

C'E' of the triangle as the long side, the short side being equal to one-half the height of the triangle C'DE'. Then the rectangle C'JLE' is equal in area to the triangle C'DE'.

Make PC' = C'J. Bisect PE' and describe semicircle PRE. Produce C'J to meet the semicircle at S.

Then C'S is the length of side of a square equal in area to the rectangle C'JLE', and consequently equal to the original figure A . . . . F.

CHAPTER V

SIMILAR RECTILINEAR FIGURES

Similar Triangles.—*DEF. When two or more triangles have the angles in one triangle equal to the corresponding angles in the others, and the corresponding sides in the figures are proportional, the triangles are said to be* "**similar.**" Two 60° set squares are shown in Fig. 79 (*a*). The angles in the one are respectively

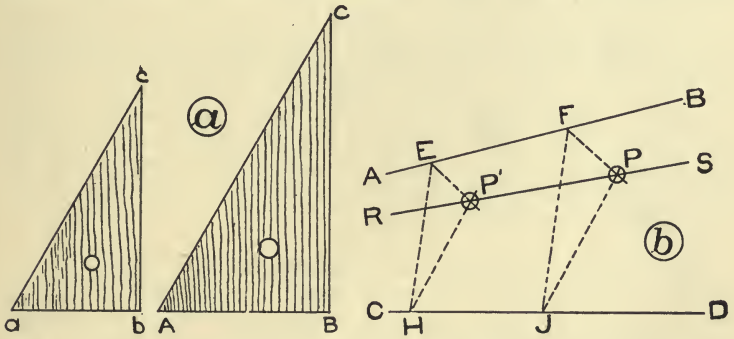

FIG. 79.—(*a*) Set squares as illustration of similar triangles. (*b*) Application of similar triangles to bisection of angle between two lines.

equal to the angles in the other, *i.e.* 30°, 60°, and 90°. The edge *ab* of the smaller set square is $\frac{3}{4}$ of the length of the edge AB of the larger set square. Measure the length, say, in millimetres, of the edge (*bc*) of the smaller set square, and compare with the lengths of *ab* and *ac*. Measure the length of the edge AB of the larger set square, and compare with the lengths of BC and AC. Arrange your statement in the form of a ratio as follows:—

Smaller set square, *ab* : *bc* : *ac*
38 : 66 : 76 units of length.
Larger set square, AB : BC : AC
51 : 88 : 102 units of length.

Note that the ratio, or comparison, between any pair of sides is the same in each set square; *e.g.* $38 : 66 = \frac{38}{66} = \cdot 57$; and similarly $51 : 83 = \frac{51}{88} = \cdot 57$ That is, *ab* is ·57 of the length of *bc*, and AB is ·57 of the length of BC. The same may be said of any pair of sides in the smaller set square as compared with the corresponding pair of sides in the larger set square. The same ratio exists between the sides of the smaller as between the sides of the larger set square. Therefore the two set squares are examples of "**similar triangles.**"

Problems involving the use of Similar Triangles.—The property of similar triangles which has been demonstrated in the foregoing paragraph is often employed in the solution of geometrical problems, some of which follow.

PROBLEM (Fig. 79 (*b*)).—Given two lines AB and CD which approach each other; given also a fixed point P. Through P to draw a straight line which would meet AB and CD, produced, at their point of intersection, which is inaccessible.

From P draw any two lines PF and PJ. Join FJ, thus completing a triangle JFP. From any point E in AB, draw EP′ parallel to FP and EH parallel to FJ, and from H draw HP′ parallel to JP intersecting EP′ in P′. Then JFP and HEP′ are *similar triangles*. If a succession of such triangles was drawn, each smaller than the last, the line RS, which passes through P and P′, would pass through the corresponding point, say P^2, P^3, etc., of each of these triangles, until the sides of the similar triangles become *infinitely small*, at the intersection of the lines AB and CD, produced.

Estimation of Heights and Distances.—In Chapter II. the method of using *trigonometrical ratios* for calculating heights and distances apart of inaccessible objects has been explained. It is sometimes possible to estimate the distance apart of two points, one of which is inaccessible, by making use of *similar triangles*.

PROBLEM (Fig. 80 (*a*)).—A and B are two points on opposite sides of a river. Determine the distance apart of A and B, *the point A being inaccessible.*

Lay down a line BE in line with AB produced; and find a point D, midway between B and E. Lay down any line EA′ indefinite in length, and on it make E*b* = DE and E*d* = EB. Join B*b* and D*d*. Through *e*, the intersection of B*b* and D*d*, draw EC. From the point *d* lay down a line *d*C in line with A on the opposite bank. Through C, the intersection of E*e* (produced) and *d*A, draw BA′ to intersect the line E*d*, produced in A′. Then *d*A′ gives the distance apart of A and B. Note that the triangles BE*b* and AE*d* are *similar*. Note also that the triangle DE*d* is *similar* and *equal to* the triangle BE*b*, and the triangle

BEA′ is similar to the triangle DE*d*. Thus it is easy to see that the triangles AE*d* and A′EB are similar and equal. Hence A′*d* = AB.

Fig. 80 (*b*) illustrates another method of finding the distance apart of two points, A and B, one of them, A, being deemed inaccessible.

Lay down a line BC in line with BA. From B lay down any line B*b*. Bisect B*b* at D, and through D draw C*c*, making D*c* = DC. Through *b* draw *cb*, and produce it to intersect the line AD (produced) in A′. Then ACD and A′*c*D are similar and equal triangles, and by the property of similar triangles, A′*c* = AC. *Take away bc* and BC respectively, and the *remainders*, A′*b* and AB, are *equal*.

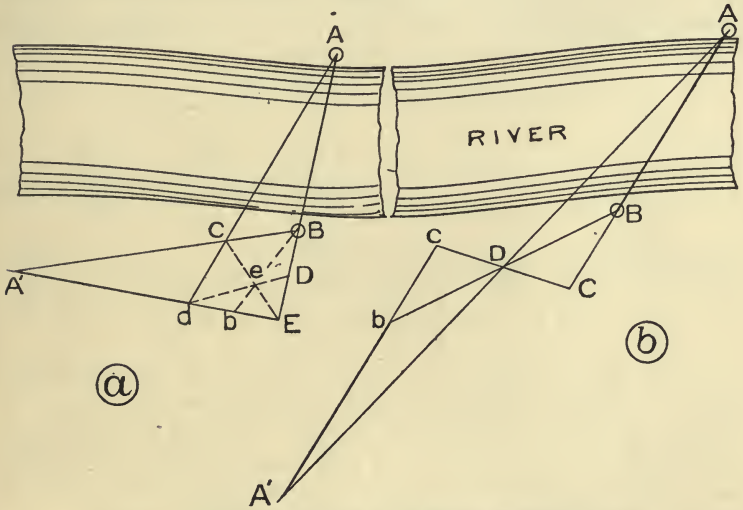

Fig. 80.—Estimation of distances by means of similar triangles.

Simple Measuring Staff for Estimating Heights and Distances.—In the Middle Ages a simple **measuring staff**, Fig. 81 (*b*), was sometimes used by the soldiers for estimating the height of an object, such as a tower, which was situated within the enemy's lines. It was a very rough piece of apparatus, consisting of a stout rod, AB, and two sliding pieces CD and EF. The bottom end of the piece CD rested on the ground while the observer knelt or lay prone on the ground. The length of each sliding piece was some well-known and easily-remembered measurement; and the rod AB was notched or marked at intervals to show inches. The rod AB served to give a rough level line, and, by sliding the piece CD nearer to, or farther

away from, EF, the observer, with his eye at B, arranged so that its top edge and the end B of the rod AB were in line with the top of the tower, or other distant object, the height of which was to be determined. When the sliding piece was in this position, note was made of the particular notch or mark on AB opposite which CD stood, and a rough calculation was made as follows :—

See Fig. 81 (*a*). The line AB, produced beyond A, is a level line which meets the tower at a height of, say, 1 ft. from its base. The line CE, marked *sight line*, produced beyond C, is in line with the top of the tower. Thus the *level line*, *i.e.* AB produced, *the tower*, and the *sight line*, are *three sides of an imaginary triangle.* This triangle is *similar* to the triangle BDC made by the *rod*, the *sliding piece* CD, and the *sight line.* Estimating his distance as, say, 400 ft. from the tower, his calculation, taking

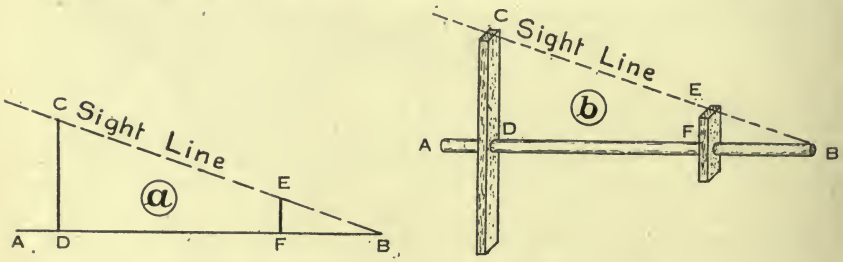

FIG. 81.—Measuring staff used in the Middle Ages.

CD as 1 ft. and DB as 4 ft., would be: In the triangle BDC, BD : CD :: 4 : 1. Estimated distance from observer to base of tower = 400 ft., ∴ height of tower : 400 ft. is the same ratio as BC : CD, *i.e.* 1 : 4. Therefore height of tower is $\frac{400}{4}$ = 100 ft., + height of observer's eye above ground. This would be about 1 ft. if observer lay prone on the ground.

It should be noted that when the observer was *very near* to the object; or when the "*angle of elevation*" of the object was *great*, the *larger* sliding piece CD was used. When the observer was *further away* from the object and the angle of elevation was so small that the piece CD, even when moved to the extreme end, A, of the rod AB, would not fall in line with the observer's eye and the top of the object, the *smaller sliding piece* EF was brought into requisition in place of CD. Thus, taking CD as 6 ins., AB as 4 ft., and estimated distance from observer to tower as 600 ft., the ratio would be as follows: Height of tower : 600 ft. as 4 ft. : 6 ins., *i.e.* $\frac{600}{8}$ = 75 ft.

Similar Rectilinear Figures.—The two irregular pentagons

shown, Fig. 82 (*a*) and (*b*), are examples of **similar rectilinear figures**, the angles in (*a*) being equal respectively to the corresponding angles in (*b*), and the ratio between any pair of sides, say, *ab* : *bc* in (*a*) being the same as the ratio between the corresponding pair of sides, AB : BC in (*b*).

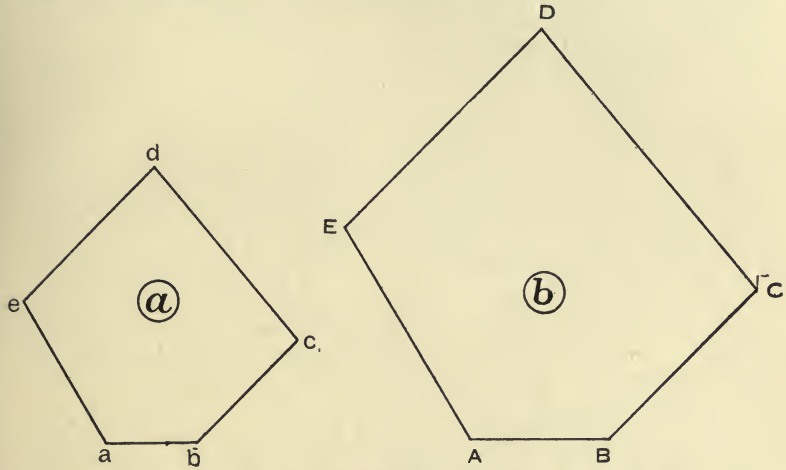

FIG. 82.—Similar rectilinear figures.

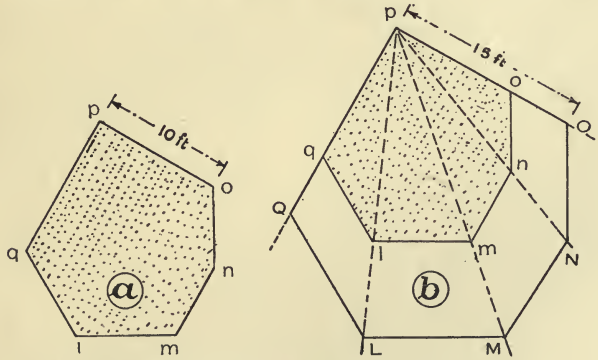

FIG. 83.—Method of enlarging a plane rectilinear figure.

Enlarging or Reducing any Rectilinear Figure.—The process of enlarging or reducing any given figure depends upon the principle of Similar Rectilinear Figures already enunciated. The following examples illustrate this principle.

PROBLEM (Fig. 83 (*a*) and (*b*)).—In the figures (*a*) represents the plan of a floor. Find the shape of a larger floor *similar in shape* to the given one, but *having the side po increased* from 10 ft. to 15 ft.

From *p* (Fig. 83 (*b*)) draw radiating lines through O, *n*, *m*, *l*, and *q*. Produce *po* making it to scale 15 ft. long. From O draw ON parallel to *on* to intersect *pn* in N. By drawing MN parallel to *mn*, find the point M on the radiating line *pm*. Do the same to find points L and Q. Then the figure *p*ONMLQ is an enlarged figure similar to the shape of the given floor, but larger in the proportion of 10 to 15.

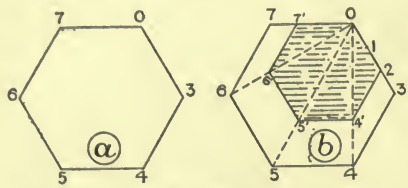

FIG. 84.—Method of "reducing" a plane rectilinear figure.

PROBLEM.—Fig. 84 (*a*) represents the cross section of a hexagonal bar. Find the correct shape and size for the cross section of a *smaller* hexagonal bar having a side $\frac{2}{3}$ of the length of side O3 in (*a*).

Draw radiating lines from O to 4, 5, 6, and 7. See (*b*). Divide O3 into three equal parts, and from point 2 draw 2,4′ parallel with 3,4, to cut the line O4 in 4′. Proceed in the same way to draw the remaining sides 4′,5′, 5′,6′, etc., parallel respectively to 4,5, 5,6, etc.

Then the *shaded hexagon* represents the size and shape of the *reduced cross section*.

NOTE.—*The area of any figure described on the hypoteneuse of a right-angled triangle is equal to the similar and similarly described figures on the sides about the right angle.*

PROBLEM (FIG. 85). — The shaded areas *abcde* and ABCDE represent two pentagonal-shaped pieces of sheet material. Both these pieces are to be used for covering a *larger* and *similar* pentagonal surface. Find the correct shape and size of this larger surface. Construct a right-angled triangle ABQ, using the side AB, of the *larger* pentagonal piece, as the *base*, and the corresponding side *ab* of the *smaller* piece as the *perpendicular*, *i.e.* BQ. Then AQ, the *hypoteneuse* of this right-angled triangle is the *length of the corresponding side* of a larger piece of material AB′C′D′E′ similar in shape to the two given pieces and having an area *equal to the combined areas* of the two given pieces. With A as centre and AQ as radius describe an arc QB′ meeting AB, produced, at B′. From A draw radial lines through CD and E. From B′ draw B′C′ parallel to BC to meet the radial line through C at C′.

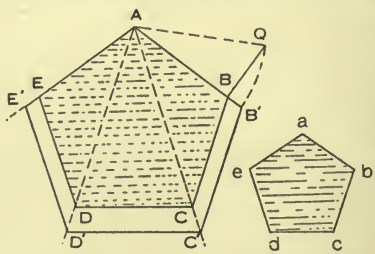

FIG. 85.—Pentagonal surface equal to the combined areas of two given pentagonal surfaces.

Proceed in the same way to obtain C′D′ and D′E′ parallel respectively to CD and DE.

Polar or Radial Projection.—In each of the foregoing cases, where an enlargement or reduction of any given outline was required, radiating lines were drawn from one corner of the figure itself. The point from which these lines radiate, *e.g.* A in Fig. 85, is called a **pole.** It is sometimes an advantage to select as a pole a point *outside the figure*, but the position is purely arbitrary. There are, however, certain practical considerations which govern the selection ; the most important being that the point chosen as pole should allow of all the radial lines being drawn *without any danger of one overlapping another.*

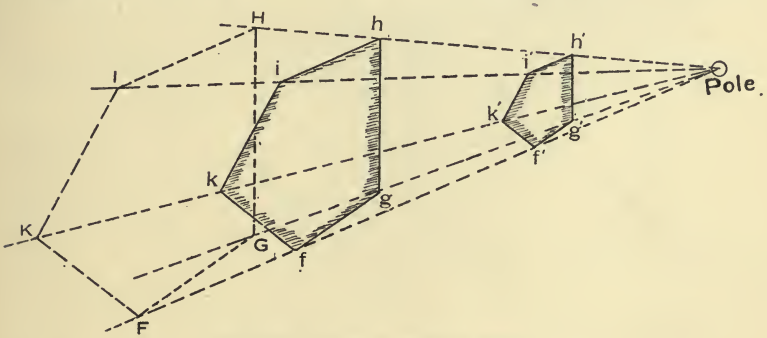

FIG. 86.—Polar or radial projection.

A piece of card, *fghik*, Fig. 86, is given. A smaller card is required, the shape of which is to be a polygon similar to the given one, but having its side *f′g′* half the length of the side *fg* of the given polygon.

Select as a pole a point P in such a position that no two radial lines, such as P*h* and P*i*, fall in the same line ; *e.g.* P should not be *in line with ih* produced, or the radial lines, P*h* and P*i*, would both be in the *same straight line.* From P draw the radiating lines P*h*, P*i*, P*k*, etc. Draw the line *f′g′* parallel to *fg* and half its length, to intersect the radial lines P*f* and P*g*. From *g′* draw *g′h′* parallel to *gh* on the original figure and between the radial lines P*g* and P*h*. Proceed in the same way to determine the sides *h′i′*, *i′k′*, and *k′f′* parallel respectively to *hi*, *ik*, and *kf* on the original figure. Then this smaller figure *f′g′h′i′k′*, reduced in size from the original, is said to be **radially projected** from *fghik*.

If an enlarged piece similar to the original is required, the radial lines should be produced *beyond* the points *h*,*i*, etc., and the

sides HI, IK, KF, etc., of the enlarged figure obtained, by drawing lines parallel to the sides of the original polygon, as shown at FGHIK.

Enlarging or Reducing any given Moulding.—It is sometimes necessary in practical work to determine the cross section of a moulding *similar in character* to a given one, but either *larger or smaller* than the given one. **Polar or radial projection** is very useful in such cases.

ABCDEFG, Fig. 87, is the cross section of a *scotia* moulding, the thickness of material from A to G being $\frac{7}{8}$ in. Determine

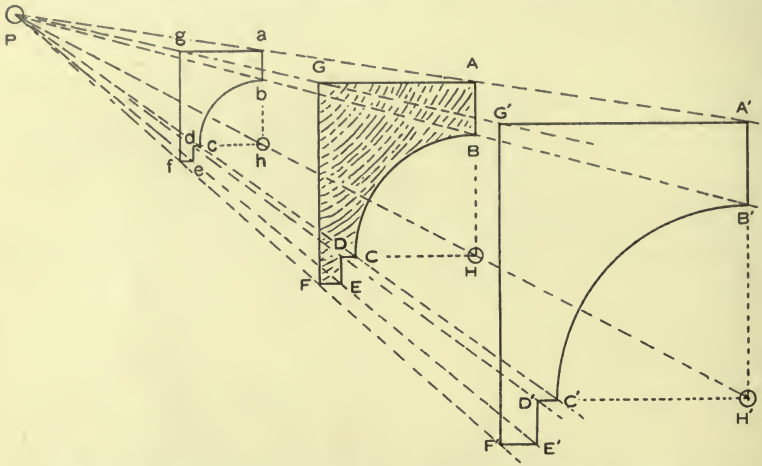

FIG. 87.—Enlarging or reducing a given moulding by means of "polar" or "radial" projection.

(*a*) the cross section of a *larger* moulding *similar* to the given one, but out of $1\frac{3}{8}$ ins. thickness material; and (*b*) the cross section of a *smaller similar* moulding made from material $\frac{7}{16}$ in. thick.

Draw a "*ray*" through G inclined to AG and FG. Select any point P on this ray as a "*pole*," and from the *pole* draw *rays* through the various points, A, B, C, etc., on the given cross section. Draw a horizontal line A′G′ $1\frac{3}{8}$ in. long to intersect the rays PA and PG, produced. This is the top edge of the enlarged moulding. From G′ draw G′F′ parallel to GF to intersect the ray PF produced at F′. This gives the *width* of the enlarged moulding. Proceed in the same way to obtain A′B′ parallel to AB, F′E′ parallel to FE, D′E′ parallel to DE, and D′C′ parallel to DC. On the original moulding the centre H of the quadrant is found by producing AB and CD to intersect at H. Similarly,

in the enlarged cross section the centre of the quadrant is found by producing A′B′ to intersect D′C′ in H′. Note, however, that a *ray from* P through H should also *pass through* H′.

Then A′B′C′D′E′F′G′ is the *enlarged cross section* required.

(*b*) To obtain the correct shape for the "*reduced*" cross section draw the horizontal line *ag* $\frac{7}{16}$ in. long, between the *rays* PA and PG. Draw *gf* parallel to GF, intersecting the ray PF at *f*, and proceed as in the last case (*a*) to determine the remaining lines on the cross section.

Then *abc* . . . *g* is the *reduced cross section* required.

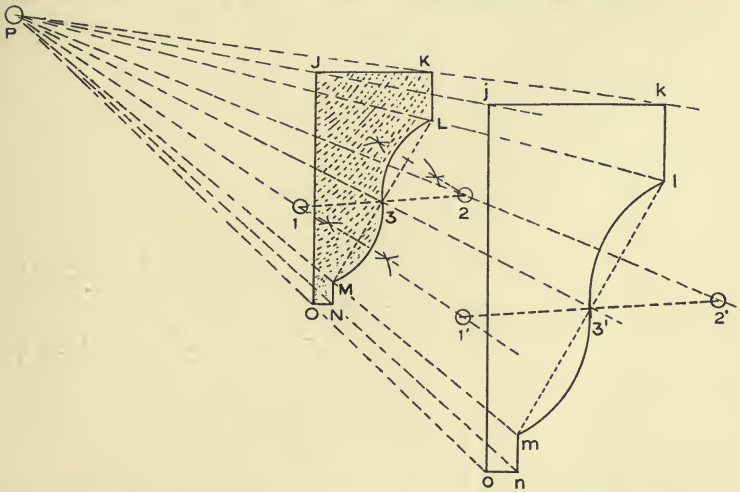

FIG. 88.—Polar or radial projection used in enlarging mouldings.

The cross section JKLMNO of a *cyma recta* or OG moulding $1\frac{1}{4}$ ins. wide, is given in Fig. 88. Enlarge it so that the width may be 2 ins.

The straight lines on the enlarged cross section are obtained as already directed (see last example), *i.e.*, *jo* 2 ins. long and parallel to JO on the given cross section ; *jk* parallel to JK and intersecting the rays PJ and PK produced. *kl*, *mn*, and *no* are obtained in the same way.

The OG curve in the original moulding is obtained by bisecting the chord line LM at 3 in order to give the point at which the *convex* part of the curve merges into the *concave* part. This point is called the **point of contraflexure**. The half chord lengths L3 and M3 are then bisected, and the centres 1 and 2 for the two parts of the OG curve are found on these bisectors.

To determine the centres 1′ and 2′ for describing the OG curve in the enlarged cross section proceed as follows: Draw the rays P1, P2, and P3, and produce them. Join L and M on the original moulding and *l* and *m* on the partly finished enlarged section. Join the centres 1 and 2 on the original moulding and through 3′, the intersection of *lm* and the ray P3; draw 1′2′ parallel to 1,2 and intersecting the rays P1 and P2 at 1′ and 2′. Then by the property of *similar figures* 2′ stands in the same relation to *l* and 3′ (on the *enlarged figure*) as 2 does to L and 3 on the *original*. Similarly with 1′ in relation to *m* and 3′ and 1 in relation to M and 3. Then 1′ and 2′ are the centres for the OG curve. This completes the enlarged cross section *jklmno*.

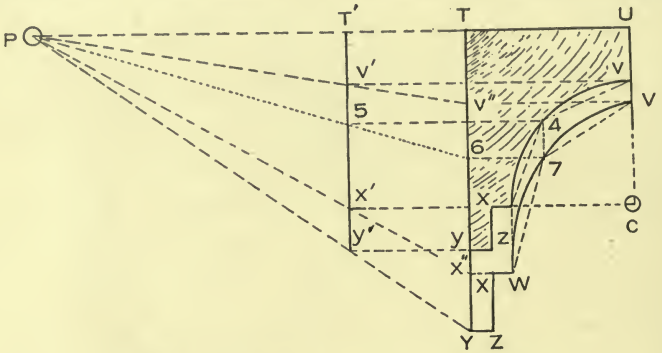

FIG. 89.—Enlargement of a moulding in width only.

Enlarging a Moulding in One Direction only.—It is sometimes necessary to enlarge or reduce a moulding one way only; *e.g.* to *increase the width* while the *thickness remains the same.* Sometimes it may be necessary to enlarge or reduce the cross section both in thickness and width, but *not to the same degree; e.g.* to increase the thickness to 1½ times the original, and the width to twice the original size. The method of working such cases is illustrated in the following problems. The resulting figures in such cases are *not similar* to the original figures. As the construction depends upon *polar or radial projection*, however, it is illustrated here.

The given shaded cross section of a moulding TU*vwxyz*, Fig. 89, is to be *increased in width* so that T*y* shall be increased to TY, *while the thickness*, TU, *remains the same.*

Select a pole, P, anywhere on the line TU produced to the left. From P draw a ray PY to the point Y, which indicates the increased width required. From T and *y* draw horizontal projectors TT′ and *yy′*, in order to find the position of the *measuring*

line T′*y*′ between the rays PT and PY. From each of the points *v*, *w*, *x*, *y*, and *z* on the original cross section draw horizonal projectors *vv*′, *xx*′, etc., to intersect the measuring line in points *v*′, *x*′, etc. Through these points draw the rays P*v*′, P*x*′, etc., and produce them to meet the line TY in points *v*″, *x*″, etc. From these points draw horizontal projectors *v*″V, *x*″X to meet vertical projectors from *v* to *x*, etc., on the original moulding. The curved part of the outline is obtained as follows: On the

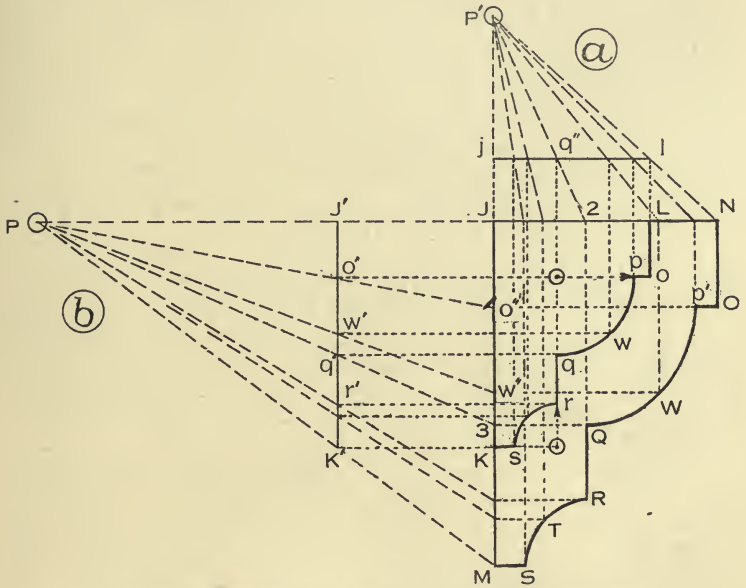

FIG. 90.—Enlargement of moulding in width and thickness.

original moulding the curve *vw* is a *quadrant*, centre *c*. *On the enlargement* this curve *will not be a quadrant* and cannot be struck from any centre. Two or more points on the new curve should be found by construction and a free curve drawn through these points. Only one point is shown numbered 4 on the drawing. From 4 a horizontal projector is drawn to meet the *measuring line* at 5. Through 5 a ray P5,6 is drawn. From 6 a horizontal projector is drawn to intersect a vertical projector from 4 at the point 7, which is a point on the required curve. Select other points on the original curve and treat in a similar manner, using a French curve to draw a curve through the points so determined. Then TUVWXYZ is the required enlargement.

Enlarging a Moulding in Two Different Directions, but in Different Degrees.—The cross section of a moulding, Fig. 90, JK*srqp*L is given. Enlarge this moulding so that the *thickness* JL *is increased* to N, and the *width* JK *is increased* to M.

In this case, as the width and thickness respectively are to be increased, *two poles are necessary*, as shown at (*a*) and (*b*), Fig. 90. The construction at (*a*) shows the method for increasing the *thickness*, and that at (*b*) for increasing the *width*.

1*st Step*.—Select a pole P anywhere on the side (*b*) of the moulding, and in line with JN the top edge of the moulding. Produce the width of the moulding JK to M to indicate the *increase of width* desired, and join PJ and PM. Project the *original width* of the moulding JK towards (*b*), in order to obtain the position of the measuring line J′K′. Having drawn J′K′ draw projectors *oo′*, *qq′*, *rr′*, etc., from all points on the original moulding to find the points *o′*, *q′*, *r′*, etc., on the measuring line at (*b*). Through these points draw rays P′*o′*, P*q′*, P*r′*, etc., and produce them to meet the line JM as indicated at 3.

2*nd Step*.—Select any pole P′ *above* the moulding as at (*a*). Produce the thickness line JL of the moulding to N, the *extra thickness* required, and draw the rays P′J and P′N. From J and L draw vertical projectors J*j* and L*l* in order to find the position of the measuring line *jl*. From *pqrs*, etc., on the original moulding draw vertical projectors as at *qq″* to meet the measuring line *jl* in points *q″*, etc. Through these points draw rays P*q″*, etc., and produce them to meet the line JN in points 2, etc.

Referring again to the 1*st step*, above: the point 3 on the line JM is the termination of the ray from P through *q′*. Similarly the point 2, on the line JN, is the termination of the ray from P′ through *q″*. Then the intersection of a *vertical projector* from 2 and a *horizontal projector* from 3 gives the point Q on the *enlarged* moulding which corresponds with *q* on the *original* moulding. Proceed in the same way to determine points RS, etc.

The curve *p*′WQ, on the enlarged moulding, as already explained in the last problem, *is not a quadrant* of a circle, because the original quadrant *pwq* has been enlarged more in a *vertical* direction than it has in a *horizontal* direction. Hence it is necessary to select points as at *w* on the original moulding, and draw vertical and horizontal projectors from this point to the respective measuring lines *jl* and J′K′ as shown at *ww′*. Then by means of rays P*w′*, etc., and horizontal and vertical projectors from JM and JN respectively, find W, the point on the *enlarged* curve which corresponds with *w* on the *original* curve. Determine points as at T on the curve RTS in the same way. Then JNO*p*′QRSM is the enlargement required.

CHAPTER VI

THE CIRCLE

Definitions.—Fig. 91 illustrates the various terms used in the following chapter in dealing with problems connected with the circle. Definitions are omitted in order to economise space.

Circles to pass through Two Given Points.

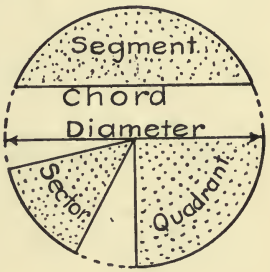

FIG. 91.

PROBLEM.—To determine the *locus* (or path) of centres of all circles which will pass through two given points A and B, Fig. 92. Join AB and bisect it by the line CD. Then AB is a *common chord* of all arcs of circles which pass through A and B. The line CD which bisects this chord at right angles is the *locus* of all points equidistant from A and B. Hence CD is the locus of the centres of *all* circles which can be described so as to pass through A and B. Two such circles are shown, but there is no limit to the number of circles of varying radii which might be drawn so as to pass through A and B.

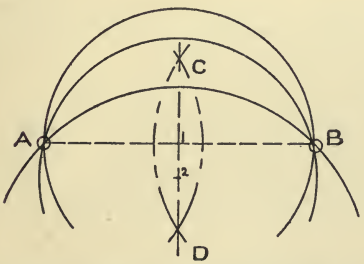

FIG. 92.—Locus of centres (CD) of all circles to pass through two points A and B.

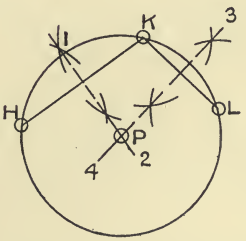

FIG. 93.—Centre of circle to pass through three given points, H, K, and L.

Circle to pass through Three Given Points.

PROBLEM.—To determine the centre of the circle which shall pass through three given points H, K, and L, Fig. 93.

The reasoning here is based on the solution of the previous problem. Join HK and draw the bisector 1,2. Then the line 1,2 is the *locus of centres* of *all* circles which pass through H and K. Join KL and draw the bisector 3,4. Then 3,4 is the *locus of centres* of *all* circles which pass through K and L. It follows therefore that P, which is the intersection of—and therefore common to—the lines 1,2 and 3,4, is the only point which would serve as the centre of a circle to pass through H, K, and L.

To determine the Centre of a Given Circle.

PROBLEM.—Given a printed circle, the centre of which is *assumed to be unknown*, to determine its centre: (*a*) using compasses, and (*b*) using only a parallel straight-edge and pencil.

(*a*) Select any three points P, R, and S on the circumference of the circle, (*a*) Fig. 94. With P and R as centres, and radius rather more

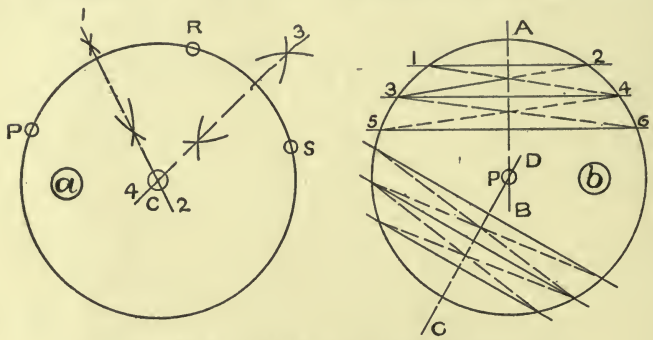

FIG. 94.—Method of finding the centre of a given circle.

than one-half PR, describe intersecting arcs and draw the diameter 1,2. Proceed in a similar manner to draw the bisector 3,4. These bisectors intersect in C which is the centre of the given circle.

(*b*) Using a parallel straight-edge, draw any three parallel chords, 1,2; 3,4; 5,6 ((*b*) Fig. 94). Draw the diagonal lines 2,3; 1,4; 3,6; 4,5. Through the intersections of these diagonals draw AB, which is one locus of the centre of the given circle. By a similar construction determine CD, which is another locus of the centre of the given circle. Then the point P, which is the *point of intersection* of these two loci, must be the required centre.

Setting out Arc of Large Circle.

PROBLEM.—To draw part of a circle of very large radius without making use of its centre, *i.e.* without using a radius rod.

In practical work, such as setting out the ribs of "*centres*" for arches, etc., a long rod (radius rod) is often used for *striking* the required curve, a bradawl being inserted as a centre upon which to turn the rod. It sometimes happens, however, that the rod required for striking a very flat arc of a circle is so long as to be unwieldy, or the centre from which the curve would have to be struck is so far away as to render its use impracticable. In such cases the following is a useful method.

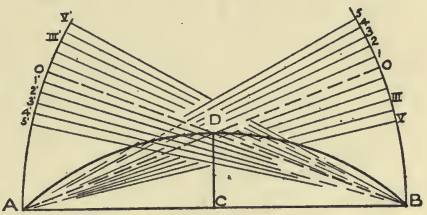

FIG. 95.—Setting out an arc of a large circle without using a radius rod.

Suppose AB, Fig. 95, to be the chord of the required arc, and CD its rise. With centres A and B respectively and radius AB, describe arcs AV', B5. Join AD and produce to meet the arc B5 at O. Join BD and produce to meet the arc AV' at O. Set off on the arcs AV and B5 respectively any number of equal distances above and below O. Join A to the points 1, 2, 3, etc., above O on the *right-hand* side and join B to the points 1', 2', 3', etc., on the *left-hand* side. The intersections of these lines will give successive points, through which the left-hand portion of the arc ADB may be drawn. In a similar manner, by joining A to points *below* O on the *right* and joining B to points *above* O on the *left*, the right-hand portion DB of the arc ADB may be drawn.

Method of continuing an Arc of a Circle.—Given any arc of a circle ABC, the centre of which is inaccessible, to continue the curve.

FIG. 96.—Continuing an arc of a circle.

Select any three points A, B, and C, Fig. 96. Join AB, BC. From any point D with radius = AB, mark off DE = AB. Set out the angle DEF = angle ABC and make EF = BC. Then F is another point on the circle of which ABC is an arc. Other points may be determined in the same manner and the curve continued beyond C by drawing a fair curve through these points.

Angle Properties of Circles.

Theorem.—**The sum of the angles in a circle is constant, and is always 360°.** Thus, if any number of radii of a circle, Fig. 97, are drawn, the *sum of the angles* in the sectors *a*, *b*, *c*, *d*, and *e* is 360°. This total is the same however many radii are drawn.

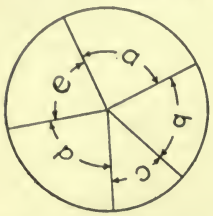

FIG. 97.—Angle properties of circle: total of angles *a*, *b*, *c*, etc. = 360°.

Theorem.—**The angle at the centre of a circle is always double that at the circumference.** Thus, in Fig. 98, if a chord PS is drawn, and the ends of this chord are joined to C (the centre of the circle), and to R (a point on the circumference), the angle PCS is **twice as large** as the angle PRS.

To set out an Arc by means of Intersecting Lines.—The following method of setting out an arc of a circle without reference to the centre of the circle is sometimes useful when it is impracticable to use a radius rod for describing the required arc. PR, Fig. 99, is the chord, and D*a* is the rise of the arc. Produce D*a* to E, making DE double the length of D*a*. Draw

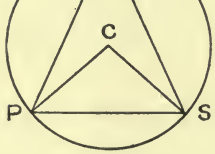

FIG. 98.—Relation of angle at the centre of a circle to that at the circumference.

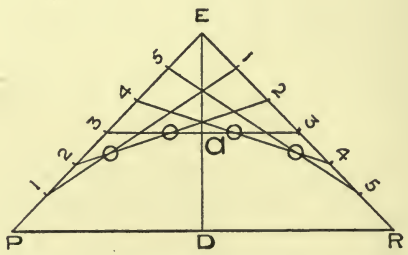

FIG. 99.—Setting out an arc by means of intersecting lines.

lines PE, RE, and divide into any convenient number of equal parts, in this case six. Number the points of division as shown, commencing to number PE near the end P and ER near the end E. Join the point 1 on PE to the point 1 on ER; 2 on PE to 2 on ER and so on. The points of intersection of the line 1,1; and 2,2; is a point through which the arc must pass. The intersection of the line 2,2; and 3,3; of 3,3; and 4,4; etc., are further points on the curve. These points are indicated by small circles on the diagram, but the arc is omitted in order to secure greater clearness of line.

Theorem.—**All angles in the same segment of a circle are equal** ((*b*) Fig. 100). Thus, if P and S, the ends of the chord PS are joined to *any* points R, R′, and R″ on the circumference, the angles PRS, PR′S, and PR″S are equal. This angle is constant *for all triangles which have* PS *as base* and have their *vertices on the circle* PRS.

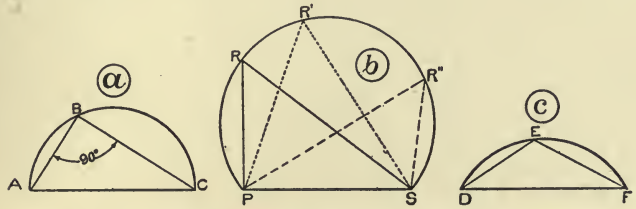

FIG. 100.—Segments of circles and the angles they contain.
(*a*) Semicircle (90°). (*b*) Greater than semicircle (less than 90°).
(*c*) Less than semicircle (greater than 90°).

Theorem.—**The angle in a semicircle is a right angle** ((*a*) Fig. 100). A semicircle is a segment of a circle of a definite size, *i.e.* half the circle. Thus, by the foregoing theorem, all angles in a semicircle are equal, and the *magnitude of the angle is constant*, viz. 90°.

Where the segment is **greater** than a semicircle ((*b*) Fig. 100), the angle of the segment PRS is **less** than 90°. Where the segment is **less** than a semicircle ((*c*) Fig. 100), the angle of the segment DEF is **greater** than 90°.

Practical Application: Setting out Rib for Centre.—A method of setting out part of a circle without using compasses or radius rod is shown in Fig. 101.

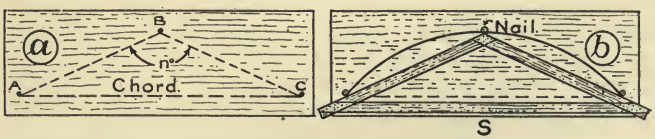

FIG. 101.—Angle property of circles applied to setting out of "rib" for a "centre."

A very simple and practical application of the angle properties of a segment of a circle is illustrated in (*a*) and (*b*). A very flat curve is required for shaping the edge of a *rib* for a *centre* for a large arch. The curve is to pass through the three points A, B, and C ((*a*) Fig. 101). It is clear, therefore that when the curve is drawn, as shown in (*b*), Fig. 101 this curve with the

chord line AC makes a *segment of a circle*. Hence, *the angle* ABC *would be constant* for all points on the curve. This fact is made use of as follows: Three strips of wood are nailed and braced together as shown at (*b*), Fig. 101, in such a way that the brace S, when nailed in position, keeps the other two strips at an angle equal to ABC in (*a*).

In practice three French nails are driven in at A, B, and C, and the strips nailed together in such a way that the framework when complete fits against the nails (*b*), Fig. 101. The nail B nearest the top is then removed. To draw the curve, the framework is held tightly against the two remaining nails A and C, and as it is moved to the left or right, a pencil is held, or fixed, at the corner B of the framework, and this traces the required curve as the framework is moved either way.

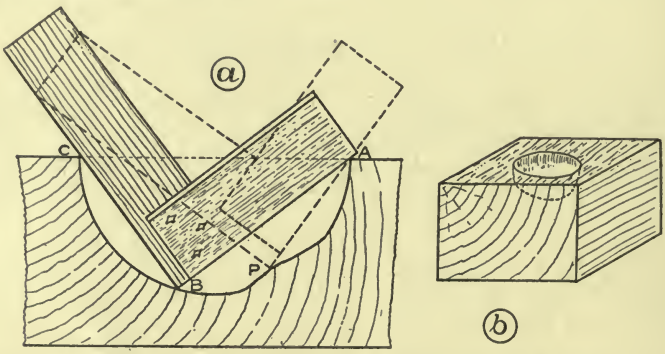

FIG. 102.—Try square (*a*) used for testing the correctness of a hemispherical sinking in a wooden till (*b*).

Try-Square as a Means of testing Hemispherical Bowl.—Fig. 102 (*a*) shows another practical application of the foregoing principle. A hardwood block (*b*) to be used for a till, is to have a hemispherical sinking made in it. (*a*) represents a cross section of such a block, in which the sinking is partly finished. In order to test whether the gouging is deep enough, a try-square is used as shown. If the depth of gouging is correct, the *corner* B of the blade of the try-square will *touch the bottom* of the sinking, while the other end of the blade rests at C and the stock rests at A. If any part of the sinking is not deep enough, as at P, the stock and blade of the try-square *will not rest* at A and C when the corner P rests upon the bottom of the sinking, as indicated by the dotted lines.

Application of Angle Properties of Circles to Outlines of Arches.—The *intrados*, or **interior** edge, of **arches** in building

construction work, other than **Dutch arches**, is usually a segment of a circle. In ordinary house construction the outline of the arch is usually a very *flat* segment, i.e. *a segment smaller than a semicircle.* The outline of the **Roman arch**—one of the earliest of which we have examples in this country—is *semicircular*, while a **Moorish arch** is usually a segment of a circle *greater than a semicircle.* (*See Chap. XI., Brickwork and Masonry Arches.*) The problems which follow should be taken as the basis of the geometrical construction necessary in that chapter.

Construction of Segments of Circles to contain Certain Specified Angles.

PROBLEM.—A **Moorish arch** ((*a*) Fig. 103) is required for a span of 10 ft. The angle in the segment is to be 40°. Draw the outline of the arch to a scale of $\frac{1}{4}$ in. = 1 ft.

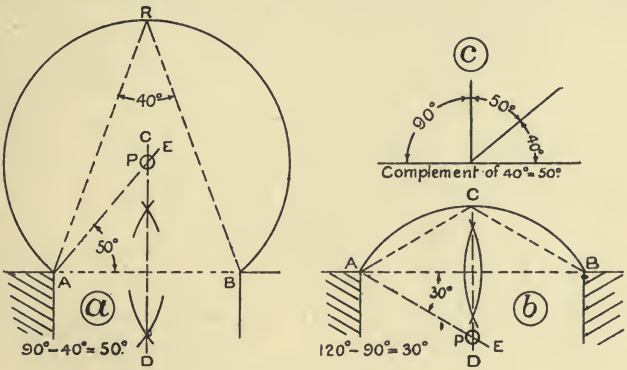

FIG. 103.—Method of setting out arches.
(*a*) Moorish. (*b*) Segmental.

Draw AB = $2\frac{1}{2}$ ins. Bisect it, and draw CD, the bisector, at right angles to AB. Then the centre of the segment will occur on CD ; *i.e.* CD is a locus (or path) along which the centre of the segment must occur. The segment is to contain 40°. Hence it will be a segment *greater* than a semicircle, and its centre must occur somewhere *above* the springing line AB. At A set out AE so that the angle at A is the **complement** of the angle which the segment is to contain. (*See inset diagram* (*c*).)

The *complement* of 40° is 50° (90° − 40°). Hence set out at A the line AE, making 50° with AB. This line is another locus of the centre of the segment, and P, the point of intersection of the two loci CD and AE, is the centre, and PE is the radius, for describing the segment ARB.

PROBLEM.—A flat **Segmental Arch** ((*b*) Fig. 103) is required for a span of 12 ft., the angle of the segment to be 120°.

Draw AB = 12 ft., and bisect it as in the last problem. Then CD is a locus of the centre of the segment. The segment is to contain 120°. Therefore it must be a segment *smaller* than a semicircle and the centre for striking it must occur *below* the springing line AB. To obtain another locus of the centre, set out at A the line AE, making an angle of 120° − 90° = 30° with AB. Then P, *the intersection of the two loci* CD and AE, is the *centre of the required segment*, of which PA (or PB) is the radius.

From the foregoing it will be observed that :—

(*a*) *For segments to contain less than* 90° the **complementary** angle of that required in the segment is used.

(*b*) *For segments to contain more than* 90°, **subtract** 90° **from the angle** which the segment is to contain.

Circumference and Area of Circle.—The circumference of the wheel, Fig. 104, is the *length* of bar iron which must be cut

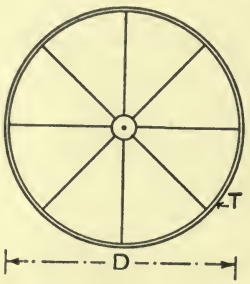

FIG. 104.—Finding the circumference of a circular wheel.

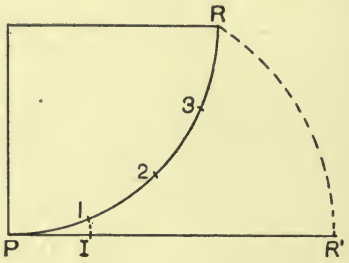

FIG. 105.—Rankine's method of finding the length of an arc of a circle. Straight line PR' = PR.

off so that when bent to shape it will make a tyre for the wheel. Given the *diameter* of the wheel D, the *circumference* is found by multiplying D × $3\frac{1}{7}$ (or 3·142 for more accurate work), *i.e.* **Circumference = D × $3\frac{1}{7}$ or 3·142.**

Meaning of π.—This factor, $3\frac{1}{7}$ or 3·142, is usually denoted by the Greek letter π, Pi ; so that **circumference = D × π.**

EXAMPLE.—A cart wheel is 3 ft. 6 ins. in diameter. What length of iron is required for making a tyre for this wheel, making no allowance for lap for welding?

3 ft. 6 ins. = 3·5 ft. Hence circumference = (3·5 × π) = 10 ft. 11·8 ins.

Method of finding graphically the Length of a Given Arc.—PR, Fig. 105, represents a piece of wire bent so as to make a quadrant of a circle. The length of this piece when straightened out could be found by calculation, taking one-fourth of the total circumference. The following method is a *graphical* method,

i.e. a method which employs *lines* rather than *calculations*. It is usually known as *Rankine's approximation.* If drawn carefully, however, it gives a result which is accurate to the first decimal place.

Divide the quadrant PR into four equal parts at 1, 2, 3. With P as centre and radius P1 draw an arc to intersect PR′ in I. With I as centre and radius IR, draw the arc RR′. Then PR′ is the length of wire which must be cut off a *straight length* sufficiently long to bend around *to make the quadrant* PR.

Area of Circle.—The foregoing illustrations should have made it clear that **Circumference is a Linear Measurement** —a *length.*

The **Area** of a Circle refers to the **surface** included within the circle, *i.e.* it is measurement in *two dimensions*: **Length × Breadth.**

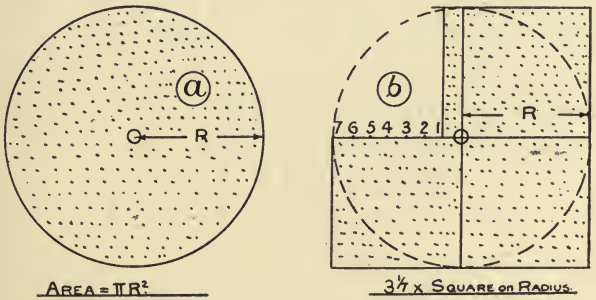

FIG. 106.—Area of circular disc.

(*a*) Rule for determining area. (*b*) Experimental proof of rule for area of circle.

Experiment.—Cut out of cardboard or sheet metal, two pieces (*a*) and (*b*), Fig. 106, the piece (*a*) being a circular disc of, say, 5 ins. diameter, and the piece (*b*) a straight-edged piece, the longest edges being 5 ins. The radius R of the disc (*a*) is 2·5 ins. The piece (*b*) is to be of such a size and shape as to be equal to three squares of 2·5 ins. side + $\frac{1}{7}$ of another such square as indicated by the shaded area in (*b*). Place the piece (*a*) in one pan of a scales and (*b*) in the other pan. Note that **they exactly balance each other,** *i.e.* they are of **equal weight.** Since they are **both cut from the same thickness** (or gauge) material, *the fact that their weights are equal* proves that their **areas are equal.**

From this we deduce the following rule for finding the area of the surface of any circular piece of material:—

Area = $R^2 \times \pi$, where R = radius and $\pi = 3\frac{1}{7}$ or 3·142. Thus, in the case of the disc (*a*), the radius, R, is 2·5 ins. Then

R^2 (radius squared, or the square on the radius) = 2·5 × 2·5 = 6·25

and $R^2 \times \pi = 6{\cdot}25 \times 3\frac{1}{7}$ or $3{\cdot}142$
$= 19{\cdot}64$ sq. ft.

Area of an Annulus of a Circle.—Fig. 107 represents the cross section of a hollow iron column, the exterior diameter of which is 9 ins. and interior diameter $7\frac{1}{4}$ ins. Find the area of the cross-sectional surface.

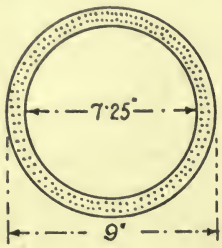

FIG. 107.—Finding the area of an "annulus" or ring.

The shaded area enclosed within the two circles representing the exterior and interior diameter respectively is called an *annulus*. The area of this annulus, which represents the cross-sectional surface, may be found as follows: (*a*) Find the area enclosed within the larger circle (9 ins. diameter) by the method shown in the previous problem. (*b*) Find the area enclosed within the smaller circle ($7\frac{1}{4}$ ins. diameter) in the same way. Subtract area (*b*) from area (*a*). The remainder is the area of the annulus represented by the shaded part of the diagram.

The working would appear as follows:—

(*a*) Exterior diameter $= 9$ ins., $\therefore$ radius $= 4{\cdot}5$
$\therefore$ Area (*a*) $= (4{\cdot}5^2 \times 3{\cdot}142)$ sq. ins.
$= (20{\cdot}25 \times 3{\cdot}142)$ sq. ins.
$= \underline{63{\cdot}625}$ sq. ins.

(*b*) Interior diameter $= 7\frac{1}{4}$ ins., $\therefore$ radius $= 3{\cdot}625$ ins.
$\therefore$ Area $= (3{\cdot}625^2 \times 3{\cdot}142)$ sq. ins.
$= (13{\cdot}1 \times 3{\cdot}142)$ sq. ins.
$= \underline{41{\cdot}287}$ sq. ins.

Subtract (*b*) from (*a*) 63·625
41·287
Remainder $= \underline{22{\cdot}338}$ sq. ins.

$\therefore$ Area of annulus, *i.e.* shaded cross-sectional area, $= 22{\cdot}338$ sq. ins.

Again, taking R = radius of outer circle.
$r =$,, inner ,,
A = area of shaded portion (annulus).

Then, from the working above, we may deduce the following formula:—

$$A = \pi R^2 - \pi r^2$$

Those with a knowledge of mathematics will see that this statement may be arranged thus:—

$$A = \pi(R^2 - r^2) \text{ or } A = \pi(R + r)(R - r)$$

Or, expressed in words: A = 3·142 *times the sum of the external and internal radii* (R + *r*) *multiplied by the difference of the radii* (R − *r*).

The working would then appear as follows :—

Sum of radii (R + *r*) = 4·5 + 3·625 = 8·125
Diff. „ (R − *r*) = 4·5 − 3·625 = ·875
Then the area, A = π × 8·125 × ·875
= 3·142 × 8·125 × ·875
Answer = 22·337 sq. ins.

Length of Arc in a Sector of a Circle.—ABC, Fig. 108, represents a plot of ground, the plan of which is a **sector** of a circle. A path DECB, 5 ft. wide, is to be made around its outer edge. How many 9-in. edging tiles would be required for lining both sides of the path ?

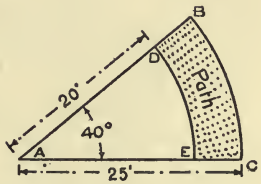

FIG. 108.—Finding the length of arc BC.

There are 360° in a circle. This sector contains 40°. ∴ length of arc DE = $\frac{40}{360}$ or $\frac{1}{9}$ *of circumference of circle of which* DE *is a part.*

Diameter of inner circle = 20 × 2 = 40 ft.
∴ Circumference „ = 40 × 3·142 = 125·68 ft.
∴ Arc DE = $\frac{1}{9}$ of 125·68 ft. = 13·96 ft.

Diameter of outer circle = 25 × 2 = 50 ft.
∴ Circumference „ „ = 50 × 3·142 = 157·1 ft.
∴ Arc BC = $\frac{40}{360}$ or $\frac{1}{9}$ of 157·1 ft. = 17·45 ft.
∴ Sum of lengths of arcs DE and BC = (13·96 + 17·45) ft.
= 31·41 ft.

Tiles are 9 ins. or $\frac{3}{4}$ ft. long.
∴ No. of tiles required = 31·41 × $\frac{4}{3}$ = 41·88
Say, 42 tiles.

Use of Two-foot Rule for determining Circumference of Circle.—A two-foot rule may be used, as shown in Fig. 109, for determining the circumference of any given circle with sufficient accuracy for workshop purposes. A distance of $3\frac{13}{16}$ ins. is measured from the joint end of each arm of the rule and a permanent mark made on each arm, as shown at D. If a letter D is stamped or scratched on the rule near this mark it will serve as a reminder that the **Diameter** of the given circle is to be used. Thus, if this *diameter* is $1\frac{1}{4}$ in. the rule is opened out as shown until the *distance between the* D *marks* on the two arms

of the rule is $1\frac{1}{4}$ in. *The* **distance apart of the ends** of the respective arms of the rule when set thus is $3\frac{15}{16}$ in., which is the **circumference** of the given circle.

Example.—The diameter of a pipe is 5 *ins., find its circumference.*

Open the rule until the distance apart of the D marks on the respective arms of the rule is 5 ins., and, with the rule set thus,

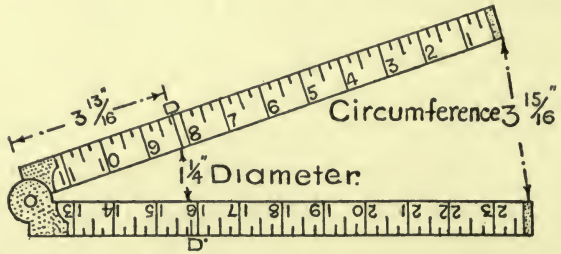

FIG. 109.—Method of using a two-foot rule for determining the circumference of a circle.

measure with another ruler the distance apart of the *extreme ends* of the rule; in this case 15·7 ins. This is the *circumference* of the given pipe. Experiment in the same way with the following cases in order to make the method familiar.

Distance apart of D marks. (Diameter, in inches.)	Distance between ends of rule. (Circumference, in inches.)
1	3·14
2	6·28
3	9·42
4	12·56

If the *circumference* is known and the *diameter* has to be determined, *reverse* the foregoing process, *i.e.* set the rule so that the *distance apart of the ends of the rule* is equal to the given *circumference*. While thus set measure the **distance apart of the D marks** on the respective arms of the rule. This distance represents the **Diameter** of the circle.

Sectorial Area of an Annulus.—The shaded area ABED, Fig. 110, represents a concrete path enclosed by two circular arcs, AD and BE, which represent the edging of the path. Find the *area of the surface* of the concrete, *i.e.* the **sectorial area** of the annulus ABED.

$$\text{Area of surface of path} = \frac{(\text{AD} + \text{BE})h}{2}$$

i.e. half the sum of the lengths of the inner and outer arcs AD and BE multiplied by the distance (h) between the arcs. The lengths of the arcs may be found as follows :—

Short radius = 15 ft. ∴ Diameter = 30 ft.

Long „ = 18·5 ft. ∴ Diameter = 37 ft

∴ circum. of inner circle = (30 × 3·142) ft. = 94·26 ft.

and circum. of outer circle = (37 × 3·142) ft. = 116·254 ft.

Angle BPD = 60°

∴ AD is $\frac{60}{360}$ or $\frac{1}{6}$ of 9·426 = 15·71 ft.

and BE is $\frac{60}{360}$ or $\frac{1}{6}$ of 116·254 = 19·375 ft.

Then area of path

$$= \frac{(15{\cdot}71 + 19{\cdot}375)h}{2} = \frac{35{\cdot}085 \times 3{\cdot}5}{2}$$

$$= \frac{122{\cdot}79}{2} = 61{\cdot}39 \text{ sq. ft.}$$

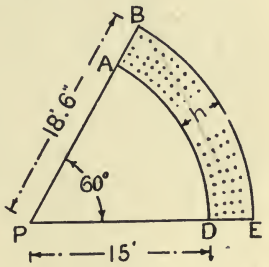

FIG. 110.—Method of finding the area of a sectorial annulus as shown shaded at ABED.

Relative Carrying Capacity of Two Cylindrical Pipes.—(*a*) and (*b*), Fig. 111, are cross sections of two drain pipes of 6 ins. and 12 ins. diameter respectively. What is their relative carrying capacity ?

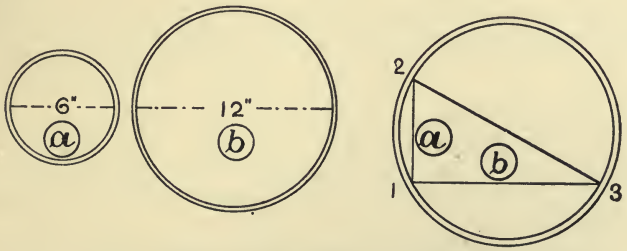

FIG. 111.—Relative carrying capacity of two pipes, (*a*) and (*b*).

Areas of circles vary as the squares on their respective diameters. If the area of cross section of the two pipes (*a*) and (*b*) varied *directly* in relation to their respective diameters, the pipe (*b*) would have *twice* the carrying capacity of (*a*). If, however, the *squares* on the diameters are compared, the ratio, or comparison, would appear thus, $6^2 : 12^2$, or 36 : 144 ; $\frac{144}{36} = 4$. Thus (*b*) has **four times the carrying capacity** of (*a*).

Pipe to have Carrying Capacity of Two Given Pipes.—Suppose it is required to do away with the two pipes (*a*) and (*b*), Fig. 111, and substitute a larger pipe which shall *have a carrying capacity equal to that of* (a) *and* (b) *combined.* The diameter of such a pipe may be found graphically, as shown at the right side in Fig. 111. The diameters (6 ins. and 12 ins.) of the two given pipes respectively are used as the base 1,3 and perpendicular 1,2 of a right-angled triangle, and the hypotenuse 2,3 is drawn. The square on 2,3 is **equal to the sum of the squares** on 1,3 and 1,2. Then, since the relative carrying capacity is in proportion to the *squares* on the diameters of the two pipes, it is evident that a pipe of 2,3 diameter will have a carrying capacity equal to that of two pipes (*a*) and (*b*) whose diameters are represented by 1,2 and 1,3 respectively.

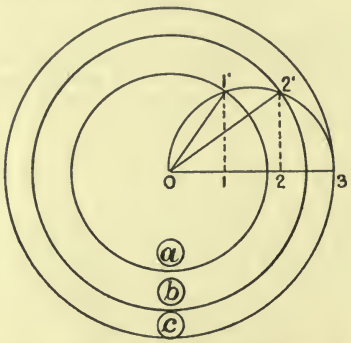

FIG. 112.—Division of a circular plot of ground into three equal areas, (*a*), (*b*), (*c*).

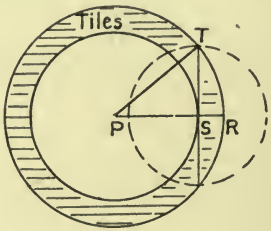

FIG. 113.—Method of determining the largest circular plot which could be covered with tiles taken from the shaded annulus.

Division of a Circle into any Number of Equal Areas by means of Concentric Circles.—The circle, radius O3, Fig. 112, represents a circular piece of ground. Find the position of two fences parallel to outer circle which will divide this ground into three *equal* spaces.

Divide the radius O3 into three *equal* parts at 1 and 2. Describe a semicircle on O3. Draw lines 1,1′; 2,2′; at right angles to O3 to meet the semicircle at 1′ and 2′.

Complete the right-angled triangles O1,1′; O2,2′. Then the *hypotenuse* O1′ is the *radius* for the circle representing one fence, and the *hypotenuse* O2′ is the *radius* for the other circle, and (*a*), (*b*), and (*c*) are the *three equal areas* required.

To find the Area of a Circle Equal to that of an Annulus.—The shaded area, Fig. 113, represents a tiled path bounded by

circles (an annulus). What is the largest circular area that could be covered with the tiles taken up from this path ?

Draw PR to intersect the inner and outer circles at S and R respectively. At S draw ST at right angles to PS. Draw the hypotenuse PT. Then the dotted circle, radius ST, represents a circular space equal in area to the shaded annulus.

Proof: $PT^2 = PS^2 + ST^2$ (right-angled triangle)
$\therefore ST^2 = PT^2 - PS^2$
i.e. ST^2 = difference between areas of outer and inner circles
or ST^2 = area of annulus (tiles).

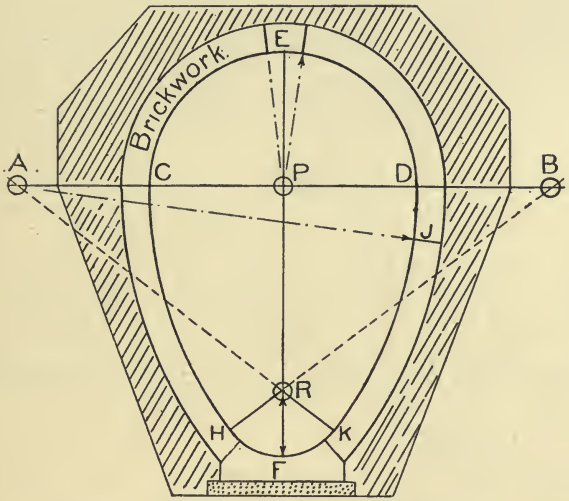

FIG. 114.—Cross section of a pear-shaped sewer.

Cross Section of Sewer.—The drawing, Fig. 114, illustrates the method of using arcs of circles for setting out the cross section of a pear-shaped brick sewer, such as is used in main drainage schemes. The height EF must be sufficient to allow a man of medium height to walk along. The distance PF, below the springing line CD, is usually made two-thirds of the height EF ; *i.e.* PF = 2PE.

Draw the lines CD and EF at right angles. With P as centre and radius PD to scale = $\frac{1}{2}$ the fullest width of sewer, describe the semicircle CED. Make PF = 2PE. Set out a distance FR along PF such that FR = $\frac{1}{2}$PD. Produce CD in both directions, and make DB = PD and AC = CP. From A and B respectively

draw the lines AK and BH to pass through R. With centres A and B and radius = AD or BC, describe the arcs DK and CH. With centre R and radius RF or RH, describe the arc HFK. The sewer is usually formed by building two rings of brick on edge, as indicated by the space marked "brickwork." The shaded area represents the earth. The space marked F represents the stoneware *invert*, resting upon a bed of concrete. The joint lines in the bricks forming the top part of the sewer radiate from P, as shown at E, while those between C and H radiate from B, and those between D and K radiate from A. The sloping slides of the invert, below H and K respectively, radiate from R. The joint lines in the brickwork at this part of the sewer, *i.e.* below H and K, also radiate from R.

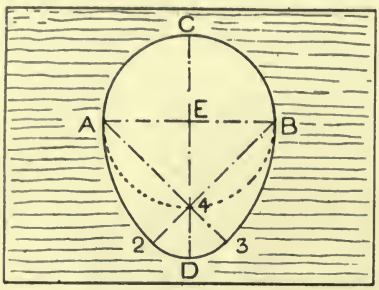

FIG. 115.—Setting out a W.C. seat.

Setting out W.C. Seat.—Fig. 115 affords a further illustration of the use of arcs of circles in setting out a W.C. seat.

Draw two lines AB and CD at right angles, and intersecting at E. With E as centre and radius, say, 4½ ins., describe a circle, ACB4. Through 4 draw A3 and B2. With centres A and B and radius AB, describe the arcs B3 and A2 respectively. With centre 4 and radius 4,2, describe the arc 2D3.

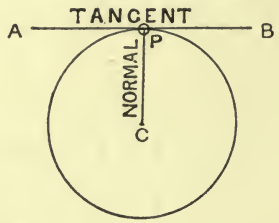

FIG. 116.—Circles (tangent and normal).

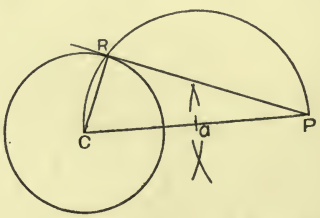

FIG. 117.—Method of drawing a tangent (RP) to a circle from a point (P) outside the circle.

Circles and Straight Lines in contact—Tangents and Normals.—A **tangent to a circle** AB, Fig. 116, is **a straight line which touches the circumference** of the circle at a point P (point of contact) and is **at right angles** to the radius CP, drawn from the point of contact to the centre of the circle. It is

clear, therefore, that there can only be *one tangent* for any point on the circle.

A **normal** to a circle is a **straight line drawn** from the **point of contact**, at **right angles to the Tangent** at that point. Thus CP, Fig. 116, is the *normal* for the given tangent AB. *All normals of a circle pass through the centre of the circle.*

PROBLEM.—**From a point outside a circle to draw a tangent to touch the circle.**

This would appear at first sight a simple thing to do. Using a straight-edge or ruler, a line could be made to pass through P, Fig. 117, and *appear* to touch the given circle centre C. Unless the correct point of contact is found, however, such a line is more likely to be a *cutter* than a *toucher*. The point of contact R, is obtained as follows: Join P to C, the centre of the circle, and on PC describe a semicircle CRP. Join CR and PR. *The angle in a semicircle is always* 90°. Then the angle CRP is a **right angle**, and since CR is the **normal** at the point R, PR, which is at right angles to CR, must be the correct **tangent** at R.

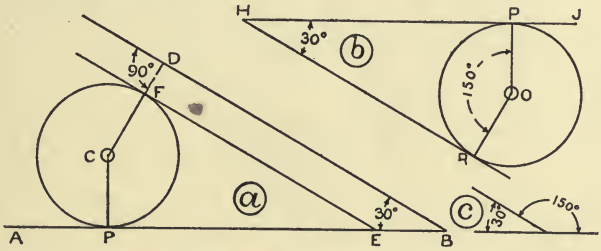

FIG. 118.—Drawing two tangents to a circle to include a given circle.

PROBLEM.—Given any circle, centre C ((*a*) Fig. 118), **to draw two straight lines tangent to the given circle** and to **include a given angle**, say 30°.

Two methods (*a*) and (*b*) are shown.

(*a*) Draw any normal CP, and draw a tangent AB at right angles to CP and passing through P. This is one of the tangents required. At any point B in AB set out a line BD making 30° with AB. From C draw CD at right angles to BD. Then the second tangent EF, when drawn, will be parallel to BD, and therefore square with CD. Hence, if EF is drawn parallel to BD and passing through F this will be the second tangent, such that it makes an angle of 30° with the first tangent AB.

(*b*) This is a quicker method than (*a*).

Draw HP as before at right angles to any normal OP. Set out at O the angle ROP = 150°, *i.e.* 180° − 30° or the **supplement** of 30°, the angle required between the tangents. (See inset diagram (*c*) for explanation of the meaning of the term *supplementary angle.*)

Draw HR at right angles to OR. Then HJ and HR are the two tangents required.

Practical application of Tangents and Normals to Joint Lines in Arches.—The joint lines between the *voussoirs* in a segmental arch, Fig. 119, are *normal* lines, *i.e.* they are parts of the radii of the circle of which the outline of the arch and the span are a segment.

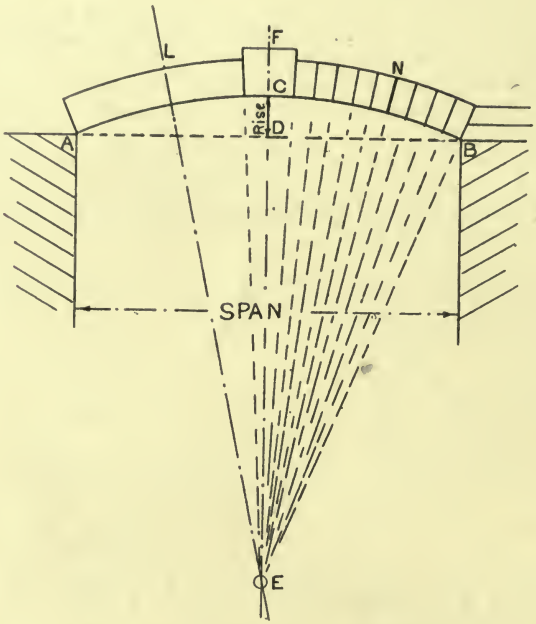

FIG. 119.—Joint lines in a segmental arch.

PROBLEM.—The *span* (AB, Fig. 119) of a segmental arch is 10 ft. and the *rise* CD = 1 ft. Draw an elevation of the arch and show the joint lines between the voussoirs.

Make AB = 10 ft. to scale, and at the centre of the span D set up CD = 1 ft. Produce CD in both directions to E and F. Then EF, which bisects the length of span AB, contains the centre from which to describe the outline of the arch. Join AC and draw its bisector LE. Then E, the point of intersection of LE and FE, is the centre from which to strike the arc ACB. Since the joint lines are to be *normal lines* to this arc they may be determined by spacing out the soffit line ACB into as many equal spaces as required, and drawing lines, as at N, etc., to radiate from E, the centre from which the arc ACB is struck.

To draw a Tangent and Normal of an Arc of a Circle without making use of the Centre of the Circle.—Fig. 120 represents a *rib* of a wooden *centre*. The centre of the circle of which APB is an arc is inaccessible. Draw a tangent and normal at P.

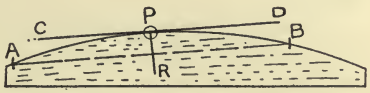

FIG. 120.—Tangent and normal to an arc of a circle.

With P as centre, and any radius, draw arcs to intersect APB at A and B respectively. Join AB. Through P draw CD parallel to AB. Then CD is the required tangent, and PR, which is at right angles to CD, is the required normal.

Fig. 121 illustrates the method of drawing a tangent and normal when the point of contact, P, occurs *near the end* of the arc PRS.

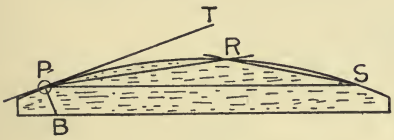

FIG. 121.—Tangent to an arc.

Select any two points R and S on the arc and draw the chords PR and RS. Join PS. At P set out PT making an angle with PR = angle RSP. Then PT is the tangent required at P, and PB at right angles to PT is the required normal.

Laying out Curves for Paths, etc.

PROBLEM.—To set out a *reverse curve*. AB and CD, Fig. 122, are two straight rails which are to be connected by a double curved rail meeting AB and CD at B and C respectively.

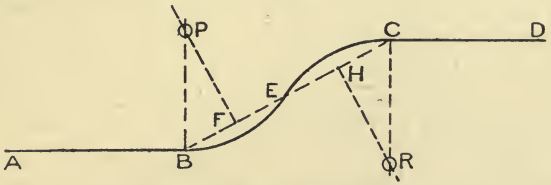

FIG. 122.—Setting out a reverse curve.

Join BC and bisect at E. Then BE and EC are the chords for the two halves of the reverse curve. At B and C set out BP and CR at right angles to AB and CD respectively. Bisect BE at F, and EC at H, and set out FP and HR at right angles to the double chord BC. Then the points of intersection, P and R, are respectively the centres from which to strike the two halves of the curve, the radius of which is RC or PB.

Great care is necessary in order to obtain a clean join at E. Practise the same curve with Indian ink, using a bow pen.

To draw a Reverse Curve composed of Arcs of different Radii.

PROBLEM.—Two straight paths AB and CD, Fig. 123, are to be connected by a path of *double curvature*, the radius of each of the two parts of the curve being different.

The construction does not differ materially from that of the last problem, except that the chord line BC instead of being divided into *two* equal parts, is divided *unequally* at E into two lengths, according to *the ratio which the curves are desired to have.*

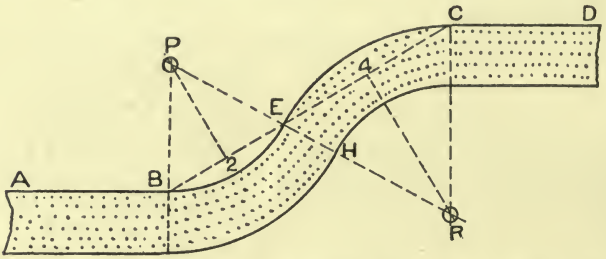

FIG. 123.—Setting out a reverse curve made up of arcs of different radii.

Draw BP at right angles to AB and CR at right angles to CD. Bisect BE at 2 and EC at 4, and draw P2 and R4 at right angles to BE and EC respectively. Then the points of intersection P and R are the centres from which to strike the two parts of the curve BEC, radii PB and RC respectively. To draw lines representing the other edge of the path join PR and set out EH = width of path required. With centre R and radius RH describe the right-hand portion of the curve and with centre P and radius PH, draw the left-hand portion of curve. Complete by drawing straight lines parallel to AB and CD respectively.

Points of Contraflexure.—The two points E and H in the foregoing problem at which *the character of the curve changes—i.e.* from *concave* to *convex*, or vice versa—are called **points of contraflexure.**

Method of setting out the Curve for rounding off an Angle of a Curb.—(*a*) Fig. 124 shows the method of setting out the curve when the two straight parts of the curb meet at an *obtuse* angle, while (*b*) shows the method of dealing with an *acute* angle intersection.

Draw AB and BC, (*a*) Fig. 124, in line with the outer edges of the two straight lengths of curb. Mark off BD and BE according to the amount of rounding required. Draw DF and

EF at right angles respectively to AB and BC and intersecting at F. With centre F and radius FD describe the arc DE. To complete the inner edge of the curb set out D5 and E7 = thickness of timber used in curb. Draw 2,5 and 7,4 parallel to AD and EC respectively. Join 5,7 if a straight back is required here

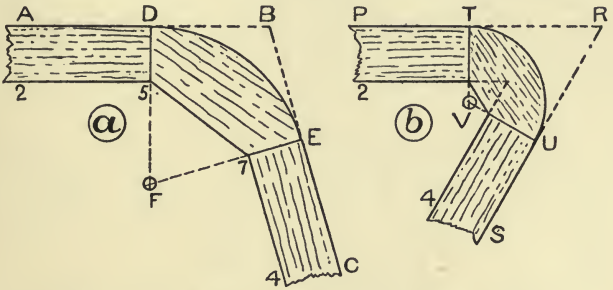

FIG. 124.—Setting out curve at intersection of two wooden curves.

for purposes of strength. If a curved back is required, describe an arc with centre F and radius F5 to connect 5 and 7.

The construction in (*b*) does not differ materially from that in (*a*). PT and SU represent the outer edge of the two straight lengths of curb. Produce PT and SU to meet at R. Mark off equal distances RT and RU according to judgment. Draw TV and UV square with the straight edges of the curb. Then V the point of intersection of TV and UV is the centre, and VT the radius for rounding off the corner. The inner edges are completed as described in (*a*).

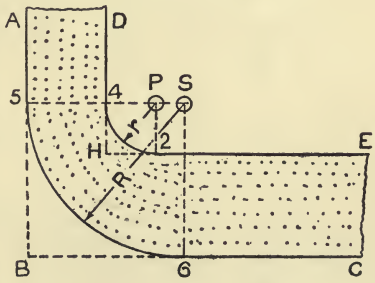

FIG. 125.—Setting out the intersection of two paths of different widths.

Method of setting out the Intersection of Two Paths of Different Widths.—Fig. 125 represents two paths AD and EC of *unequal* width, the intersection of which is to be rounded off.

Draw straight lines to represent the sides of the two straight paths and intersecting at H and B respectively. Mark off equal spaces, H4 and H2, according to the amount of rounding required. Through 4 draw 4P at right angles to D4, and through 2 draw 2P square with E2. Then P is the centre and *r* the radius for

rounding off the inner edges of the paths. Produce P4 in each direction and make 5S = 5B. Then S is the *centre* and R the *radius* for rounding off the outer edges of the paths.

Tangents to Unequal Circles.—*Two pulleys* A *and* B ((*a*) Fig. 126), 1 *ft.* 6 *ins. and* 3 *ft. diameter respectively, are fixed at* 6 *ft. centre to centre. An endless belt passes over the pulleys. Draw the elevation of the pulleys and belt. Find the length of the belt and estimate its cost at* 2*s.* 6*d. per ft. run.*

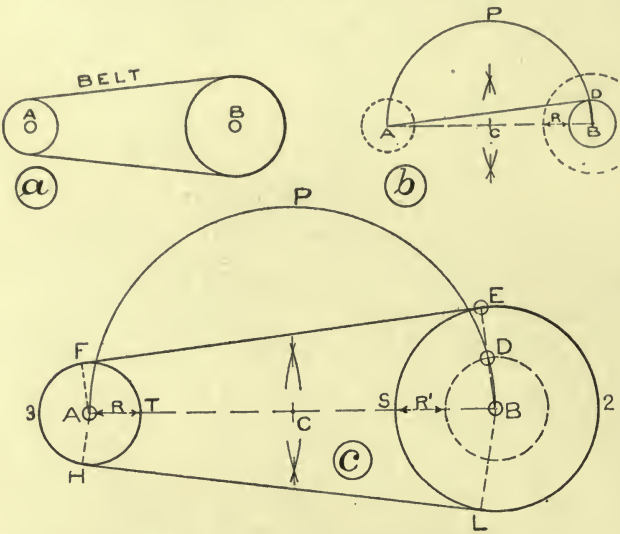

FIG. 126.—(*a* and *b*), tangents to two unequal circles. (*c*) Endless belt to pass over two unequal pulleys, A and B.

A belt such as that illustrated would "sag" slightly between the pulleys. Theoretically, however, the upper and lower parts of the belt may be considered as **tangents** to the **two unequal circles**—the pulleys.

1*st Step*, (*b*) Fig. 126.—Imagine the smaller pulley to *shrink* until it is a *dot*, A. This being so, the smaller pulley has shrunk by a radius R = 9 ins. *Reduce* the diameter of the larger pulley *by an equal amount*, R. It would then appear as shown by the circle, radius BD ((*b*) Fig. 126). Draw the *trial tangent* AD to touch the smaller pulley (which is now a dot, A) and to touch the larger pulley at D. To do this it is necessary to describe a semicircle APB with centre C, midway between A and B. This gives the correct point of contact D, such that the angle ADB is a right angle. Produce the normal BD to meet the outer edge

of the larger pulley at E ((*c*) Fig. 126). This is the point of contact of the tangent (the belt) and the larger pulley. Draw AF parallel to BE. Then F is the point of contact of the tangent and the smaller pulley. Join FE. This is the tangent required and represents the upper portion of the belt. With centre S and radius SE mark off SL. Similarly, with centre T and radius TF mark off TH. Join HL. This represents the lower straight part of the belt.

Length of Belt.—If the diagram has been drawn to scale, the lengths of the straight pieces of belt FE and HL can be measured direct from the drawing (5 ft. 11 ins. approximately). The lengths of the parts of the belt in contact with the pulleys are found thus: Using the protractor, measure carefully the the magnitude of the angle EBL between the normals EB and LB ((*c*) Fig. 126). It measures 163° on the diagram. Then the *re-entrant angle* EBL = 360° − 163° = 197°. The ratio of the length of the arc E2L in the sector EBL2 as compared with the circumference of the whole circle (the larger pulley) is 197 : 360. The working would appear as follows:—

Diameter of larger pulley = 3 ft.

∴ Circumference of larger pulley = $(3 \times \pi)$ ft.

$= (3 \times 3{\cdot}142)$ ft. $= 9{\cdot}426$ ft.

Then 360° : 197° :: 9·426 : Length of arc E2L

$$\frac{9{\cdot}426 \times 197}{360} = \frac{1856{\cdot}92}{360}$$

$= \underline{5{\cdot}15}$ ft.

Similarly for smaller pulley:—

Angle FAH in the sector FAH3 = 163°

Diameter of smaller pulley = 1 ft. 6 ins.

∴ Circumference „ „ $= (1{\cdot}5 \times 3{\cdot}142)$ ft. $= 4{\cdot}71$ ft.

Then 360° : 163° :: 4·71 ft. : Length of arc F3H

$$\frac{4{\cdot}71 \times 163}{360} = \frac{767{\cdot}73}{360} \text{ ft.}$$

$= \underline{2{\cdot}13}$ ft.

Arrange your results neatly in tabular form as shown:—

No. of pieces.	Piece.	Length.	Total.
2	FE and HL	5 ft. 11 ins.	11 ft. 10 ins.
1	E2L	5·15 ft.	5 ft. 1¾ ins.
1	F3H	2·13 ft.	2 ft. 1½ ins.
		TOTAL . .	19 ft. 1¼ ins.

Cost of Belt.—Total length required = 19 ft. 1¼ ins.
∴ 2*s.* 6*d.* × 19·1
Cost = £2 7*s.* 9*d.*

Interior Tangents to Two Unequal Circles.—Two pulleys, Fig. 127, diameter and distance apart as in the last problem, are to be fitted with a belt arranged so as to run over one pulley and under another interiorly, as shown.

Describe the semicircle APB as in the previous problem. Imagine that in this case the larger pulley *grows* instead of *shrinking* as in the previous case. Let the increase of radius R = radius

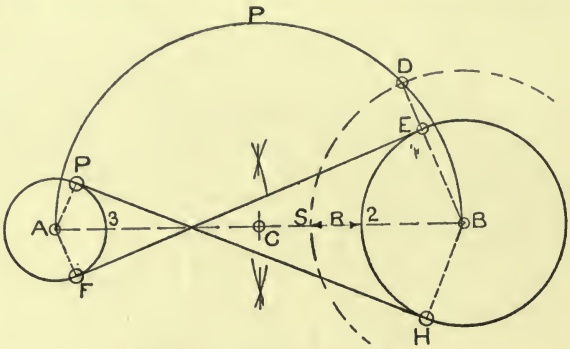

FIG. 127.—Interior tangents to two unequal circles.

AP of the smaller pulley. With B as centre and radius BS describe the arc SD. Join BD. Then BD gives the direction of one of the normals for the interior tangent, and the point E is its point of contact. Draw AF parallel to BE. This is another normal. Join FE. Make 2H = 2E and 3P = 3F. Join PH. Then FE and PH are the interior tangents and represent the straight parts of the belt.

The total length of belt required may be found as shown in previous problem.

Circle to touch Two Given Circles and include them.—The two shaded areas, (*a*) Fig. 128, represent two circular ponds, diameter 60 ft. and 30 ft. respectively. A circular enclosure is to be made to fulfil the following conditions: (1) *Diameter of enclosure* 150 *ft.* (2) *The fence surrounding the enclosure is to touch the circumference of each pond.* (3) *Both ponds are to be included within the enclosure.*

Join the centres A and B in (*a*) and produce each way to cut the circles at 2 and 4. With 2 as centre and radius 75 ft. (half

diameter of enclosure), mark the point E. With 4 as centre and same radius, mark the point D. With A as centre, and radius AE, describe the arc ES. With B as centre and radius BD, describe the arc DS. From the point S draw the lines SA and SB and produce to F and H respectively. Then S is the centre of the enclosure, and F and H are the points at which the fence of the enclosure touches the ponds. The circle representing the fence of the enclosure is struck with S as centre and radius SF or SH.

Circle to touch Two Given Circles and exclude them.—(*b*) Fig. 128 represents a variation of the foregoing problem. Two pulleys, 2 ft. and 1 ft. 3 ins. diameter, centres A and B, are fixed as shown. Another pulley, 3 ft. diameter, centre S, is to be

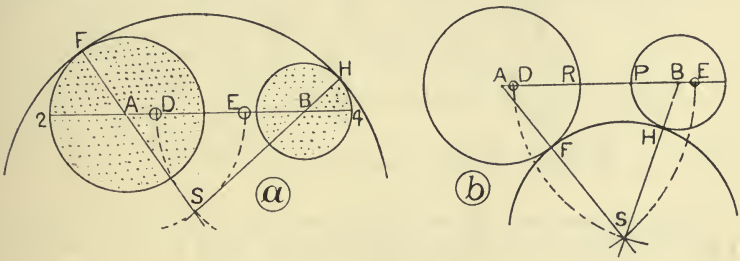

FIG. 128.—Circles in contact.

fixed so that its *outer edge* is *in contact* with those of the *two smaller pulleys*.

In the previous problem the required circle—the fence of the enclosure—had to *touch* and *include* the two given circles. In this case the required circle has to *touch* and *exclude* the two given circles.

From P mark off PD = 1 ft. 6 ins. (the radius of the required pulley), and from R mark off RE = PD = 1 ft. 6 ins. With centre A and radius AE describe the arc ES, and with B as centre and radius BD describe the arc DS. Join S (the intersection of the two arcs) to A and B. Then S is the centre of the required pulley, and F and H are the points of contact of the required circle and the two given circles. Part of the required circle is shown, the radius for it being SF or SH.

To draw a Series of Circles to touch One Another and to touch Two Given Straight Lines.—The sketch (*a*) Fig. 129 represents a piece of wood which is pierced and carved for the purpose of a small easel support. This operation involves the necessity of setting out a series of circles to fulfil the conditions

stated above. The enlarged diagram (*b*) shows the necessary working.

Draw the centre line A5. The centres of all circles which touch the lines PS and RT must be along this centre line. Assuming that A is to be the centre of the first circle, draw a line AB at right angles to RT. With A as centre and AB as radius draw the first circle to touch RT and PS. From D draw D2 at right angles to the centre line A5. With 2 as centre and D2 as radius, draw the arc D4 intersecting RT at 4. From 4 draw 4,5 parallel to AB. Then 5 is the centre, and 5,4 or 5D the radius for the next of the series of circles, and so on.

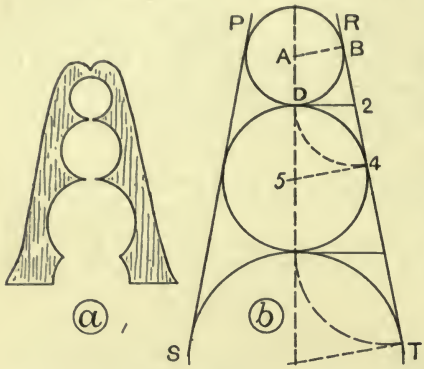

FIG. 129.—Series of circles to touch two straight lines.

Intersection of Curved and Straight Mouldings in Show-case.—(*a*) Fig. 130 is the plan of one corner of a show-case. A *straight* length of moulding, the section of which is shown at D, is to be fixed to the outer front edge of the base of the show-case, as shown at CA. A *curved* length of moulding AK is to be made to intersect the straight length, the mitred joint being at AB.

Case I. If the moulding at AK is made of the *same section* as that at CA, the line of intersection or *mitre* will be a **curved line.**

Draw projectors from the various points on the sectional view of the moulding as 1, 2, 3, 4, 5, and thus obtain lines, as FH, parallel to CA. At any point O on the curved length of moulding draw a line 0,5′, normal to the curve AK, *i.e.* radiating from the centre from which AK is struck. On 0,5′ mark off distances 0,1′, 0,2′, 0,3′, etc., equal to 0,1, 0,2, 0,3, etc., on the sectional view D, and draw arcs, such as 2′H parallel to the outer edge of the moulding AK. The intersections of the straight lines (FH, etc.) and the curved lines (2′H, etc.) will not occur in a straight line between A and B. Use a French curve to draw the line AHB through these points of intersection. This represents the *curved line of intersection* or mitre.

Case II. In (*b*) Fig. 130 a similar case is shown. In order

to do away with the labour of preparing curved surfaces for the mitres, the line of intersection PR is required to be straight. Assuming that the section of the straight length of moulding OP is the same as that shown at D (*a*), a slightly *different section*, S (*b*), will be *required for the curved length* of moulding.

Draw the plan of the front and back edges of the straight and curved lengths OP, PQ. Draw the straight line of intersection PR. From the sectional view D draw line as TV parallel to the front edge of the moulding OP. These lines intersect the mitre line PR at points V, etc. From these points draw lines VN, etc.,

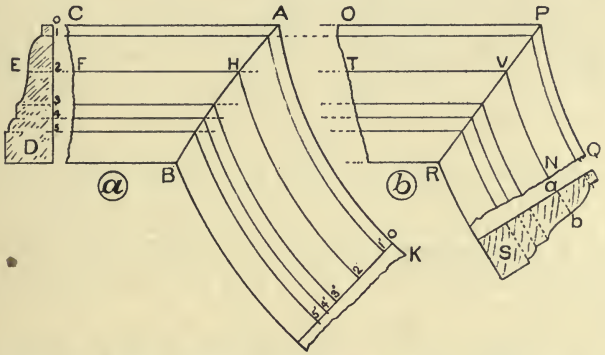

FIG. 130.—Plan of intersection of straight and curved mouldings in a show-case.

parallel to the front edge of the curved moulding PQ. Set up perpendiculars *ab*, etc., in line with VN, etc., making $ab = 2E$ on the sectional view D, and so on for each of the other projectors indicated by dotted lines on the sectional view D. Complete the sectional view D by drawing lines through the points so obtained. Then S represents the *correct section for the curved moulding* PQ such that it shall make a *straight mitre* with the straight length OP, the section of OP to be as shown at D.

Intersection of Two Curved Mouldings in Show-case.—Fig. 131 (*a*) and (*b*) indicate the method of dealing with a case in which *both lengths of moulding are circular in plan.* Assuming that the *same section* of moulding S is to be used *for each of the pieces* OP and PQ in (*b*), the *mitre line will be a curved line* PR, which may be determined in a manner similar to that already described in (*a*) Fig. 130. If a *straight line of intersection* is required, AB (*a*) Fig. 131, the moulding DA must have a slightly *different section* F, in order to intersect correctly with a moulding of section E fixed at AC. The method of obtaining this section

is similar to that described in dealing with (*b*) Fig. 130, and should be compared.

Lines are drawn parallel to AC from points 3, etc., on the section E. These lines meet the straight line of mitre AB at

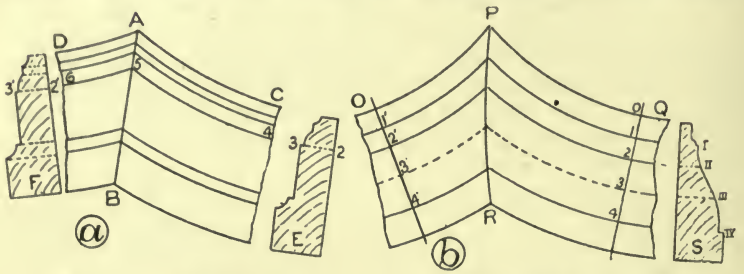

FIG. 131.—Plan of intersection of two curved mouldings in a show-case.

5, etc. From the various points on the mitre draw lines 5,6, etc., and set up perpendiculars 2′,3′, in line with 5,6, etc., and equal in length to the respective heights 2,3, etc., on the section E.

Then F is the **correct section** for the length of moulding DA, and E for the length AC, in order that the mitre AB may be a **straight line.**

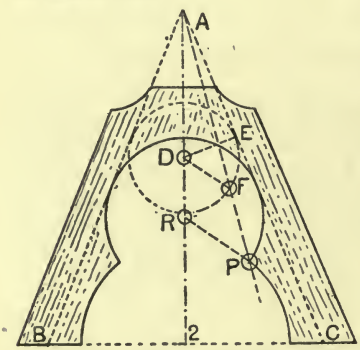

FIG. 132.—Drawing a circle, centre R, to pass through a given point P and touch two straight lines AB and AC.

Circle to pass through a Given Point and touch Two Given Straight Lines.—A piece of wood is to be ornamented and shaped as shown in Fig. 132, so that the circumference of the circular hole which is pierced through the wood shall pass through P and touch AB and AC; *i.e.* P is the *given point*, and AB and AC are the two given *straight lines.*

Bisect the angle BAC, and draw the centre line A2. From A draw the line AP to pass through P. At any point D on the centre line draw a line DE at right angles to AC. With centre D and radius DE, describe a circle to touch AC at E. Join D to F, the point at which the circle intersects the line AP. From P, the given point, draw PR parallel to FD. Then R is the centre, and RP the radius, of a circle to pass through P and touch AB and AC.

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Circle to touch a Given Circle and a Given Straight Line.—Given a straight line AB, Fig. 133, and a circle, centre C, to draw a *smaller* circle of radius R, to touch the straight line AB and the given circle. The diagram is intended to suggest how this problem might be used in setting out a pattern for an angle bracket.

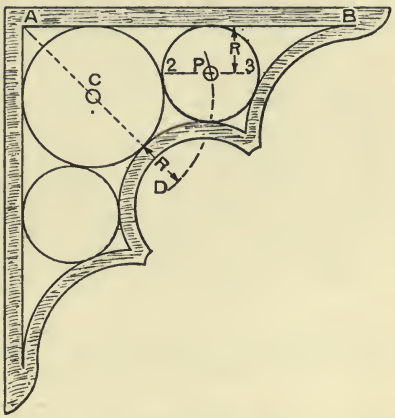


FIG. 133.—Drawing a circle, centre P, to touch a given circle (C) and a straight line AB.

Draw the two straight parts of the bracket at right angles to each other. Fix upon a suitable diameter and draw the larger circle, centre C, which has its centre on the bisector of the right angle. Mark off a distance R outside this circle equal to the radius which the smaller circle is required to have, and with C as centre draw the arc DP. Measure a similar distance, R, away from AB, and draw the line 2,3 parallel to AB. Then P (which is the intersection of the arc DP and the straight line 2,3) is the centre of the required circle, and R is the radius.

Circle to touch a Given Straight Line at a Given Point, and touch a Given Circle also.—The small sketch, (*b*) Fig. 134,

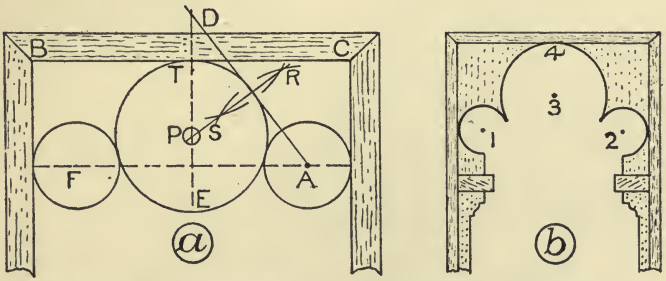


FIG. 134.—Setting out circles for ornamental wooden arch.

illustrates the use of this problem as applied to an ornamental wooden arch. The size of the circles 1 and 2 has been decided upon and their position fixed. It is required to describe another circle (3) which shall touch the head of the frame at (4), and touch the circle (2) also.

(*a*) Fig. 134 shows the construction. Fix the position of the circles, centres A and F, as required. Draw DE. the centre line of the opening. Mark off a distance TD, above the head, equal in length to the radius of the given circle, centre A. Join DA, and draw the line RS to bisect DA, and intersecting DE at P. Then P is the centre of the required circle, and PT is its radius.

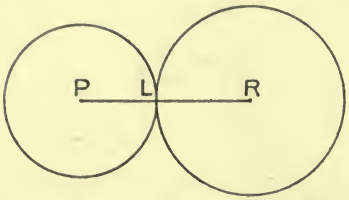


FIG. 135.—Point of contact of two circles.

THEOREM.—*If two circles*, P *and* R (Fig. 135), *touch each other, the point of contact of the two circles must occur upon the line* PR *which joins the two centres*, i.e. *at* L. *Hence, if the centre* P *were known and any point, such as* L, *was to be the point of contact, the line* PL, *produced, would contain the centres of all circles which would touch the given circle, centre* P, *at the point* L.

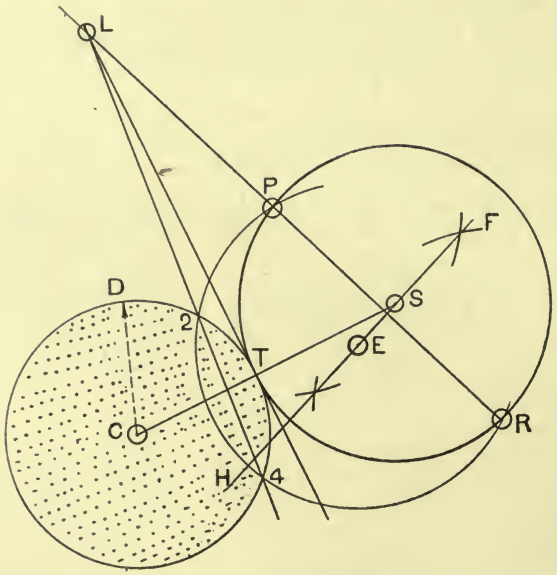


FIG. 136.—Circle (PTR) to pass through two points, P and R, and touch the shaded circle.

Circle to pass through Two Given Points and touch a Given Circle.—The shaded circle, centre C, Fig. 136, represents a circular field, and P and R indicate the position of two trees.

A circular enclosure is required, such that (*a*) the fence surrounding it shall *include* the two trees P and R; (*b*) the circumference of the enclosure shall just *touch* that of the given field.

Join P and R, and bisect by an indefinite perpendicular, FH. Any point on this line is equidistant from P and R. Take any point E on this line, and with radius ER or EP, describe a circle P2,4R to pass through P and R, intersecting the given circle at 2 and 4. Join 2,4 and produce indefinitely. Produce RP to intersect 2,4, produced, at L. From L draw the tangent LT to touch the given circle at T. Then T will be the point of contact of the given circle and the one required. Join CT and produce so as to intersect FH in S. Then S, being on FH, is equidistant from P and R. If the distance from S to T (the point of contact) is taken as radius, a circle PTR may be drawn which will pass through P and R and touch the given circle (centre C) at T.

Radical Axis of Two Circles.—*Definition.—The radical axis of two circles is a straight line which contains all points from which equal tangents can be drawn to the two given circles.*

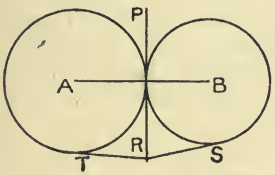


FIG. 137.—Radical axis (PR) of two circles.

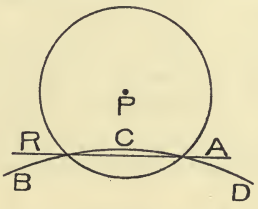


FIG. 138.—Radical axis (RA) of two circles which intersect.

If the two given circles (centres A and B) touch each other, as in Fig. 137, the common tangent PR at their point of contact is the Radical Axis of the two circles. In other words, PR is a *locus of all points from which equal tangents, such as* RS, RT, *can be drawn* to the given circles.

If the two given circles, centre P and BCD, *cut each other*, as in Fig. 138, the *common chord*, RA, of the two circles is the Radical Axis of the given circles.

PROBLEM.—Given any two circles, centres A and B, Fig. 139, which do not meet or cut each other, to find their radical axis.

Draw any *trial circle*, CED, to cut the two given circles. Then by the last problem, the common chord CE of the trial circle and the circle centre A is the radical axis of these two circles. Similarly, the common chord of the trial circle and of the circle centre B is the radical axis of these two circles. Produce these two common chords to intersect at F.

Then F is a point on the radical axis of the two given circles. Join the centres A and B, and through F draw PF at right angles to AB. Then PF is the radical axis of the two given circles. From any point such as P, on this line, equal tangents PR and PT can be drawn to the two given circles.

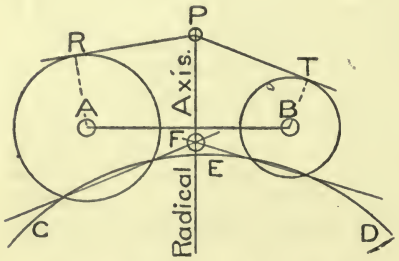


FIG. 139.—Method of finding radical axis.

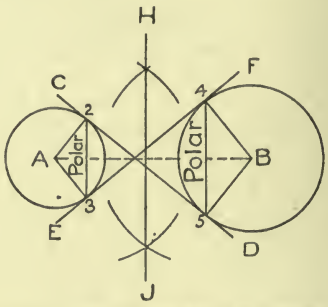


FIG. 140.—Radical axis by means of "polars."

Radical Axis by means of Polars.—Alternative method. The circles, centres A and B, Fig. 140, are given. Draw the interior tangents, CD and EF, to these two circles. These tangents touch the circles at 2,5 and 3,4, respectively. Join the points of contact, 2,3 and 4,5. The lines joining these points are called **Polars**. Join the centres A and B. Draw the line HJ *midway between the two polars*, and *at right angles to* AB. Then the line HJ is the radical axis of the two given circles.

CHAPTER VII

LOCI

In the preceding chapters the student has learnt to form an idea of a *simple locus* or *path traced by a moving point.* The following examples are intended to amplify and elaborate the knowledge already gained.

Suppose a stake S, Fig. 141, driven into the ground, a rope SP attached to this stake, and a donkey, P, tethered to the stake by means of the rope. As the donkey moves around in the direction indicated by the arrow, keeping the rope constantly taut, the *locus* or path which he traces is a *circle.*

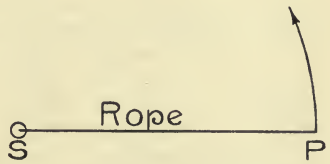


Fig. 141.—Example of simple locus.

From this simple example we note that *a circle is the path of a point which moves in a plane at a constant distance from a fixed point.*

In mechanical work this branch of the subject is of much importance, as it helps towards a knowledge of the correct method of shaping various parts of machines, such as cams, wheel teeth, etc.

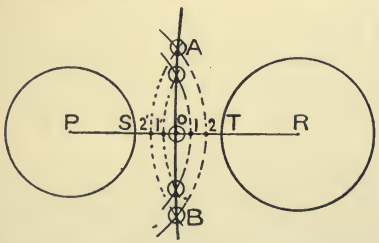


Fig. 142.—Locus of centres of circles to touch two given circles, P and R.

Locus of Centres of all Circles to touch Two Given Circles.—The circles, centres P and R, Fig. 142, represent two pulleys. Assume that another pulley, centre B, is required to touch the two given pulleys exteriorly. To find the position of B. Join PR the centres of the given circles. Bisect the distance ST

between the two circles at O. Mark off on either side of O equal distances O1, 1,2, and O1′, 1′2′, etc. With centre P and radius = P2 describe an arc A2B, and with centre R and radius = R2′ describe an arc A2′B, cutting the arc A2B at A and B. Proceed in the same way, by using radii P1 and R1′, to obtain other arcs. Through A and B and O and the various points of intersection of the respective arcs draw a free curve AOB. Then any point on AOB may be taken as the centre of a circle to be drawn to touch the given circles P and R.

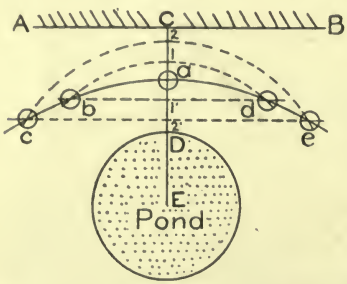


FIG. 143.—Locus of centres of circles to touch a given circle and a straight line.

Locus of Centres of all Circles which touch a Given Straight Line and a Given Circle.—The straight line AB, Fig. 143, represents a *wall*, and the shaded circular area a *pond*. Show the position of a fence *which shall be at all points equidistant from the wall and the pond.*

From the centre of the circle E draw a straight line EC at right angles to AB. Bisect CD at *a*, and mark off equal distances *a*1, 1,2 and *a*1′, 1′2′, etc., on either side of *a* along CD. With E as centre and radii E1, E2, etc., draw the arcs *b*1*d*, *c*2*e*, etc. Through 1′ and 2′ draw straight lines parallel to AB. Then the points *c*, *b*, *a*, *d*, *e*, at which the respective arcs and straight lines intersect, are all equidistant from the circle and the straight line. A free curve through these points *c* . . . *e*, may be taken to represent a fence which is at all points equidistant from the pond and the wall.

Locus traced by Point on a Rod.—A rod AB, Fig. 144, is so arranged that pins or dowels inserted in the rod at A and B are constrained to move along two grooves CD and DE at right angles to each other. *Find the locus of P, the centre point of the rod.*

First method. Select any points, such as A′, etc., to left and right of A, along CD, and with radius = AB, mark off the respective positions of the other end, B, of the rod. Draw lines as indicated by the dotted lines, A′B′, etc., to represent the various positions of the rod. With radius = AP mark the respective positions of P on these lines, as at P′. Draw a smooth curve through the points PP′, etc. Note that this curve, the locus of P, is a *quadrant*, centre D, radius DP.

Second method. Using tracing paper or thin celluloid, trace

off the line AB and mark the position of P on the traced line. Shift the tracing paper in such a way as to bring the ends A and B in various positions on the lines CD and DE, and mark the

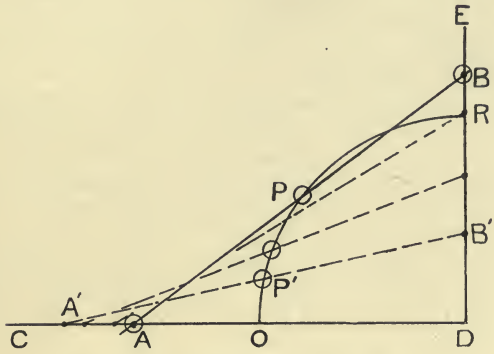


FIG. 144.—Locus of point on moving rod.

successive positions of P by pricking through the point P on the tracing paper. This method is much more expeditious and should be adopted in the various positions which follow.

Locus of Point on Connecting Rod.—AB, Fig. 145, is a connecting rod, the end A of which is constrained to move *horizontally* along between two guides, D. The other end of the rod, B, is attached to a crank BC in such a way that the end

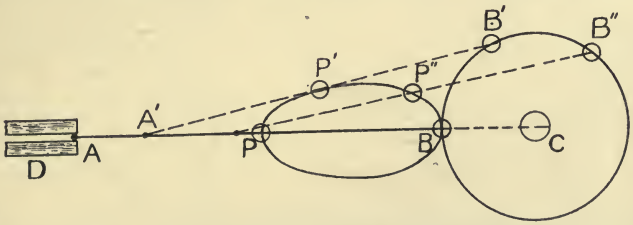


FIG. 145.—Locus of centre point of connecting rod.

of the connecting rod B moves *around in a circle*, BB′, etc., while A moves *horizontally*. Find the locus of P, the *centre point* of the connecting rod.

The dotted lines A′B′, etc., indicate successive positions of the rod, with the centre point P′P″, etc., marked in each case. A free curve through the various points so found represents the path traced by P as the rod moves under the given conditions. As suggested in the preceding problem, a much quicker method

is to trace off the line AB and mark the centre point P of the line. Arrange the tracing paper in such a way that while the end A of the traced line falls on any point A′ on the line AB, the other end B of the traced line falls upon the crank circle BB′. Mark the successive positions of P by pricking through the tracing paper with a needle, or with the pricker specially provided in some sets of instruments.

Having satisfied himself that the locus of the centre point P is an *ellipse*, the student should select other points, say, midway between A and P, and midway between P and B. Find the locus of each of these points while the rod moves under the same conditions as before. The path traced by P will be an *ellipse* in both cases. Note that if a point is selected near the B end of the rod the two axes of the ellipse so traced are *nearly equal* in length, *i.e.* it is nearly a circle. On the other hand, if a point is selected near the A end of the rod, *the axes of the ellipse differ greatly in length.*

Involute and Evolute of a Circle.—Suppose the circle, centre O, Fig. 146, to represent the end of a cylinder or drum from which a piece of wire or rope is being unwound, *the rope, meanwhile, being kept taut.* The free end of the rope moving under such conditions would describe a locus as shown A, 8, 7, 6 . . . 1P, while one coil of rope was unwound. Such a locus or path is known as an **Involute** and the curved surface from which the rope is unwound is the corresponding **Evolute** for this particular *Involute.* The Involute is drawn as follows. At A draw AP tangent to the evolute (the circle, centre O) and make AP = $3\frac{1}{7}$ times the diameter of the evolute. Divide AP into twelve equal parts, and divide the circle into the same number of equal parts as at OA, etc. At each of the points on the circle, such as *a*, draw tangents to the circle *a*2, etc.

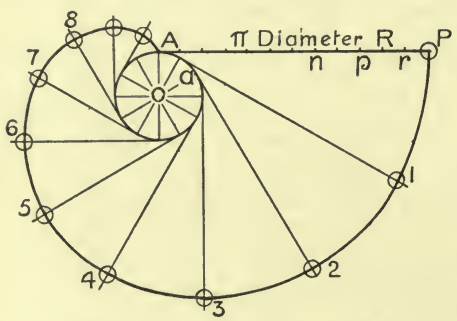


FIG. 146.—Involute of a circle.

When the rope is completely unwound for *one convolution*, the length of rope unwound is = AP. Then when three-fourths of one convolution of rope has been uncoiled the end would be at 3 ; when

half a convolution has been uncoiled the free end would be at 6, and so on. In order to determine the position of the points 1, 2, 3, etc., it is only necessary to measure $\frac{1}{12}$, $\frac{2}{12}$, etc., of the line AP *i.e.* A*r*, A*p*, A*n*, etc., and set this distance off from the point of tangency. One example will suffice. The length *a*2 is found by measuring $\frac{10}{12}$ of the length AP, *i.e.* AR. Care is required in drawing a free curve through the points 1, 2, 3, etc., particularly between the points 8 and A, where the curvature changes more quickly than between the points P1 ; 1,2, etc.

CHAPTER VIII

SPIRAL CURVES

A *spiral curve* is the locus of a point P, Fig. 147, moving on a *plane surface*, around a *fixed point*, or **pole**, in such a way as to approach nearer or recede further from the *pole*.

Archimedean or Equable Spiral.—Imagine the point P, Fig. 147, to move around the *pole* in the direction indicated by the arrow, and to approach nearer to the pole in such a manner that, for *each equal angular space a, b, c, etc., passed through*, the point P has *moved an equal distance nearer to the pole*. Suppose, for example, that the limiting conditions are as follows: P is to move $\frac{1}{16}$ in. nearer to the pole for every angular space of 15° passed through. Then if OP = $1\frac{1}{2}$ ins.; OP′ = $1\frac{7}{16}$ ins.; OP″ = $1\frac{3}{8}$ ins., and so on. The point P is then said to be moving **uniformly** towards the pole O. A free curve through the points PP′ . . . , etc., is part of a spiral curve, which is known as an **equable spiral** ("equable," *i.e.* equal distances passed through). It is also named after **Archimedes**, an ancient Greek who made much use of the properties of this curve.

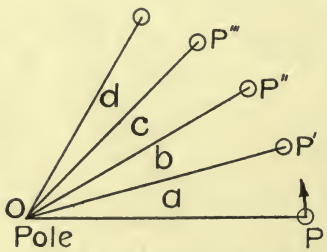


FIG. 147.—Movement of point P around a pole O to illustrate "uniform" movement.

To draw an Archimedean Spiral of Two Convolutions, the longest Radius Vector being 2 ins.—The lines OP, OP′, OA, OC, OB, etc., Fig. 148, which mark off the equal angular spaces through which P is to pass, are called **radius vectors** (*vector* = a line). Suppose P to move around the pole towards A, *moving uniformly* nearer the pole. Then, since P has to reach the pole after having gone twice completely round the pole—i.e. *two convolutions*—it is clear

that the total distance which P has to travel towards the pole is 2 ins., *i.e.* the length of the longest radius vector. Hence, by the time P has travelled once completely round the pole, it will be at R, halfway towards O. Similarly, it is obvious that, after passing through one right angle POA, the point P is at A, having moved nearer the pole by a distance P3 equal to one-fourth of the total distance PR covered in one convolution. In the same

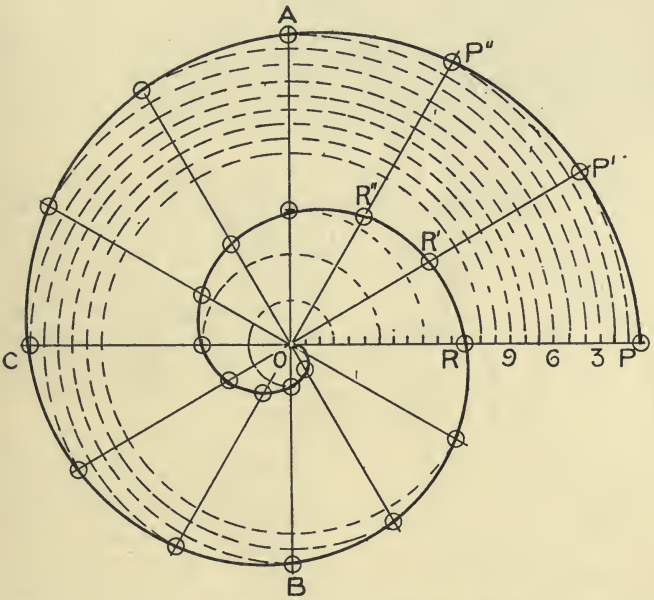


FIG. 148.—Archimedean or equable spiral.

way when P reaches C it has moved nearer the pole by a distance equal to P6, *i.e.* half PR, and so on.

Construction.—Set out OP = 2 ins. for the longest radius vector. Bisect OP at R, and divide RP into twelve equal parts at 3, 6, 9, 12. Using the 30° set square, set out the remaining radius vectors OP′, OP″, OA, etc., making twelve angular spaces, POP′, P′OP″. Note that the number of angular spaces must correspond with the number of equal parts into which RP has been divided. With centre O and radius O1 (R1 = $\frac{1}{12}$ of RP), describe an arc to intersect OP′ at P′. Similarly with centre O and radius O2, describe an arc to intersect OP″ at P″. Proceed in the same way—using O3, O4, etc., as radii—to obtain the remaining points A, etc. Draw a smooth curve PP′P″A . . . R

through these points to complete the first convolution. In order to obtain points R′R″, etc., on the second convolution divide OR into twelve equal parts. Using O as centre and radius $\frac{1}{12}$ of OR draw an arc to intersect OP′ at R′. Centre O and radius $\frac{10}{12}$ of OR will give an arc cutting OP″ in R″, and so on. Great care is necessary in drawing the curve through the last few points as the spiral approaches the pole O, as at this point the curve is much "quicker," and more difficult to draw freehand. One of the "quicker" curves on a *French curve* should be used for this purpose, the flatter parts of the curve being used to guide the pencil in filling in the part of the spiral between B and R, while the very flat parts of the curve may be used in drawing the parts of least curvature CB, AC, and PA.

Cams.—A **cam** is a piece of metal plate or other material having its outline shaped in such a way as to impart a definite motion to another member of a machine, such as a slider. The designing and shaping of cams is so intimately connected with the various spiral curves that a few instructions as to their properties may suitably be considered at this stage.

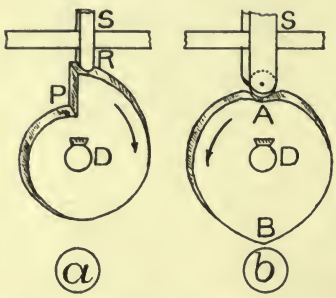


FIG. 149.—Cams.
(*a*) Snail cam. (*b*) Heart cam.

Snail Cam and Heart Cam.—(*a*) and (*b*), Fig. 149, are two very well known types, the design of which is based upon the Archimedean or equable spiral. (*a*) represents a **Snail Cam** and (*b*) a **Heart Cam.** The shape of the outer edge—shaded in the diagram—of either of these may be so modified as to produce any required movement. In (*a*) the cam P is fixed securely to a rotary spindle D in such a way that as the spindle moves in the direction indicated by the arrow, the cam is constrained to move with it. The outer edge of the cam is in contact with the lower end of a slider S, which moves up and down vertically between guides as indicated.

The outer edge of such a cam is a *spiral curve* of an *equable* character, such that the cam will impart a *uniform upward movement* to the slider throughout a *whole* revolution, with a drop to the initial position at P at the end of each revolution of the spindle D.

(*b*) The outline of the cam in this case is made up of *two equal symmetrical spiral curves*, such that a *uniform upward*

movement is imparted to the slider S during *half* a revolution of the spindle D. After the part B has passed under the end of the slider, the latter moves *uniformly downward* again to the lowest position at A.

PROBLEM.—*To set out a snail cam, (a) Fig. 150, which shall impart a uniform rise of* ·5 *in. to a slider. Diameter of spindle* ·2 *in.* Using the 30° and 60° angles of the set square draw a series of lines AB, CD, FK, etc., intersecting at O. At O describe a circle of ·1 in. radius. Mark off OE of such a length as to give sufficient strength of material at the weakest part of the cam. Make EC = ·5 in., *i.e.* the required *rise* of the slider. Divide EC (the rise) into as many equal parts as there are angular spaces between the radius vectors OC, OF, etc.—in this case twelve.

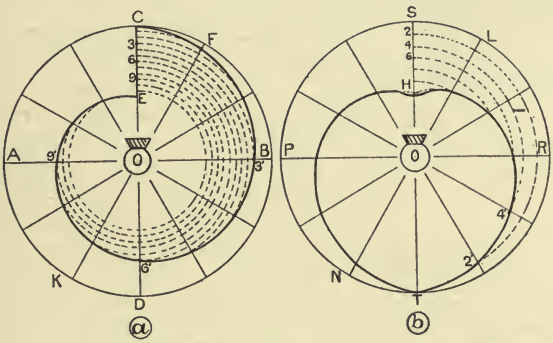


FIG. 150.—(*a*) Designing a **snail** cam to fulfil certain conditions.
(*b*) Designing a **heart** cam.

With O as centre and radius O11 describe an arc cutting OF. (*Note:* only numbers 3, 6 and 9 are shown on the drawing.)

Similarly, with centre O and radius O3 describe the arc 3,3′ cutting the vector OB in 3′. In the same way, using O as centre and radii O6 and O9 respectively, the points 6′ and 9′ on the vectors OD and OA are obtained. Other points on the spiral curve are obtained in a similar manner.

PROBLEM.—*To set out a heart cam, (b) Fig. 150, which shall impart a uniform rise of* ·5 *in. during half a revolution, and a uniform drop to the initial position in the second half revolution. Diameter of spindle* = ·2 *in.*

Set out the radius vectors OS, OL, OR, etc., in the same manner as already described in the preceding case. Set out diameter of spindle O, and mark off sufficient strength of material OH as before. Divide SH into as many equal spaces as there are angular spaces between the vectors OS, OL . . . OT in half the complete circle of revolution—in this case six. With centre O and radius O2, describe the arc 2,2′, and

so determine the point 2′ on the required outline of cam. In the same way, with centre O and radius O4, draw an arc 4,4′ to intersect the vector O4′ at 4′. Proceed in the same way to obtain the remaining points of the curve on the vectors OR, OL, etc. The other half of the curve is *symmetrical* with that already drawn and may be obtained in the same manner, using the radii O2, O4, O6 as before for determining points on the left-hand half of the curve.

Cam as Bench Stop Adjuster.—The various modifications of the two types of cam already shown, Fig. 150, are not of special interest to students of Building Construction, and are therefore omitted. It may be of interest, however, to note that a special type of cam bench stop adjuster (Fig. 151) has been designed and fitted as shown at A. The adjuster is made in cast iron, and is fixed to the leg of the bench by means of a stout coach screw. When the adjuster is turned in a *clockwise* direction the bench stop B is constrained to move in a *vertical* direction, and remains at any height above the bench as may be most suitable for the work in hand.

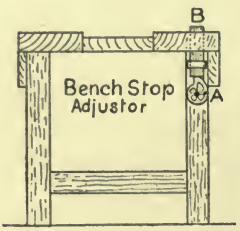


FIG. 151.—Cam used for raising or lowering bench stop.

Such a bench stop adjuster may be made in $\frac{3}{4}$ in. hard wood, the correct shape for the edge being obtained as shown in the design of a snail cam, Fig. 150. A small wooden pin or a screw may be driven into the face of the wooden adjuster to be used for greater convenience in turning it when it is required to raise or lower the bench stop.

The Logarithmic Spiral.—The principle upon which the Archimedean or equable spiral is based—viz. *uniform progression towards a fixed point or pole*—is fairly easily understood. The principle of the **Logarithmic Spiral**, however, is somewhat more difficult to understand, since the moving point which traces the curve, although it is constantly *approaching* the pole, *does not move uniformly towards it.* A practical illustration may serve to make the meaning clearer. The following lengths increase uniformly from the lowest to the highest: 2 ft., 4 ft., 6 ft., 8 ft., 10 ft. Similarly the decrease in length in the following rods is uniform 18 ft., 15 ft., 12 ft., 9 ft., 6 ft., 3 ft., 0 ft. Suppose the joiner who had the 18-ft. rod was told to make another rod three-fourths of the length of the 18 ft. rod. The length of such a rod would be $\frac{18 \times 3}{4} = \frac{54}{4} = 13\frac{2}{4}$ ft., or 13 ft. 6 ins. He is now told to make another rod $\frac{3}{4}$ of the length of the one just made, *i.e.*

13 ft. 6 ins. The length of this rod would be $13\frac{2}{4} \times \frac{3}{4} = \frac{54}{4} \times \frac{3}{4} = \frac{162}{16} = 10\frac{1}{8}$ ft. The lengths of the three rods are 18 ft.; 13 ft. 6 ins.; and 10 ft. $1\frac{1}{2}$ ins. The difference in length between the first and second is 18 ft. − 13 ft. 6 ins. = 4 ft. 6 ins.; the difference in length between the second and third is 13 ft. 6 ins. − 10 ft. $1\frac{1}{2}$ ins. = 3 ft. $4\frac{1}{2}$ ins. In the same way, if a series of rods were made, each in turn being $\frac{3}{4}$ (or some other proportionate amount) of the length of the last, the decrease in their lengths *would not be uniform* but would *follow a certain definite law* according to the *factor* (or multiplier) that was used—in this case $\frac{3}{4}$. In the same way, if the properties of the Logarithmic Spiral are compared with those of the Archimedean Spiral it will be noted that in the former, the moving point *does not approach uniformly* towards the pole, but follows some definite law; and yet, while it *approaches* the pole—more or less quickly according as the factor used is large or small—it *never* actually *reaches the pole*.

The diagram, Fig. 152, will make the matter clearer. Draw a line OP to represent a rod 10 ins. long. Set out the equal vector angles *a*, *b*, *c*, etc., as shown. Insert a drawing pin in the drawing board at O. Suppose the end P to move in the direction indicated by the arrow, the end P meanwhile moving towards O, while the other end of the rod is kept meanwhile tight against the drawing pin at O in such a manner that when the rod has passed through the vector angle *a*, the end P is at R, having moved 1 in. nearer to O, *i.e.* $\frac{1}{10}$ of the length of the rod OP. Then OR is ·9 or $\frac{9}{10}$ of the original length OP. In the same way let the rod move through another vector angle *b*, equal in magnitude to *a*, while P approaches nearer to the drawing pin at O until at S it is $\frac{9}{10}$ of distance from O that it was at R, *i.e.* OS = $\frac{9}{10}$ of OR. In the same way when P has moved to T let OT = $\frac{9}{10}$ of OS. Under these conditions of movement, P does not advance an *equal distance* towards O for each equal vector angle passed through, *e.g.* OR = 9 ins., OS = ·9 × 9 ins. or 8·1 ins., and OT = ·9 × 8·1 or 7·29 ins. Moving in this way P would be *constantly moving around* the pole O, *constantly getting nearer* to the pole, and yet would **never reach the pole.** The locus or path traced by P as it moves under the given conditions would be a **Logarithmic Spiral.** It follows, therefore, that in order to draw a Logarithmic Spiral, it is

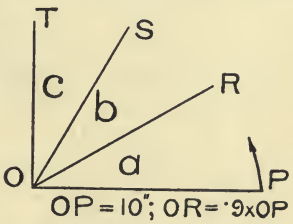


FIG. 152.—Movement of a point P around a pole O in logarithmic spiral.

necessary to specify three items: (1) *Length of longest radius vector.* (2) *Vector angle.* (3) *Vector Ratio.*

PROBLEM.—*To draw a Logarithmic Spiral of two convolutions having given* (1) *Longest radius vector* = $2\frac{1}{2}$ *ins.;* (2) *Vector angle* = 30°; *and* (3) *Vector ratio* = ·9. Set out OP ((*a*) Fig. 153) = $2\frac{1}{2}$ ins. for the longest radius vector, and draw the other vectors OA, OB, OC, etc., indefinite in length. The distance OA from the pole O to the point A on the curve is to be ·9 ($\frac{9}{10}$) of OP, and OB is to be ·9 of OA and so on. In order to simplify the operation of obtaining these respective lengths an *auxiliary drawing* ((*b*) Fig. 153) is employed. Set out two lines *op*, *on* at any angle *nop*. Make *op* = OP, *i.e.* $2\frac{1}{2}$ ins. Divide *op* into ten equal parts and thus make *oa* = $\frac{9}{10}$ of *op*. With centre *o*

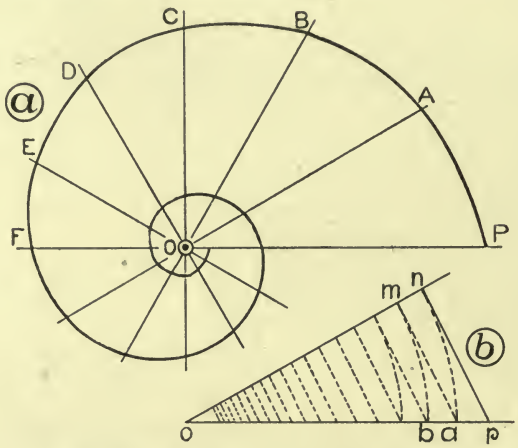


FIG. 153.—(*a*) Logarithmic spiral. (*b*) Construction for obtaining lengths of radius vectors in (*a*).

radius *oa* describe an arc *an*, cutting *on* at *n*. Join *pn*. Then in the triangle *nop*, the side *on* is ·9 of the length of *op*. From *n* draw *am* parallel to *pn*. With centre *o* radius *om*, describe an arc *mb*, cutting *op* at *b*. Then *nop*, *mop*, *dob*, etc., are *similar triangles*. Hence the ratio between the lengths of the respective sides of these triangles is *constant*, *i.e.* *om* : *on* :: *oa* : *op*, and *od* : *om* :: *ob* : *oa*. In other words, *ob* is $\frac{9}{10}$ of *oa*. But *oa* = $\frac{9}{10}$ of *op*. ∴ *ob* = $\frac{9}{10} \times \frac{9}{10}$ of *op*; or ·9 × ·9 × 2·5 ins., *i.e.* 2·25 ins. Then *oa* is the correct length required for marking off the length OA (*a*) of the next radius vector to OP, and *ob* is the correct length for the next radius vector OB. The lengths of the remaining vectors of the spiral (*a*) are obtained in the same manner by drawing arcs (*b*) and straight lines, similar to *an* and *ma* respectively. Note that as these straight lines *np*, *ma*, etc., approach *o* they are gradually getting nearer and nearer together.

The spiral is obtained by drawing a smooth curve through the points P, A, B, C, etc.

The Log Spiral as a Means of Calculation.—One important property of the Log Spiral is that **Each radius vector is the Mean Proportional or Geometric Mean between the radius vectors on either side of it.** Thus OE is the mean proportional between OD and OF. (*For Mean Proportional, see Chap. II.*) OE is, therefore, the length of side of a square having an area equal to that of the rectangle on OD and OF.

Hence, if the length and breadth of any rectangle are given, it is possible to make use of a log spiral as a means of calculating the length of side of a square equal in area to the given rectangle. Taking the longer and shorter sides of the rectangle (say 2·1 ins. and 1·8 ins. respectively) as radii and O as centre, describe arcs O*r*, O*t*, cutting the spiral at *r* and *t* respectively. (Points *r* and *t* are not shown.) Bisect the angle *r*O*t* and draw the bisector OS cutting the spiral at S. Then OS is the mean proportional between O*r* and O*t*, and is the length of side of a square (1·94 ins. side), equal in area to the rectangle of 2·1 ins. and 1·8 ins. sides.

Versed Sinic Spiral.—This curve, Fig. 154, is of special interest to students of architecture, as it forms the basis for the **volutes** which are a distinctive feature of certain well-known orders of architecture.

In the inset diagram, Fig. 154, bottom right-hand corner, the line *hk*, which is a perpendicular from the centre of the chord *bc* to meet the arc *bkc*, is the **versed sine** of that particular *chord* and *arc*. This spiral, Fig. 154, derives its name from the use of the *versed sine* in its construction.

The relative size of the eye of the spiral, radius AP, having been fixed, and the longest radius vector, AB, drawn, decide upon the number of convolutions which the spiral is to have—in this case three. With A as centre and AB as radius describe a circle BDFK and divide into any convenient number of parts, say 8 or 12. In the diagram the circle is divided into eight equal parts at CDEF, etc. Draw the diameters BF, CH, DK, etc.

Draw BR at right angles to AB, and make BR = BP. With B as centre and radius BR describe the arc RP. Divide this arc into as many *equal spaces* as the spiral is to have *convolutions*—three in this—at *a* and *b*. Divide each of the arcs R*a*, *ab*, *b*P, into eight equal spaces as indicated at 1, 2, 3, etc. This number of parts must always be the *same number* as that into which the *circle* BDFK has been divided. Draw the ordinates 1*r*, 2*s*, etc.,

at right angles to BP. With A as centre and radius A*r* describe the arc *rn*, cutting AC in *n*. Similarly, with radius AS, obtain the point *d* on AD. Proceed in the same way to obtain points on each of the radii AE, AF, AH, etc., and through the points *r*, *n*, *d*, etc., so found, draw a smooth curve by means of a flexible straight-edge.

Alternative method.—Determine points *r*, *n*, *d*, etc., as before. With centres B and *d* and radius equal to the mid radius A*n*,

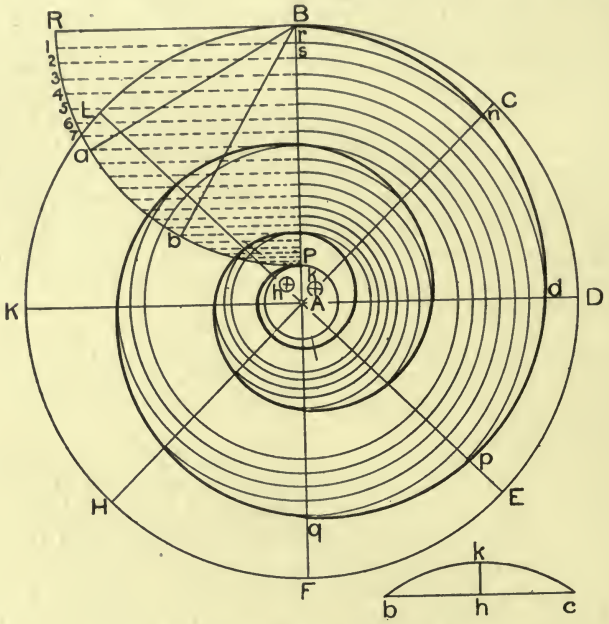


FIG. 154.—Versed sinic spiral.

describe arcs intersecting at *h*, within the *eye* of the spiral. With *h* as centre and radius A*n*, describe the part of the curve B*nd*. With centres *d* and *q*, and radius AP, describe arcs intersecting at *k*. With *k* as centre and radius A*p*, describe the next quarter of a convolution *dpq* of the spiral. Proceed in the same way to obtain a new centre for describing each of the remaining quarter convolutions of the spiral.

Versed Sinic Volute.—Volutes such as those shown in Fig. 155 are a distinguishing feature of the **Ionic Order** of **architecture**, and formed a very effective means of ornamentation at the top of the columns in this Order, as shown in (*b*)

Fig. 155. The number of volutes varies in the different orders. The sketch (*b*) represents a column in the **Grecian Ionic** order of architecture. The method of setting out the volute is shown at (*a*), and is based upon the construction already described in detail in the foregoing problem. Two separate *sinic spirals*, the longest radius vectors of which are respectively AC and AD, have to be drawn as shown. These two spirals gradually approach each other and eventually meet each other at the eye of the spiral B. The circle, radius AC, is divided into eight equal parts as in the last problem, and CE made equal to CB and the

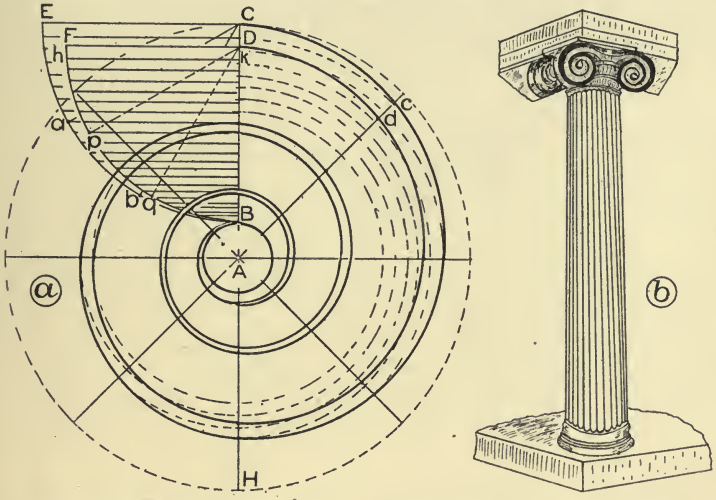


Fig. 155.—(*a*) Versed sinic volute. (*b*) Volute used for ornamentation of ionic column.

quadrant E*ab*B drawn. The three equal arcs E*a*, *ab*, *b*B are each divided into eight equal parts and ordinates drawn parallel to EC. The distances of these ordinates respectively from A as measured along AC give the radii for obtaining points such as *c* on the outer spiral. DF is then made equal to DB and the quadrant F*pq*B is drawn and trisected at *p* and *q*, each of the equal arcs F*p*, *pq*, etc., is then divided into eight equal parts as at *h*. From these points *ordinates*, such as *hk*, are drawn *parallel to* FD. With radius A*k* and centre A an arc is drawn to cut AC in *d*. Other points on the inner spiral are obtained in a similar manner, and the various curves C*c*, D*d*, etc., put in with a French curve, or *struck* from centres, as described in the previous problem.

Ionic Volute (alternative method).—(*a*) and (*b*), Fig. 156, illustrate an alternative method of describing an Ionic Volute. The straight line 0,7 (*a*) is the extreme distance over all the volute, measured from P to S. Divide this line 0,7 into seven

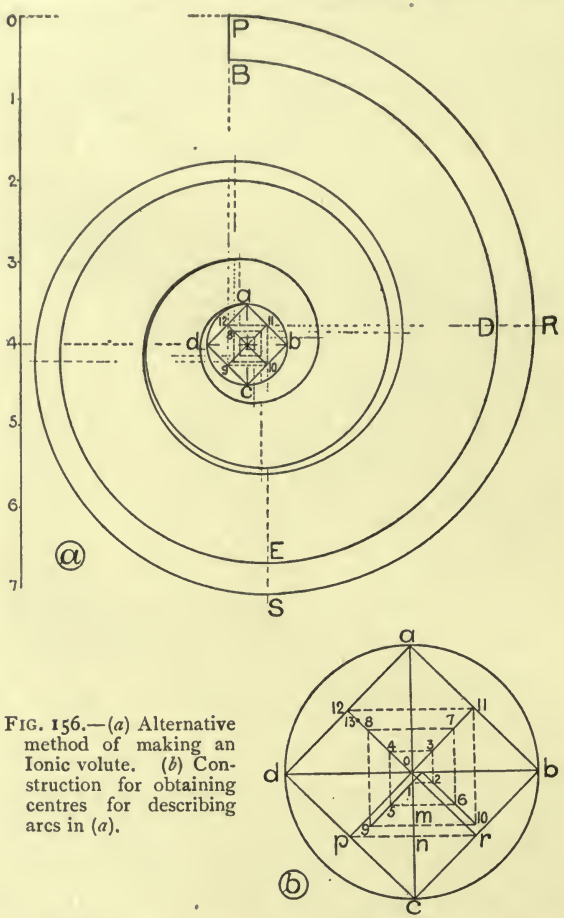


FIG. 156.—(*a*) Alternative method of making an Ionic volute. (*b*) Construction for obtaining centres for describing arcs in (*a*).

equal parts as shown. From the fourth point of division draw any line *db*. On this line mark a point for the centre of the *eye* of the spiral. The diameter of the *eye* is made equal to one of the divisions of the first line 0,7. The eye of the spiral is shown, drawn to a larger scale, in (*b*). Having drawn the diameters of the eye *ac* and *db*, construct the square *abcd*. Bisect the sides

of this square at 12, 11, *r*, *p*, and draw the diameters 12*r*, 11*p* of the square. Divide the line 12,0 (*i.e.* half the diameter 12*r*) into three equal parts at 8 and 4. Join 12, 11 and *pr*, and draw a line 9,10 parallel to *pr* and at a distance *mn* away from it, *mn* being = half the distance between the parallel lines 12,11 and 8,7. From 11 draw the lines 11,10 to meet the line 9,10. From 10 draw 10,2 parallel to 12*r* to meet the diameter *db*, and from the point of intersection of 10,2 and *db* draw 9,1 parallel to *p*11. The points 12, 11, 10, and 9 are four **centres** from which to describe the arcs which make up the outer convolution of the spiral. Proceed as follows to obtain the remaining centres. From 9 draw 9,8 to intersect 12*r* at 8. In the same manner draw 8,7, and draw 7,6 to intersect the line 2,10 at 6, and 6,5 to intersect the line 9,1 at 5, and so on for the remaining points 4, 3, 2, 1.

With centre 12 and radius 12P describe the arc PR to cut a line passing through 12,11. With centre 11 and radius 11R, describe the arc RS to cut a line through 11,10. Proceed in the same manner to use the remaining centres 10, 9, 8, etc., drawing lines in line with 9,10; 8,9; 7,8, etc., in order to obtain the correct point where one arc finishes and the other begins.

The centres for describing the *inner* portion of the spiral occur on the lines 12*r*; 11*p*, etc. Only *one* such point is shown, in order to avoid confusion. Thus the point 13, midway between 11 and 8 in (*b*) Fig. 156, is the centre for describing the arc BD; a point *intermediate between* 11 and 7 would be the centre for describing the arc DE of the spiral, and so on.

CHAPTER IX

PLANE CURVES

The Ellipse.—This curve is almost as important and as frequently used as the *circle.* A knowledge of its properties is essential both to the draughtsman and to the practical man. It was frequently used by the ancients in the design of their architectural work, enrichments, and mouldings, and is in every way a most beautiful curve. In a later chapter, the ellipse is considered as the *section of a cone or cylinder* cut *by a plane inclined to the axis of the solid.* Considered as a plane curve, however, it may be defined as *the locus of a point which traces a continuous curve, every point on which is such that the sum of the distances from two fixed points or foci is always constant.*

Several methods of drawing the ellipse are shown, of which one of the simplest and most practical is by the use of a **trammel,** *i.e.* a strip of wood or thin card.

(1) **Trammel Method.**—Fig. 157 illustrates the method of **trammelling** in order to determine points on an ellipse. The lengths of the major axis CD and minor axis EF are given. These axes intersect at O. Draw CD and EF at right angles to each other, making CO = DO. Take a strip of cartridge paper or thin card as shown, AP, measuring from one end, P, of this strip make PA = DO or CO, *i.e. half the length of the major axis.* Measuring again from the same end, P, make PB = EO or FO, *i.e. half the length of the minor axis.* Then the distance AB on the trammel is equal to PA – PB, *i.e.* the *difference* between the lengths of the major (or greater) axis and the minor (or lesser) axis. Arrange the trammel, as shown, in such a way that the marks A and B fall upon the minor axis and major axis respectively, and at each new position make a dot with the pencil at P. Then as P is moved from D towards E, the mark A moves *down* the minor axis from O towards F, and the mark B moves *along* the major axis in the direction DO. A free

curve is then drawn through the successive positions of P. When one quarter of the ellipse has been determined in this manner, reverse the trammel so that B falls upon the semi-major axis CO, keeping A still on OF, and mark new positions

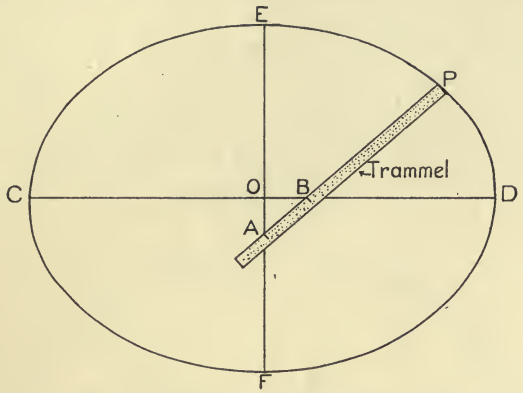


FIG. 157.—Trammel method of drawing ellipse.

of P intermediate between C and E, and thus complete the quarter, CE, of the ellipse. In a similar manner the remaining half of the ellipse may be drawn by reversing the trammel in such a way that the point A moves along the semi-minor axis EO. In this way, while B moves along the semi-major axis DO, the end P traces the quarter ellipse, CF ; similarly while B

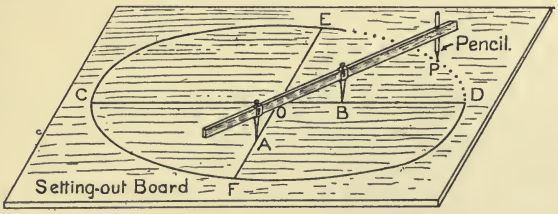


FIG. 158.—Method of using joiner's trammel rod and points for describing an ellipse.

moves along CO, the end P traces the remaining quarter, FD, of the ellipse.

This is a very neat, clean, practical method of describing an ellipse, as there are no working lines left on the paper after the curve has been drawn through the successsive positions of P.

Joiner's Trammel Rod and Points.—Fig. 158 shows the application of the trammel method to workshop practice. Two

metal **trammel "points,"** A and B, provided with set screws, are set in position on a wooden trammel rod as shown. The major and minor axes, CD and FE, are drawn on the *setting-out board* or on the actual work as shown. A pencil, P, is inserted in a hole in the trammel rod, and the distances PA and PB between the pencil point and the trammel points A and B are made respectively equal to the semi-major axis DO and the semi-minor axis EO. The trammel points are then inserted in the wood, on the axes CD and EF, as shown, and the position of P marked. The trammel points are then re-inserted in a new position, such that B, say, is near O, and A nearer F, and the

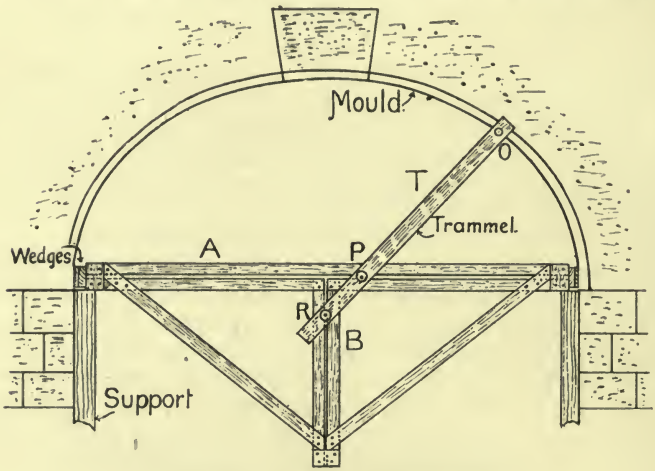


FIG. 159.—"Striking" an elliptical arch.

position of P marked again by the pencil. When a succession of dots has been drawn in this way, a free curve may be drawn through these dots by bending a thin lath in such a way as to coincide with the dots EPD.

Trammel Rod for Mouldings of Arch.—Fig. 159 illustrates a further practical adaptation of the *trammel method* of describing an ellipse. A **moulding** (or bead) is to be "*stuck*" on the angle at the soffit of an **elliptical arch,** as shown. A light wooden framework of $2\frac{1}{4}$ ins. × 1 in. battens is nailed together in such a manner as to provide a $\frac{3}{8}$ in. space between the two battens which make up the horizontal part, A, and the vertical part, B, of the construction. These wooden *guides* are suitably braced and wedged and supported in position, so that the space or groove in the horizontal guide A is in line with the *springing line* of the

arch, and the groove in the vertical guide B is *central* between the supports. Another piece of batten, T, is used as a *trammel.* Two $\frac{3}{8}$ in. bolts are inserted at R and P in such a position that OR = *semi-major axis* of the ellipse (*i.e* $\frac{1}{2}$ span) and OP = *semi-minor axis* of the ellipse (*i.e.* the rise of arch). The bolts P and R are provided with nuts at the back of the guides, to prevent the trammel from becoming detached from the guides. The plasterer's zinc mould, cut to the required shape, and suitably mounted on wood, is attached to the trammel at O. As the mould at O is moved around the soffit line of the arch in order to remove the superfluous plaster, the bolts P and R, moving along the guides A and B, act as directors which ensure that the mould shall preserve an elliptical outline.

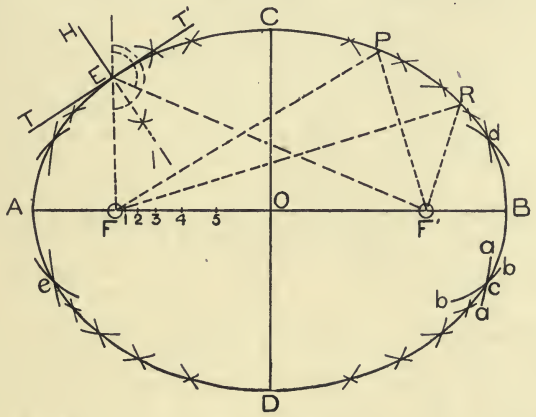


FIG. 160.—Method of setting out an ellipse by means of intersecting arcs.

Describing an Ellipse by means of Intersecting Arcs.—Fig. 160 illustrates another method of describing an ellipse of comparatively small dimensions on paper. In the *trammel* method, the *two fixed points*, or **Foci**, were not used. In this case it is necessary to determine the **Foci** of the ellipse. These points are obtained as follows: Draw the major and minor axes, AB and CD, as before. With centres C or D, and radius = AO or BO—*i.e.* $\frac{1}{2}$ major axis—describe arcs cutting AB at F and F′ respectively. Then F and F′ are the required **Foci** for an ellipse to pass through ACBD. On the semi-major axis AO, mark off points 1, 2, 3, etc., on the right of F towards O. These points may be at any distance apart, but experience will prove that it is necessary to select points closer together as they approach towards F, and further apart as they approach O.

The reason for this is that the *curvature* of the ellipse *changes more rapidly* as it approaches the ends of the major axis at A and B, and it is necessary, therefore, to determine *points closer together* on the ellipse if the curve is to be drawn correctly at these places. As the curve approaches CD it is "*flatter*," *i.e.* the curvature is not changing so rapidly, and it is not necessary to determine so many points on the ellipse, nor so close together, at this part of the curve. Hence the reason for the greater space between points 3, 4 ; 4, 5 on the axis AO. Having marked points 1, 2, 3, etc., at suitable distances apart, set the compass to A1, and with F and F′ as radii, describe four arcs, as at *a*, *d*, *e*, and near E. Set the compass again to the remainder of the length of the major axis, 1B, and with centres F and F′ describe four arcs cutting those already drawn, as shown at *c* (where the arcs *a* and *b* intersect), and at *d*, *e*, etc. The intersection of the four pairs of arcs drawn in this manner gives four points on the ellipse required. To obtain another set of four points, set the compass to A2, and with F and F′ as centres describe four arcs. Re-setting the compass to distance 2B, and using F and F′ as centres, describe four arcs cutting those just drawn. The intersection of these four pairs of arcs gives another four points on the ellipse. Proceed in the same way, taking distances A3, 3B ; A4, 4B ; A5, 5B, and using F and F′ as centres, draw other intersecting arcs as shown. A free curve through these points is the required ellipse.

Important Property of the Ellipse.—Select any point P on the ellipse (Fig. 160), and join PF, PF′. These lines are the focal distances for the point P. Select any other point, R or E, on the ellipse, and draw the focal lines RF, RF′ ; EF, EF′. Measure the length of FP + PF′, and write it down. Measure the length of FR + RF′. Write it down, and compare with the length FP + PF′ already found. *The lengths are the same.* In the same way measure the distance FE + EF′ and compare with the lengths FP + PF′ and FR + RF′. *The sum of the two focal lengths is the same in each case.* By construction, the focal lengths FC and CF′ are *each equal to the semi-minor axis* AO or BO. Then FC + CF′ = major axis ; and since C is a point on the ellipse, the sum of the focal lengths FC + CF′ is equal to the *sum of the focal lengths* FP + PF′, etc. Hence the following important principles may be deduced :—

(*a*) *The sum of the focal lengths for any point on an ellipse is constant*, i.e. *the sum of the lengths is the same in each case.*

(*b*) *The sum of the focal lengths for any point on a given ellipse is always equal to the length of the major axis.*

Tangent and Normal to an Ellipse—Joint Lines in Arches.—In order to determine correctly the **joint lines** for an elliptical arch in brickwork or masonry, it is necessary to determine **normals** to the curve, such as HE, Fig. 160. The *tangent* and *normal* at any point E on the ellipse may be determined as follows: From the point E draw the focal lines EF and EF′. Bisect the *interior* angle FEF′ as shown. Produce this bisector outwards, as shown HE. Then HE is the **normal** at this point, and gives the *correct slope for the joint* in the arch which occurs at that point.

The **tangent** TT′ at the same point, E, is obtained as follows: Produce the focal line FE outwards as shown, E7. Bisect the *exterior* angle F′E7 between the *focal lines* E7 and EF′. Then TT′, the line which bisects this angle, is the required **tangent** at E.

Describing an Ellipse by means of a Looped String.—In the sketch (Fig. 161) another method of describing an ellipse is shown. *Experiment:* Drive two drawing pins, or small nails, into the paper or drawing-board, say, 6 ins. apart. Make a loop at each end of a piece of string or thread of such a length that, when the loops are made, the thread measures, say, 7½ ins. from end to end. Pass the loops over the drawing pins already fastened to the drawing point. Hold a pencil tightly against the thread, in such a way as to pull it *taut*, *i.e.* to take up the slack. Keeping the thread taut, move the pencil around so that it describes a curve on the paper. *This curve is an ellipse*, or would be so, if it were possible to keep the pencil *uniformly upright* and the thread *equally taut* while the whole curve was drawn.

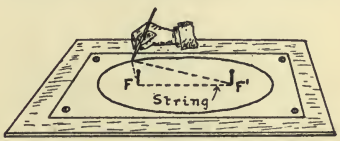


FIG. 161.—Describing an ellipse by means of a looped string.

After experimenting in this way it will be obvious that *it is quite impracticable to describe a small ellipse with any degree of accuracy by this method.*

Practical cases sometimes arise, however, in which a *large* ellipse, say 10 ft. × 7 ft. has to be described, and *where no great degree of accuracy is required.* For such cases the string method may be used, and will prove in every way a practicable one.

Method of setting out an Ellipse of Large Dimensions by means of a Looped Cord.—(*a*) and (*b*), Fig. 162, will help to illustrate the application of this method to a practical purpose. A piece of ground (*b*) is to have an **elliptical bed** made in it for ornamental purposes, or may have to be **excavated** prior to

commencing the brickwork of an **elliptical wall.** For such a purpose a divergence of half an inch or so from the line of a true ellipse is not noticeable. Suppose the lengths of the axes of the required ellipse are to be 11 ft. and 6 ft. 3 ins. respectively. Two battens AB (11 ft.) and CD (6 ft. 3 ins.) are nailed together at right angles to each other ((*a*) Fig. 162). A piece of cord RT is looped up at each end so that RT = AB, *i.e.* the major axis of the required ellipse. Two nails are driven into the batten AB at points *c* and *p*, such that the distance D*c* or D*p* = half the major axis AB. Thus, for the given dimensions, the nails *c* and *p* would be approximately 11·7 ins. from the ends of the batten at A and B respectively. The wooden construction is now laid on the ground ((*b*) Fig. 162) and arranged in such a position that the longer batten is laid in the direction required for the longest axis of the elliptical excavation, and the battens are fixed

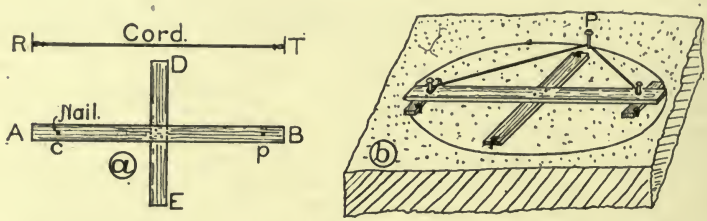


FIG. 162.—Rough method of setting out an elliptical space for garden bed or excavation.

temporarily in position with a few spikes or pegs. The ellipse is then *scratched* on the surface of the ground by means of a spike, P, or a bricklayer's line pin, the cord meanwhile being held taut, as shown.

For rougher purposes the wooden construction is dispensed with, and two round pegs or pieces of gas barrel are driven into the ground at the required distance apart, *cp* in (*a*) Fig. 162. The cord is looped up as before to the total length of the major axis AB, and the ellipse marked, or, in the case of turf, *cut* with an old table knife in such a way that when the excavation is made the edges of the turf around the bed are neatly cut through and are not torn or ragged.

Fourth method: **By making use of the auxiliary circles** of the ellipse.

In (*b*) Fig. 163, with centre O and radii = semi-major and semi-minor axes OH and OP respectively, describe the circles HTR and P2B. These are the **major** and **minor auxiliary circles** of an ellipse, having HR for major axis and PB for minor

axis. Select any number of points, such as 4, on the circumference of the larger circle, and draw the radii O4, etc., cutting the smaller circle at 2, etc. Draw 4,5, at right angles to HR, and 2,5 parallel to HR and cutting 4,5 at 5. Then 5 is a point on the required ellipse. Other points on the ellipse are determined in the same manner, as shown.

Elliptical Opening in Rectangular Framework.—(*a*) Fig. 163 shows a simple method of setting out an ellipse for a panel or opening to occupy a certain definite size in a rectangular framework. Draw the centre lines BA and OP. Divide OP into any number of equal parts (say 4) at 1′, 2′, 3′, and divide OR into the same number of equal parts at 1, 2, 3. Join Bo, B1, B2, B3. From A draw A1′, A2′, etc., to intersect B1, etc.,

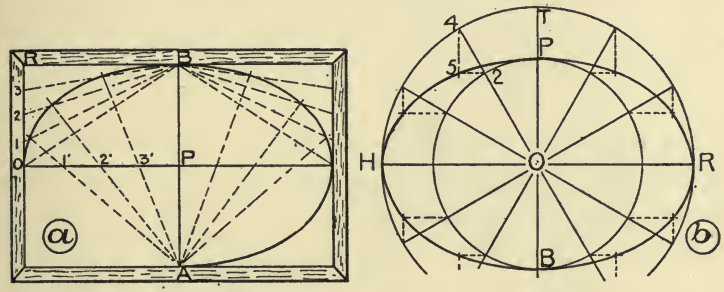


FIG. 163.—(*a*) Describing an ellipse to fill a rectangular opening. (*b*) Ellipse obtained by use of major and minor auxiliary circles.

as shown. A smooth curve drawn through these points of intersection gives one quarter of the required ellipse. The top right-hand quarter of the ellipse may be determined as shown. Points on the lower half of the ellipse are found by drawing lines from B through 1′, 2′, 3′ to intersect lines drawn from A to points on the lower half of the short side of the rectangle. These construction lines are omitted in the illustration.

Special Adaptation of Trammel Method.—Fig. 164 shows a special method of using a trammel for the purpose of describing an ellipse, the *major and minor axes* of which are **almost equal in length**. AB is the major axis and F2 the minor axis of the required ellipse. With centre O and radii OA and OC, describe the semi-major and semi-minor auxiliary circles A1B, C2D. Bisect the angle DO1 and draw the normal O*p*, cutting the arc D2 at 3. At *p* draw the tangent 4*p*5. From 3 draw 3P parallel to AB, and from *p* draw *p*P parallel to O2, and intersecting 3P in

P. Through P draw rPt parallel to 4p5, and at P draw Pa at right angles to rPt. Then the angle Pa2 = angle Pt2, and P, the

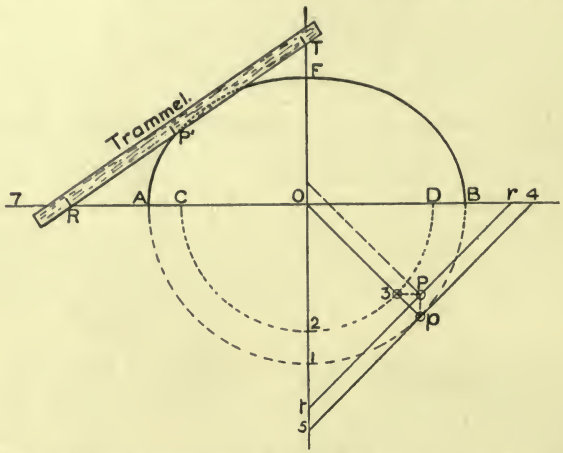


FIG. 164.—Method of using trammel when axes are nearly equal in length.

point of contact of the lines rt and Pa, is a point on the required ellipse. The points rP and t should be accurately transferred to a strip of card or paper, as shown at R, P′, T, and used as a trammel, keeping the point R on the line A7, *i.e.* the major axis produced, and the point T on FT, *i.e.* the minor axis produced, and marking successive positions of the point P′ as the trammel is moved into various positions on the paper.

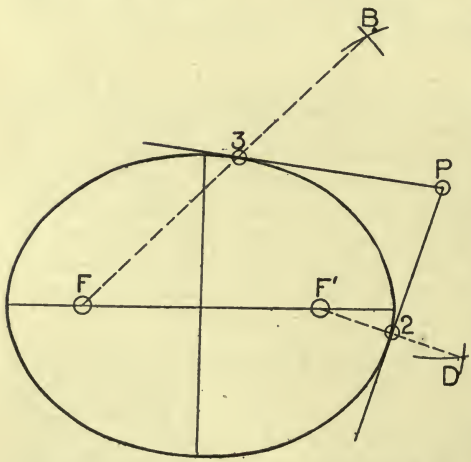


FIG. 165.—Two tangents from a point outside an ellipse.

Tangents from a Point Outside an Ellipse.—It is sometimes necessary in practical drawing to determine the true tangents from a point such as P, Fig. 165, *outside* an ellipse. The construction in such a case is as follows:—

Measure the distance from P to the nearest focus F′, *i.e.* PF′. With P as centre and radius PF′ draw an arc at B and D respectively. Set the compasses to the length of the major axis, and with this radius, and using F as centre, describe a second arc at B and D respectively, intersecting those already drawn at B and D. From D draw DF′ intersecting the ellipse at 2, and from B draw BF intersecting the ellipse at 3. Then 2 and 3 are the points of contact of the required tangents from P which may be drawn from P through 2 and 3 respectively.

Conjugate Diameters of an Ellipse.—*Definition.—Any two lines* AB and PR, Fig. 166, *whose extremities lie on the ellipse, and whose intersection occurs at the intersection of the major and*

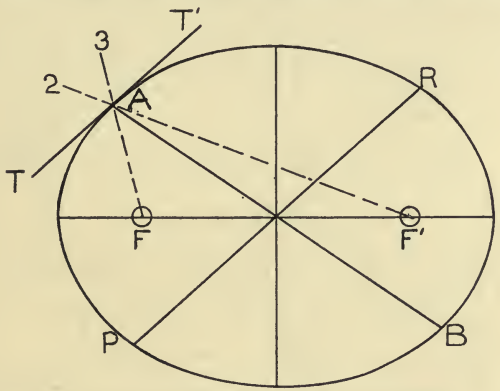


FIG. 166.—Conjugate diameters of an ellipse.

minor arcs, are, in a general way, spoken of as **conjugate diameters** *of the given ellipse.* Strictly speaking, however, it is necessary to determine by construction the particular diameter which is a *conjugate* of any given diameter. Let AB be the given diameter, and it is required to determine the conjugate diameter PR which pairs with AB. Draw the focal lines FA, F′A and produce to 2 and 3. At A determine the tangent by bisecting the angle between the focal lines. Draw PR parallel to TT′ to pass through the intersection of the major and minor axes. Then PR is the *true conjugate diameter* for AB the given diameter.

Determination of Ellipse from Conjugate Diameter.—Two straight lines AB and RP, Fig. 167, are given as a pair of conjugate diameters of a certain ellipse. To draw the ellipse the following construction is one that is sometimes employed.

Through R and P draw lines parallel to AB, and through A and B draw lines parallel to RP, thus completing a parallelogram (a rhomboid). Divide BE into any number of equal parts, say four, at 1′, 2′, and 3′, and divide OB into the same number of equal parts at 1, 2, 3. Join R1′, R2′, R3′. From P draw lines P1, P2, P3, and produce to intersect the lines R1′, etc., at K, etc. A smooth curve through these points of intersection gives the part of the ellipse RKB. The portion RA of the ellipse may be determined in a similar manner, as shown.

Points on the remaining half of the ellipse may be found by proceeding in the manner already described, *e.g.* join P4′ and

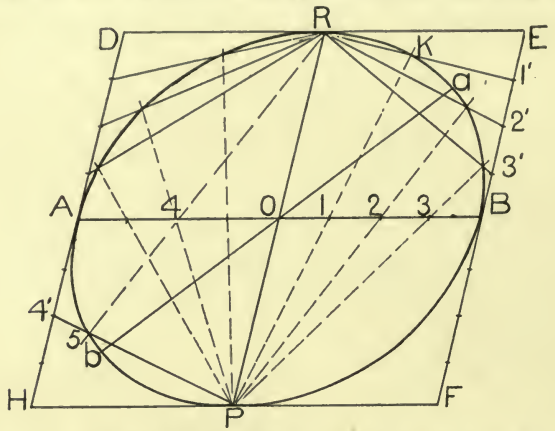


FIG. 167.—Drawing an ellipse when conjugate diameters are given.

produce R4 to intersect P4′ at 5, which is a point on the part of the ellipse AP.

Another method is based on the fact that *all diameters are bisected at* O. Thus if the curve RKB has already been determined, the curve A*b*P may be determined by drawing a series of diameters—one of which AO*b* is shown—and marking off a distance O*b* equal to O*a*, etc.

Parallel to Ellipse for Bridge Construction.—An elliptical arch, Fig. 168, is sometimes used in bridge construction, and gives a *very strong type of arch.* The objection to using a *true ellipse* for the outline of the arch is that when the water rises rapidly to flood level in the rainy season, as it frequently does in mountainous districts, there is *not sufficient clearance* for the water near the springing line B4 of the arch. Hence it is necessary to design the arch in such a way as to give greater

clearance at this point. For this purpose a **parallel to the true ellipse** is sometimes employed, as shown B3CD. The span of the required arch is BD, and the rise from the normal level of the water is shown at AC. Equal distances BE, CF, and DH are measured from the *springing points* B and D and the *crown* C respectively, and a semi-ellipse EFH is struck by one of the methods already described. In order to obtain a parallel BCD to this ellipse, *normals* are set out, 5, 6, etc., at intervals around the ellipse, the distances 6, 5, etc., are made equal to BE or CF, and a free curve is drawn through these points. The advantage gained by this method is demonstrated in the diagram, where the *dotted curve* B4 represents part of a *true ellipse* struck with BD as major axis and AC as minor axis. By comparing

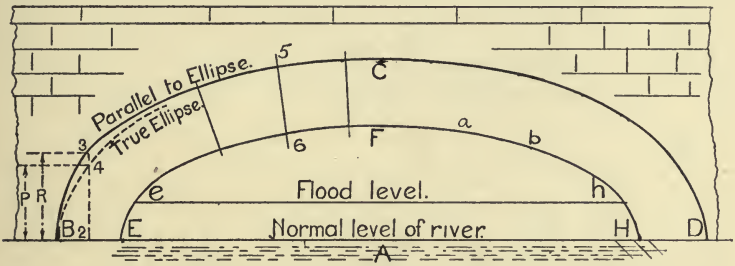


FIG. 168.—Use of parallel to an ellipse in bridge spanning a river. Note the extra clearance gained at R as compared with P.

the heights P and R it is evident that there is an **extra clearance of water way** equal to 3,4, by using the **parallel to the ellipse** P3 as compared with the clearance secured by means of the **true ellipse** P4.

Substitutes for an Ellipse—Quasi-Ellipses.—Various methods of construction have been devised with a view to securing a curve which shall have the appearance of an ellipse but which can be set out by means of the compass, and so do away with the need for employing the usual methods for setting out a true ellipse. In Chapter XI (Brickwork and Masonry Arches) several practical examples of the use of such curves are given, and the advantages of each type explained. It is only necessary, at this point, to give one or two typical examples of curves which may be used as *substitutes* for true ellipses. It should be noted, however, that no such curve can be truthfully called an ellipse, since *no part of the curve of a true ellipse can be struck by means of a compass.*

These substitutes, or **quasi-ellipses**, as they are sometimes

called, are more properly classed as *three-centred ovals*, *five-centred ovals*, etc., according to the number of centres used in striking the curve. Two such curves are shown at (*a*) and (*b*) Fig. 169, and reference should be made to the chapter on Arches (Chapter XI) for further examples.

The diagram (*a*) represents a **60° three-centred curve** for the outline of an arch, the span of which is AB and the rise CD. The construction is as follows:—

Draw AB to scale to represent the *span*, and at C, the centre of AB, set off CD (the *rise*) at right angles to AB. Produce CD

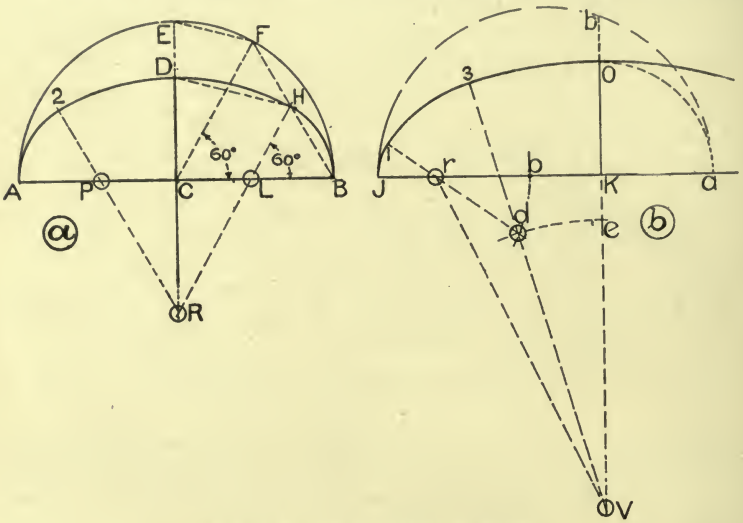


FIG. 169.—Substitutes for an ellipse.
(*a*) 3-centred oval shaped curve. (*b*) 5-centred oval.

indefinitely in each direction, *i.e.* towards E and R. With centre C and radius CB describe the semicircle AEB which intersects CD, produced, at E. At C set out CF making 60° with CB. Join EF, FB. From D draw DH parallel to EF to cut FB in H. From H draw HR cutting CB in L, and DC (produced) at R. Set off CP = CL. Draw RP and produce to 2. Then R, P, and L are the three centres required for striking the curves A2DHB, which make up the outline of the arch. With P as centre, radius PA or P2, describe the arc A2. With R as centre, radius R2, describe the arc 2DH ; and with L as centre, radius LB or LH, draw the arc HB.

Curve which most nearly resembles a True Ellipse.—In

(*b*) Fig. 169 a method of construction is shown by means of which a **five-centred oval** is obtained which perhaps *resembles a true ellipse as closely as any such curve can do.* The construction is based upon a somewhat elaborate algebraical formula into which there is no need to enter.

Set out the span (½ span JK, shown in diagram) and the rise KO. Find the *mean proportional* between JK the *half span* and KO *the rise*, as follows: With K as centre and radius KO describe the arc O*a*. Then K*a* = OK, *i.e.* the rise. Describe a semicircle on J*a*, and at K, the junction of the longer line JK with the shorter line K*a*, produce KO to meet the semicircle at *b*. Then K*b* is the *geometric mean* or **mean proportional** between JK and KO, *i.e.* Kb^2 or K*b* × K*b* = JK × KO. From O set out O*e* = K*b* (the *geometric mean*). (N.B. The method of obtaining the geometric mean is treated more fully in Chapter IV.) From O set out OV = span, *i.e.* JK × 2. With J as centre and radius K*b* describe an arc *bd*, and with V as centre and radius V*e* describe an arc *ed* cutting the arc *bd* at *d*. From J set out J*r* = ½ the rise, *i e.* $\frac{OK}{2}$. Join V*d* and produce to 3. Join V*r*. Join *dr* and produce to 1. Then V, *d*, and *r* are the three centres for describing the three arcs O3; 3,1; and 1J respectively, with radii VO, *d*3, and *r*1. Two other centres, corresponding with *r* and *d*, may be determined in a similar manner for describing the arcs which make up the remainder of the outline of the arch to the right of OK.

Simple Substitute for an Ellipse.—Fig. 170 illustrates an alternative method of drawing a substitute for an ellipse, the construction for which is somewhat simpler than that just given.

AB and CD are the longer and shorter axes, respectively, for the *oval.* These lines are at right angles to each other and are bisected at H, *i.e.* AH = HB and CH = HD. Having set out the lines AB and CD, with H as centre and radius HB, describe an arc BE intersecting HC, produced, at E. Join CB and with C as centre and radius CE describe an arc EF, intersecting CB at F. Bisect BF and draw the bisector PR which intersects the axis CD, produced, at R, and crosses the line HB at 1. Then the points R and 1 are respectively the centres for striking the two parts C4 and 4B of the curve C4B. Thus with R as centre and radius RC describe the arc C4, the point 4 being that at which the curve crosses the bisector PR. With 1 as centre and radius 1,4, describe the arc 4B. Make H2 = H1 and draw the line R,2,5. Then the point 2 is the centre for describing the arc A5, and R is the centre for striking the arc 5C. The

remaining half of the *quasi-ellipse* ADB may be found by determining the position of a point—not shown in the diagram—intermediate between C and E, the distance of this point from C

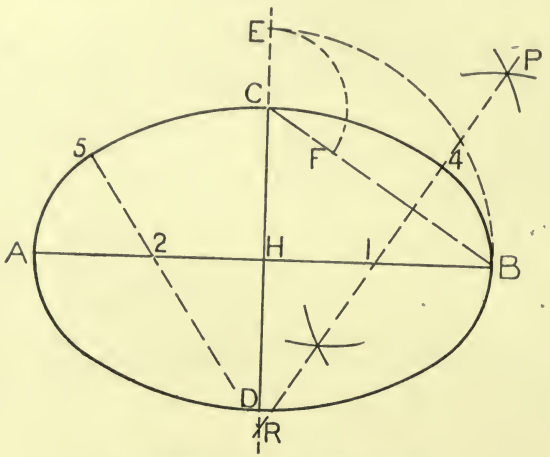


FIG. 170.—Substitute for an ellipse.

being made equal to DR. This point would serve as the centre for the *flatter* part of the curve ADB near D, while 2 and 1 are the centres for the remainder of the curves DA and DB.

Relative Advantages of 5-centred and 3-centred Ovals.—A *5-centred oval*, such as the one just described, is preferable to a 3-centred one when *the rise of the arch is less than ⅓ the span.* On the other hand, when the rise is *greater* than ⅓ the span, the *3-centred oval*, with 60° arcs, as described in the previous problem, is preferable to a 5-centred oval, and in bridge construction **affords greater capacity for the flow of water** than does a **semi-ellipse** with the same span and rise, since the radius of the curve near the springing line of the arch is greater.

Circumference and Area of Ellipse.—The **circumference** of an ellipse $= 2\pi\sqrt{ab}$ approximately, where a and b are the semi-major and semi-minor axes of the ellipse.

Referring again to Fig. 159, determine the number of feet run of plaster mould which would have to be worked on the soffit of the elliptical arch, taking the span of the arch as 15 ft. and the rise as 6 ft.

Semi-major $= \frac{15}{2} = 7$ ft. 6 ins, semi-minor $= 6$ ft.

Then $2\pi\sqrt{7{\cdot}5 \times 6}$ = circumference of whole ellipse

$$\therefore \frac{2\pi\sqrt{7{\cdot}5 \times 6}}{2} = \text{circumference of semi-ellipse}$$

$$= \frac{2 \times 3{\cdot}142\sqrt{45{\cdot}0}}{2} = \text{circumference of semi-ellipse}$$

$$= \frac{6{\cdot}284 \times 6{\cdot}7}{2} = \frac{42{\cdot}1028}{2} = 21{\cdot}05$$

Ans. 21 ft. run, approx.

Fig. 171 illustrates graphically the rule for determining the **area of an ellipse**, of which OA is the semi-major axis and OB is the semi-minor axis. The area of the ellipse is found from the formula

$$\pi(\text{OA} \times \text{OB})$$

Referring to (*b*) Fig. 162, which illustrates a method of setting out an elliptical bed or excavation: taking the major axis of

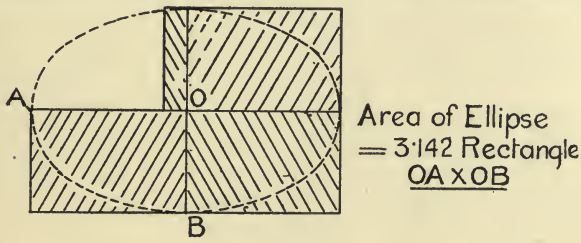


Fig. 171.

the elliptical excavation as 8 ft. 6 ins. and the minor axis as 5 ft., the area of the surface of the ground to be removed within the ellipse would be found as follows:—

Major axis = 8 ft. 6 ins. ∴ semi-major = 4 ft. 3 ins.
Minor axis = 5 ft. ∴ semi-minor = 2 ft. 6 ins.

$$\therefore \quad \text{Area} = \pi(4\tfrac{1}{4} \times 2\tfrac{1}{2}) \text{ sq. ft.}$$
$$= 3{\cdot}142(10{\cdot}625) \text{ sq. ft.}$$
$$= 33{\cdot}38 \text{ sq. ft.}$$

If the elliptical excavation was carried down to a depth of 4 ft. 6 ins., how many cubic feet of earth would be taken out?

Area of surface of excavation = 33·38 sq. ft.
Depth of excavation = 4·5 ft.
∴ Cubic contents of excavation = (33·38 × 4·5) cub. ft.
Cubic contents = 150·2 cub. ft.

$$\text{or } \frac{150{\cdot}2}{27} = \underline{5{\cdot}5} \text{ cub. yds.}$$

Eccentricity of an Ellipse.—A point V, Fig. 172, is situated so that its distance from a straight wall AB as compared with its

distance from a stake F driven into the ground is as 10 : 7. Suppose V to move in the direction 3,5, in such a way that its distance (F3, F5) from F as compared with its distance (3,2 ; 5,4) from AB is *always in the same ratio*, *i.e.* $\frac{F3}{3,2} = \frac{7}{10}$; $\frac{F5}{5,4} = \frac{7}{10}$. Then the path 5,3,V,6,7 traced by V is an *ellipse*, the **eccentricity** or *change of curvature* of which is expressed as $\frac{7}{10}$, 7 : 10, or ·7.

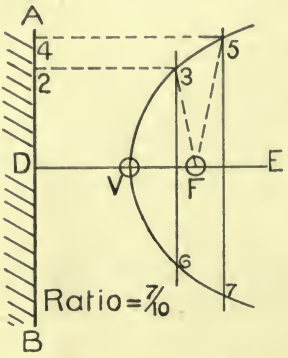


FIG. 172.—Eccentricity of an ellipse.

The **eccentricity** of an ellipse may be *any fraction less than unity*, thus 3 : 5 ; 2 : 7 ; etc., according as the change of curvature is small or great.

The Parabola.—A point V, Fig. 173, is situated so that its distance from the wall AB and the stake F is equal. Imagine V to move in either direction, *i.e.* above or below the axis DE, in such a manner that its distance from the wall at any points such as 4, 2, 2′, 4′, is always the same as its distance from F. Thus, F2 = 2,3 ; F4 = 4,5 ; etc. The path of V moving under these conditions is a parabola. **The eccentricity of a parabola is always unity** ; *i.e.* 2 : 2 ; 10 : 10 ; etc. The *initial position* of V on the axis DE is termed the **vertex** of the curve ; the *straight line* AB (representing the wall), is the **directrix**, and the stake (or *fixed point*) F is the **focus** of the parabola. Lines such as 2,2′ ; 4,4′ ; drawn at right angles to DE, are **ordinates** to the curve.

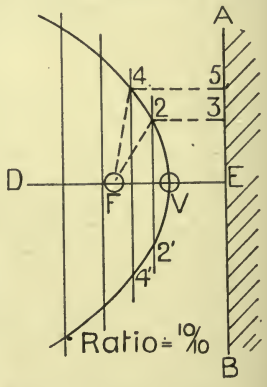


FIG. 173.—Eccentricity of a parabolic curve.

PROBLEM ((*a*) Fig. 174).—*Given the axis* AB, *focus* F, *and directrix* DE, *of a parabola to draw the curve.* Bisect AF at V to find the position of the *vertex*, and draw any number of *ordinates a′a″*, *b′b″*, etc. These ordinates may be drawn at *any distance apart*, but experience will prove that it is necessary to determine a *greater number* of points *near the vertex of the curve* V than are required as the curve becomes flatter in character towards *a′b′*. Hence the ordinates should be more numerous towards V and spaced farther apart to the right of the focus F.

To obtain points on the curve, measure the distance from the directrix at A to the intersection of the ordinate and the axis, as at *a*. Thus,

taking Aa as radius and using F as a centre, describe an arc cutting the ordinate in two places at a' and a''. These are two points on the parabola. Similarly, with radius = Ab and centre F, describe two arcs cutting the ordinate through b at b' and b''. These two points are also on the curve. Any number of points may be determined in the same way, *i.e.* two points for each ordinate drawn. A fair curve drawn through these points in the required parabola.

Tangent and Normal to Parabola.—The **tangent** and **normal** at any point P, (*a*) Fig. 174, may be drawn as follows: Join PF. This is one of the *focal lines* required. Strictly

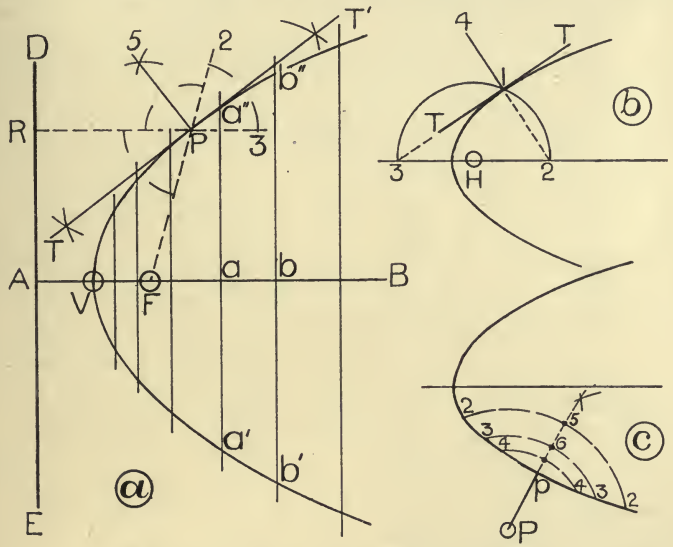


FIG. 174.—(*a*) Method of drawing parabola. (*b*) and (*c*) Alternative methods of determining tangent and normal to a parabolic curve.

speaking, the parabola has only *one focus*, but for practical purposes, the other focus may be considered as being at *an infinite distance* from the first focus F. Hence PR, which is parallel to AB, is a small part of the focal distance from P to the second focus. Produce RP to 3 and FP to 2. Bisect the *interior* angle 3P2, and draw the bisector TT′. This is the **tangent** to the curve at the point P. Bisect the *exterior* angle RP2 between the focal lines RP, P2, and draw P5. This is the **normal** to the curve at P.

Alternative Method of determining Tangent and Normal.—In the diagram, (*b*) Fig. 174, H is the *focus* of the

given parabola. To determine the **tangent** and **normal** at any point 1, *without making use of the focal lines.*

With H as centre, and radius H1, describe a semicircle 3,1,2. Join 2,1 and produce to 4. Join 1,3 and produce to T. Then 1,4 is the *normal* and TT the *tangent* at the point 1. This is a very simple and expedient method of finding the tangent and normal at any point on the curve *when the focus of the parabola is known.*

To draw a Normal to a Parabola from a Point outside the Curve.—In the diagram, (*c*) Fig. 174, P is the given point *outside* the parabola. With P as centre and any radius greater than P*p* describe arcs 4,4; 3,3; 2,2; etc. Bisect these arcs at 5, 6, etc., and draw a curve 5,6 through the centre points of the arcs. This curve meets the parabola in *p*. Then P*p* is the required *normal* to the curve from P.

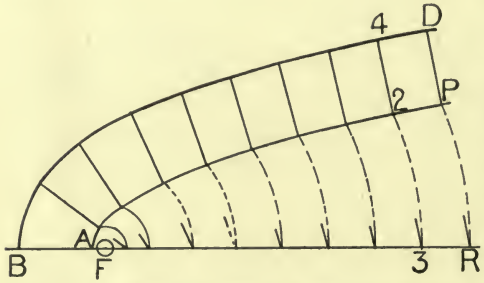


FIG. 175.—Practical method of determining joint lines in parabolic arch.

Method of determining Joint Lines in Parabolic Arch.

PROBLEM.—*Given a part of the soffit line of a parabolic arch* AP, Fig. 175, the axis BR, *and the focus* F, *to draw a parallel to the curve to represent the extrados of the arch, and to determine the correct joint lines between the voussoirs.* Step off the *intrados* line A2P into a number of equal parts varying according to the size of the blocks of stone for making the *voussoirs*. With centre F and radius FP describe an arc PR. Join RP and produce towards D. In the same way, with centre F and radius F2, draw an arc 2,3; join 3,2, and produce to 4. Proceed in the same way to determine the correct inclination for the joint lines at each successive point on the intrados line. Set off the distance PD to the length of joint line required between the voussoirs.

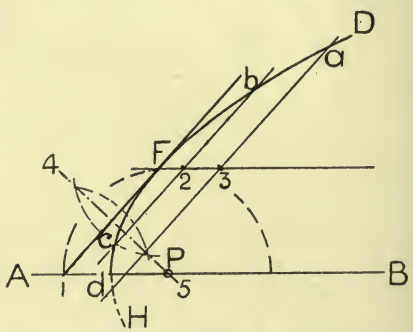


FIG. 176.—Method of finding focus of a parabolic curve.

Make 2,4 = PD. Mark off the *same length* for each of the joint lines from PD to AB, and draw a fair curve through the points D4 . . . . B to represent the *extrados* of the arch.

PROBLEM.—*Given the axis of a parabola, Fig.* 176, *and part of the curve, to determine the focus.* Draw any two parallel lines *ad*, *bc* to cut the curve at *a*, *d*; *b*, *c*, respectively. Bisect *ad* and *bc* at 3 and 2, and draw F3 through the points 2, 3. At F draw F1 parallel to *ad*, *bc*. Then F1 is a tangent to the curve at F. Bisect F1 and draw the bisector 4,5 to intersect the axis AB in P. Then P, which is the centre of the semicircle which passes through the points F and 1, is the **focus** of the parabola.

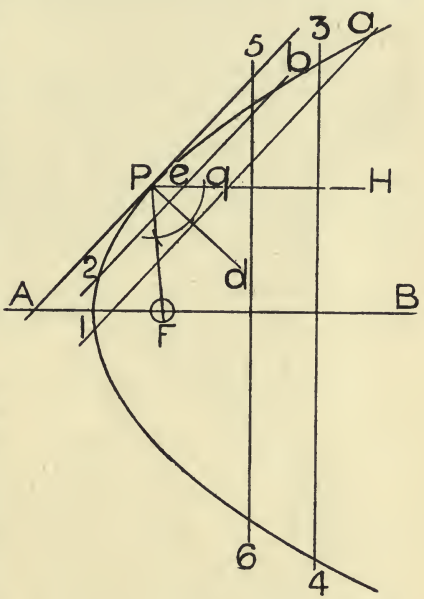


FIG. 177.—Alternative method of finding focus of a parabolic curve.

Alternative Method of determining Focus (Fig. 177).—Draw the parallels *a*1, *b*2, as before, and bisect at *e* and *q*. Draw PH through *e* and *q*. Through P, the point of tangency, draw the tangent AP. Then P*d*, which is drawn at right angles to AP, is the normal at P. Set out a line PF such that the angle FP*d* = the angle HP*d*. This line, which intersects the axis AB at F, is the required focus. This is the converse of the method of drawing tangent and normal already described, Fig. 160, where it was shown that *the normal bisects the angle between the focal lines*. In this case *one focal line* PH is known, and the *normal* P*d*, and since P*d*, the normal, must bisect the angle between the focal lines, the angles FP*d* and HPD are made equal, and PF is thus the *second focal line* and F is the **focus.**

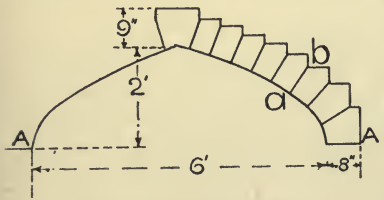


FIG. 178.—Parabolic arch.

EXERCISE.—*The diagram, Fig.* 178, *represents a parabolic arch, the intrados being formed of two parabolic arcs which have* AA *as the*

common axis. Set out the half arch ring to the figured dimensions, scale 1 inch to 1 foot. The distances between the joint lines *ab*, etc., measured along the intrados line, are *equal*, and the *joints ab*, etc., between the voussoirs *are also of equal lengths* and are **normals** to the parabolic curve. These normals must be accurately determined by geometrical construction.—(*Bd. of Ed. Exam.*, 1915.)

Parabola to fill a Rectangular Space.—To design a parabolic arch, made up of two parabolic curves AD and BD, Fig. 179, having a *span* = AB and a *rise* = CD.

Draw the rectangle AEFB. Divide the *rise* AE into *any number of equal parts* (in this case 5) and draw 1,1 ; 2,2 ; etc., parallel to AB. Divide the *half span* ED into the *same number of equal parts* at 1′, 2′, etc., and join A1′, A2′, etc. The

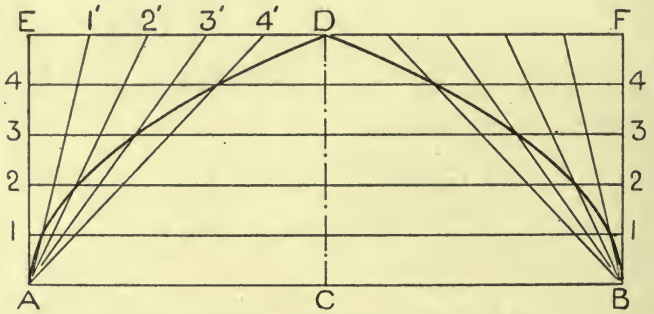


FIG. 179.—Parabolic arch to fill a rectangular space.

points of intersection of these lines (*e.g.* the intersection of A4′ and 4,4) give a succession of points, a fair curve through which is the required parabolic arc for one half of the arch. Points on the curve for the right-hand side of the arch are determined in a similar manner, as shown.

Parabolic Path of Projectiles.—(*a*) and (*b*), Fig. 180, illustrate the fact that the path of a shell, or other projectile fired from a gun, is a parabolic curve. Suppose a shell is fired from P (*a*) towards R. The distances 1, 2, 3, etc., represent equal distances through which the shell would pass in equal times if not affected by gravitation. The distances P1′, 1′,2′, etc., represent the drop of the shell towards the earth as a result of the force of gravitation, these distances P1′, 1′2′, etc., being in the order 1, 3, 5, 7, 9, etc. From 1, 2, 3, etc., draw lines parallel to P5′, and from the points 1′, 2′, 3′, etc., draw lines parallel to PR. The intersections of these two sets of lines, *i.e.* I, II, III, etc., are points in the path of the projectile.

If the shell is fired from A (*b*) towards B the same laws would hold good, and by taking into account the velocity of the shell and the effect of gravitation upon it, the two sets of lines

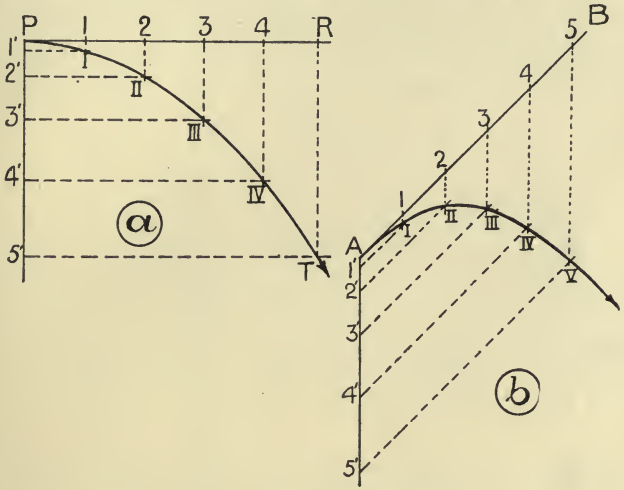


FIG. 180.—Parabolic paths of projectiles.

which intersect at I, II, III, etc., are drawn. A parabolic arc drawn through these points represents the path of the projectile.

Paraboloidal Reflector.—Fig. 181 illustrates a very practical use to which some of the properties of the parabola have been applied. A hollow metal reflector is so shaped that all longitudinal sections through the axis BD are parabolæ. An opening in the lower side of the reflector is so arranged that the *light* F is *at the focus* of the parabola. The source of light should be as *small* and actinic as possible. The nearer it approaches to a *point* of light, the more effective is the reflector. Rays of light from F such as F2, F4, are reflected along lines 2,3 ; 4,5, *parallel to the axis* BD. Thus, theoretically, *there is no loss of light;* every ray is conserved and reflected forward in a powerful pencil of rays which is quite out of proportion to the small and insignificant flame of the illuminant.

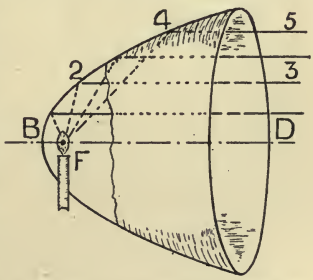


FIG. 181.—Paraboloidal reflector.

Loss of Light in Spherical and Plane Reflectors as compared with Paraboloidal Reflector.—Referring to (*b*) Fig. 182, if a light L is placed in front of a **plane** reflector as shown, any ray of light such as LP is reflected from the mirror along a line PR, such that the angle RPT (between the *reflected* beam PR *and the normal* PT) is equal to the angle LPT between the *incident* beam LP *and the normal* PT. Thus, rays such as

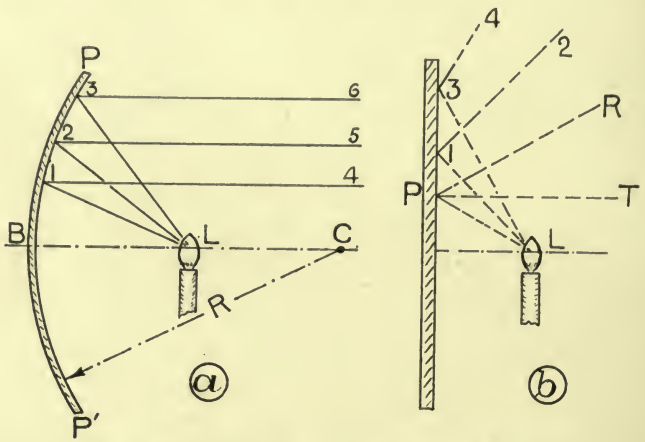


FIG. 182.—Reflectors : (*a*) Spherical ; (*b*) Plane.

L3, which meet the reflector very obliquely, are reflected along a line 3,4, and are so diffused as to be of no effective value for illuminating any small area directly in front of the reflector.

The **spherical** reflector, (*a*) Fig. 182, overcomes this difficulty to a certain extent, provided that the light L is placed *midway* between C, the *centre of curvature* of the reflector, and the point B, where the axis meets the reflector. It is a fact, however, that the *radius of curvature* R of such a reflector must be *fairly large* to be effective, so that the light L would have to be removed much further from the reflector than it would be in the paraboloidal mirror.

CHAPTER X

GEOMETRY AS A BASIS OF ORNAMENTATION

Geometrical details may often be used as the basis of ornamental designs for carving, plaster and stucco work, stencilled friezes, panelled ceilings, wainscotting, and many other artistic embellishments to be found in the best class of house decoration. The designs which immediately follow are not difficult, and may easily be arranged to suit one or other of the purposes already enumerated.

Designs based on the Circle.—Fig. 183 (*a*) shows a method of using the **semicircle** as a detail of design, and serves as a pattern for **wall decoration,** or for the arrangement of artistic **roof coverings.**

(*b*) illustrates the use of **interlacing circles** for various ornamental purposes, such as **chip carving** and **inlay work** in various coloured woods.

(*c*) is a somewhat more daring attempt to employ groups of semicircles, one large and two smaller ones, arranged in groups of three in alternate vertical and horizontal positions, the three semicircles in each group making up a *lunar* type of ornamentation, which may be usefully employed for **stencilling** and **marqueterie** patterns.

(*d*) In this case a **semicircle** and two **quadrants** are used in building up each unit in the design. This pattern may be used for ornamental roof coverings and **ornamental plaster work** on panelled walls.

(*e*) The **interlacing circles** in this design are all of *equal radius*, and are so arranged that they intersect at six points at equal distances apart on the circumference of any given circle. This pattern may be most usefully employed for **marqueterie** and other similar work.

(*f*) The **arcs of circles** which make up the separate units of this design are arranged upon an **equilateral triangle** as a base, as in the case of a Gothic arch.

The faint straight lines in each of the foregoing cases are added in order to give some idea of the basis of construction of the pattern. In (*a*), (*b*), (*c*), and (*d*) the straight line base is arranged in *squares*, while in (*e*) and (*f*) they are so arranged as to form *rectangles*.

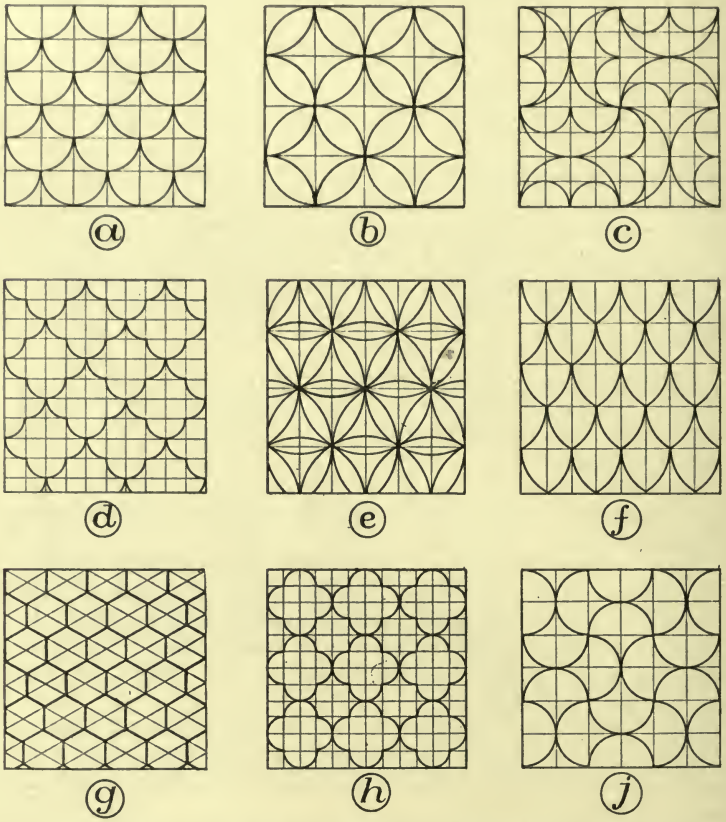


FIG 183.—Circles, semicircles, arcs of circles, etc., used as details of ornamentation of tiles and various square spaces.

(*g*) In this case the straight lines are drawn *diagonally* at 30° towards the right and left side of the design. By thickening in certain portions of each line a series of *hexagons* is obtained which makes a very useful design for **tiled floors** and for **mural decoration**.

(*h*) Faint lines are drawn at right angles to each other and at equal distances apart to form squares, and the design, which makes use of four semicircles for each unit, is so arranged that the centres of the respective semicircles occur at the intersection of two straight lines forming a square.

(*j*) This is a somewhat unconventional design built up of semicircles arranged in pairs, and in such a way that the *chords* of one pair of semicircles are at right angles to the chords of the next pair to the right or left, or above and below any given unit of the design.

It should be evident that each of the foregoing designs *may be extended in length and width* by adding more units of the design on the right and left or above and below those shown, and in this way each design may be so arranged as to fill a *square panel* or a *rectangular space* of any given dimensions.

Repeat Patterns for Banding and Rails—Fig. 184 (*a*) to (*m*) are illustrations of the use of simple geometrical details in building up a *running* **repeat pattern,** suitable for the ornamentation of narrow spaces, such as *rails*, *friezes*, and *bands.*

(*a*) and (*b*) are simple designs in which *semicircles* are employed, the centres for the successive semicircles occurring upon the straight lines which form a margin to the pattern.

(*c*) In this case two series of semicircles are so arranged about a horizontal centre line that this line is a common tangent to all the semicircles.

The three designs (*a*) (*b*) and (*c*) are simple and effective, and may be used for the ornamentation of wooden **chair rails,** the pattern being more or less *flat*, *i.e.* without any *relief*, and made by forming incisions with a carving gouge of the correct curvature, or by following the outline with a carver's V tool.

(*d*) In this design the circles which make up the design are so arranged as to have a diameter equal to the space between the flat parallel margins of the rail, while the extreme point on the circumference of each large circle coincides with the centre of the next circle. The parts of smaller circles are added to give greater effect to the ornamentation. The faint lines at the left-hand end of the diagram give an idea as to how the design is constructed. In executing this design in wood or other more plastic material, the work should be cut away in such a manner as to make it appear that the design is made up of *discs*, each of which is partially hidden by the next disc to the right of it.

(*e*) This design is also based upon the circle. The centres of the circles which form the main part of the design occur upon the straight line which runs centrally between the parallel flat

portions on either edge of the banding. The straight construction

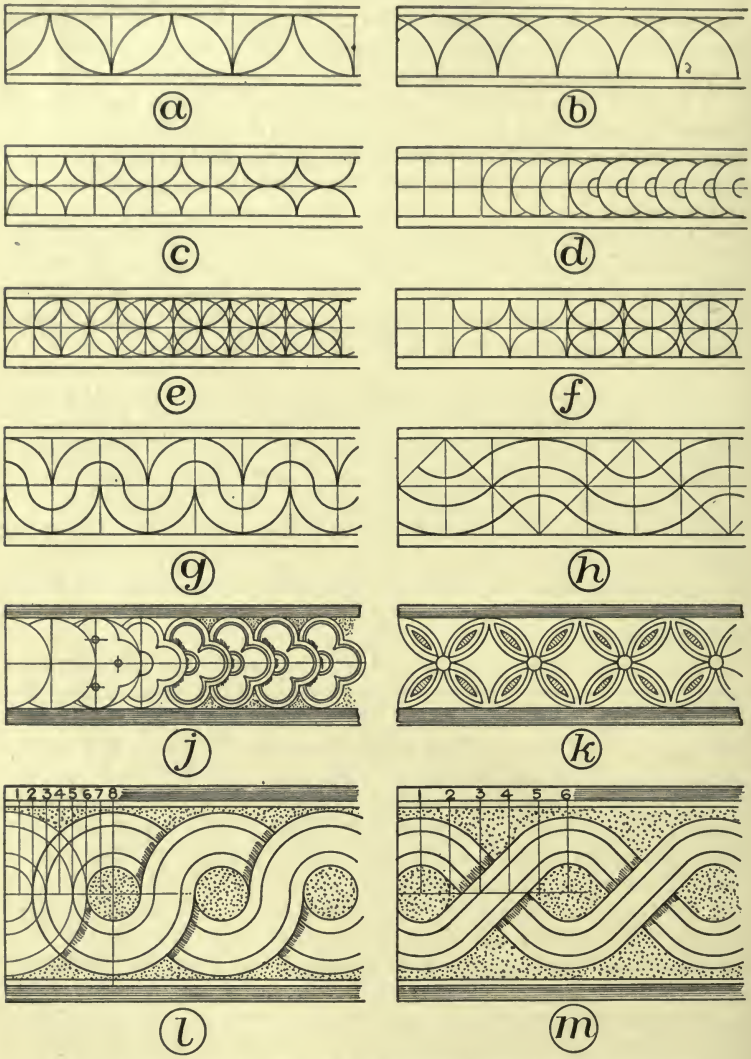


FIG. 184.—Circles and arcs used for ornamentation of banding and rails. (Repeat patterns.)

lines running across the pattern are so spaced as to have a clear

distance between them equal to half the diameter of the circles. A double series of semicircles is drawn, the centres of which lie upon the inner edge of the *banding*. These semicircles *interlace* the circles in such a way as to make up a series of **quatre foils**, *i.e.* four-leaved details.

(*f*) The straight lines used in the construction of this design are arranged in a manner similar to that in (*e*), but the whole circles in the central series are arranged in such a way that they are tangential to each other.

(*g*) This design is perhaps more suitable than any of the previous examples in cases where longer lengths of banding or moulding are to be ornamented. The whole design is made up of a series of semicircles, all the centres of which are upon the straight centre line of the banding. Each pair of semicircles is *concentric*, *i.e.* both semicircles are struck from a common centre.

(*h*) This somewhat unconventional design is made up of circular arcs, the centre for each group of three arcs occurring at the point of intersection of two diagonal lines. This and the previous example afford a very useful exercise in the careful manipulation of the bow-pen. Both (*g*) and (*h*) are perhaps most suitable as the basis of a coloured design, the spaces between the sinuous curves made up of the respective arcs being picked out in various colours. In such a case the diagonal lines would not appear in the finished colour scheme.

(*j*) represents a more elaborate design in which circles are employed. The finer lines at the left-hand side indicate how the design is built up, the method of arrangement being somewhat similar to that in (*e*). After the main circles are arranged in position as shown at the extreme left, the various details are added, as shown. The right-hand side of the design indicates how the wood or other material is cut away to secure the necessary relief, and to make the pattern stand out more plainly.

Ornamental Plaster Work.—(*k*) This design may be used for **ornamental plaster work** and its structure should be compared with that shown in greater detail in (*e*).

(*l*) This and the next design (*m*) are examples of the **shallow relief strap-work patterns** that figure so largely in the *carved* work of the churches and other old buildings in Scandinavia. Both are exceedingly useful designs for the woodcarver, and may be employed either for the ornamentation of a longer length of rail or banding, or for filling smaller spaces, such as panels, in which case all that is necessary is to make use of two or more details of the pattern according to the space to be filled. All the curved lines in this design are arcs of circles, the centres in every

case occurring upon the main centre line of the banding. The finer lines to the left and the numbering will help in the construction of such a pattern.

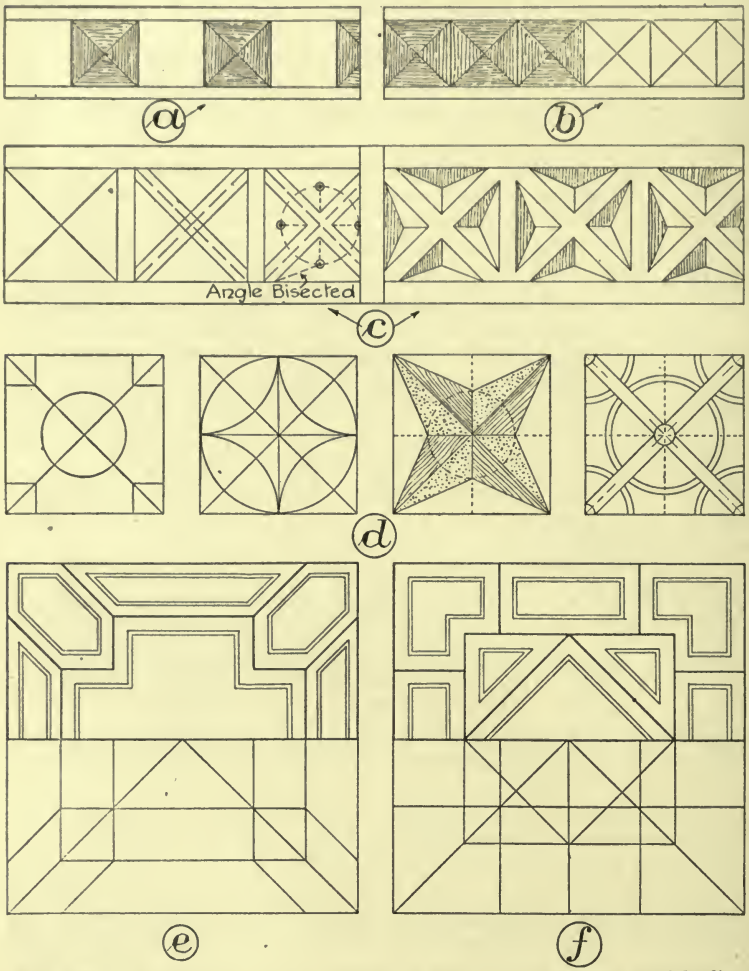


FIG. 185.—(*a*), (*b*), and (*c*) Ornamentation of rails, etc., by means of straight line patterns. (*d*) Patterns to fit square spaces. (*e*) and (*f*) Suggestive treatment for square ceilings.

(*m*) is a **strap work pattern** made up of *circular arcs* and *straight lines*, the centre for striking all the curves in the first unit being on the centre line immediately below the point 1,

while the centre for all the curves in the next unit is immediately below 6. Both the patterns (*k*) and (*m*) are exceedingly effective for the **decoration of rails** and **small panels** in antique oak work.

Curved lines are usually more pleasing than *straight* lines in design, but in some of the examples which follow it will be noted that a very effective design for certain specific purposes may be built up almost entirely of straight lines.

Designs involving the use of Straight Lines.—Fig. 185 (*a*) shows a method of ornamenting a rail by glueing down upon it a series of **pyramid shape blocks** which are represented by the shaded portions in the diagram.

(*b*) is a simple method of embellishing a wooden rail by first **chamfering** the centre portion in such a way as to produce a ridge-like *arris* along the centre and then chiselling away the wood to form a series of *stops* to the *chamfer* as shown.

(*c*) represents a simple and effective design for the type of **incised woodwork** usually classed as *chip-carving*. The left-hand portion of the sketch indicates the method of setting out the design, while at the right-hand side the shaded parts indicate the triangular sinkings where the wood has been chiselled away.

(*d*) The first two designs here illustrate a method of ornamenting a square panel either with colour or inlay work. They are equally suitable as suggestive designs for the arrangement of the sash bars in an ornamental door. The third design is often used as a basis for inlaying a panel or table top, the shaded portions which make up the star representing the different coloured woods employed for the purpose. The fourth design is suitable for ornamental plaster work, and does not call for any comment.

Designs suitable for the Ornamentation of Square and Rectangular Spaces.—(*e*) and (*f*) are designs made up entirely of **straight lines**, the whole design being so arranged as to fill a **square space**. Both these designs should prove suggestive in connection with *panelled ceilings* and *moulded panels* in *wainscotting*. The lower half of the drawing in each case indicates the basis of the structure of the design. Fig. 186 (*a*), (*b*), (*c*) and (*d*) are intended to show how rectangular spaces may be ornamented in a pleasing manner. That in (*a*) makes a very suitable design for an ornamental chest with recessed panelled sides or lid, the part shown in bold lines being first fretted away and the panelled part (faint lines) being afterwards glued on the back of the fretted portion.

(*b*) may be employed in various ways for embellishing a

rectangular space. In its simplest form the straight lines forming the pattern may be simply *incised* in the wood with a V tool, while in a more elaborate method of execution the pattern may be built up by means of narrow *astragals* or *beads*

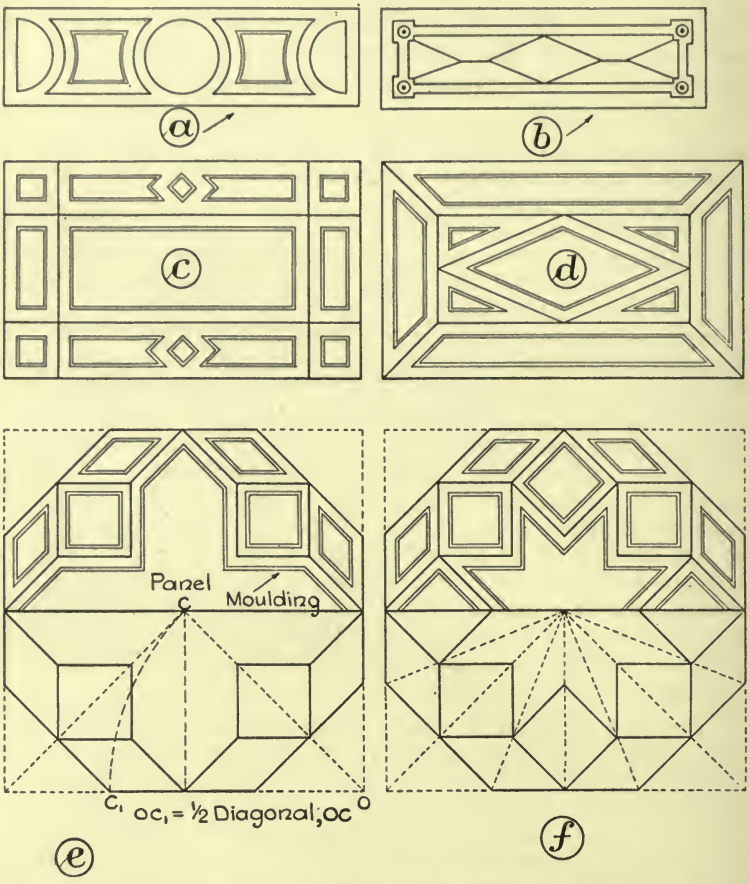


FIG. 186.—(*a*) Ornamentation of front of wooden chest. (*b*) Lid of box. (*c*) and (*d*) Rectangular panelled ceilings. (*e*) and (*f*) Octagonal panelled ceilings.

planted down upon the surface of the main part of the woodwork.

Panelled Ceilings.—(*c*) and (*d*) are illustrations of the decorative treatment of **panelled ceilings** of a *rectangular* character, the panels being formed by means of a wooden framework upon

which moulding of a bold character is mitred and fixed. (*e*) and (*f*) show the method of treatment of *octagonal* ceilings panelled and moulded as already described. The lower half of the diagram in each case is so arranged as to show the initial stages of the construction of the design.

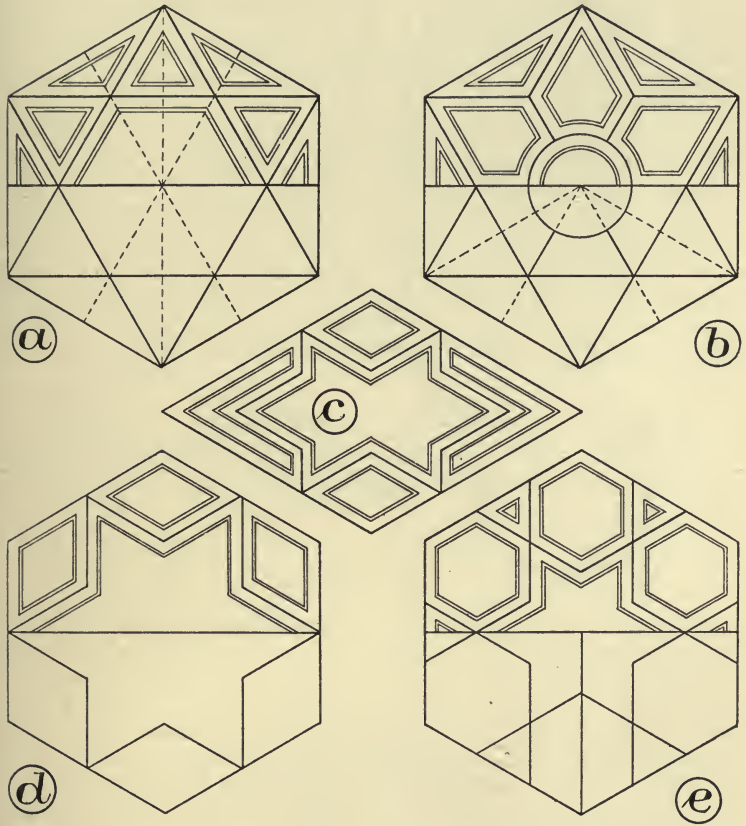


FIG. 187.—(*a*) and (*b*) Treatment of hexagonal panelled ceilings based upon interlacing triangles. (*c*) Treatment of diamond-shaped ceiling. (*d*) and (*e*) Alternative methods of ornament for hexagonal ceilings.

Fig. 187 (*a*) shows the method of treatment of a *hexagonal* panelled ceiling. The main lines of the design are shown in the lower half of the diagram, the first step being to draw the two large equilateral triangles interlacing each other. The centre hexagonal space obtained in this way is the main panel,

while at the apex of each of the larger equilateral triangles a small equilateral triangle is formed, and each of the sides of the hexagonal outline of the ceiling serves as the base for an isosceles triangle.

(*b*) In this case, which also illustrates a *hexagonal* panelled ceiling, the two larger interlacing equilateral triangles are drawn as in the last example, but a circular central panel is introduced and the outline of the surrounding triangular panels is finished by drawing curves concentric with the outline of the central circular panels.

(*c*) shows the main lines in a design suitable for a panelled ceiling, or decorated panel in a framework, the basis of the design being a rhombus.

(*d*) and (*e*) are further examples of the decorative treatment of hexagonal panelled spaces or of panelled ceilings. In each case the main structure of the design is so arranged as to form a central six-point *star* with a series of six outer rhombus-shaped spaces arranged in (*a*), and in (*b*) a series of six hexagonal spaces disposed symmetrically around the central star-shaped panel.

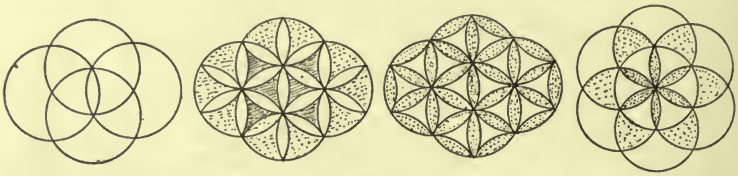


FIG. 188.—Progressive stages in the evolution of a design based on interlacing circles.

Designs involving the use of Interlacing Circles.—Fig. 188 is introduced as a further example of the use of *circles* as a basis of ornamentation. The first of four figures shows four *interlacing circles*. These circles are the basis for the design of the second pattern, which may be usefully employed for **marqueterie and inlaid work**, the different types of shading indicating the various coloured woods. In the third design small *lune-shaped* details are added at the circumference of each of the circles, thus making a more complex pattern for marqueterie. The fourth design also shows another adaptation of the circular *motif* for a design which may be extended indefinitely in such a way as to fill a larger space. Practice with the bow pen upon such examples as these will afford the student a very useful introduction to those examples which immediately follow.

Gothic Tracery.—Designs for Circular Windows.—Fig. 189 (*a*) to (*d*) are examples of different types of **circular windows** in

which geometrical details are employed as the basis of ornamentation. The method of arranging the design will be better understood if the small sketches at the bottom of the sheet (*a'*) to (*d'*) are compared with the larger sketches above, thus (*a'*) should

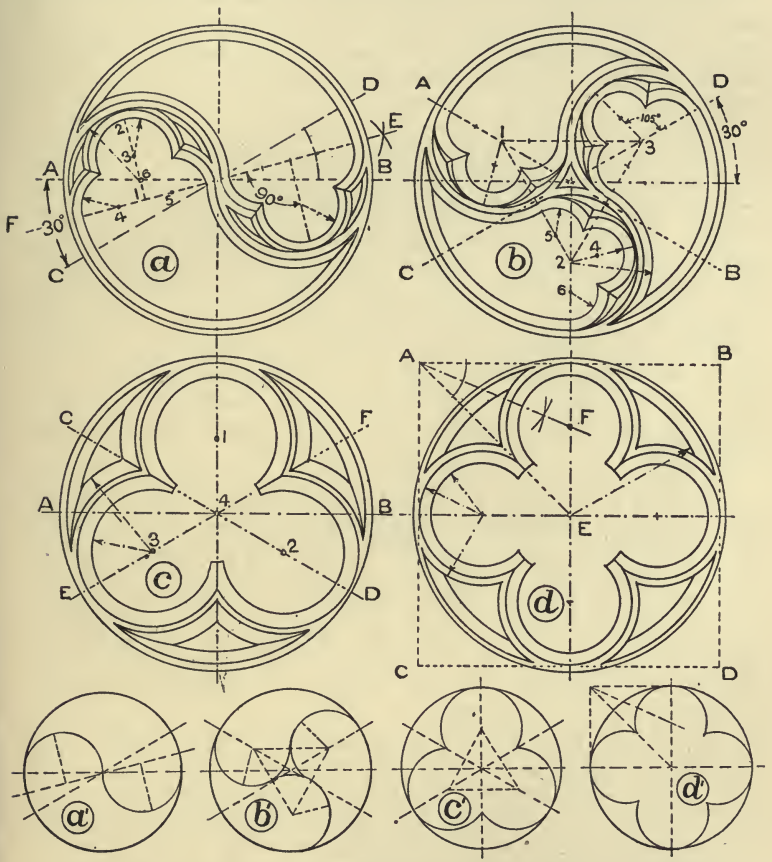


FIG. 189.—(*a*), (*b*), (*c*), (*d*) Designs for circular Gothic tracery windows. (*a'*), (*b'*), (*c'*), (*d'*) Sketches showing basis of construction employed in the respective designs (*a*), (*b*), (*c*), (*d*) above.

be compared with (*a*); (*b'*) with (*b*), and so on. The first step in designing (*a*) (see *a'*) is that the diameter AB of the circular opening is drawn and divided into four equal parts. Using the first and third points of division as a centre the two semicircles in (*a'*) may then be drawn. The centre for one of

these semicircles is shown at 6 in (*a*). A line CD making 30° with AB, and passing through the centre of the circular window, is then drawn. The angle between AB and CD is then bisected and the line EF is drawn as shown. Lines such as 2,1 are then drawn at right angles to EF. The point 6, at which 2,1 intersects AB, is the centre required for the larger arcs, while the points 3, 4, and 5 are the centres respectively for the smaller arcs which make up the foliation at the widest part of each of the two openings.

(*b*) In this case the details are similar to those in (*a*), but the *foliated* design occurs three times. Two diameters, AB and CD, are drawn, each making 30° with a horizontal line through the centre of the window, as indicated in (*b'*). An equilateral triangle, 1,2,3, is drawn, and the three points are used as centres for the outer curves in each of the three lights. At the points 1, 2, and 3, lines making 105° with the respective diameters are drawn, as shown at 3. The three points, 4, etc., are used as centres for the smaller curves at the centre of the head of each light, while points 6, etc., on the respective diameters, and 5, etc., on the three sides of the triangle, 1, 2, 3, are the centres for striking the smaller arcs on either side of the centre of each light.

Trefoil and Quatrefoil.—(*c*) The design in this example is based upon a **trefoil,** the simple outline of the construction of which is shown at (*c'*). Three diameters are drawn, as shown, AB, CD, and EF. The three points, 1, 2, 3, are the corners of an equilateral triangle, as shown at (*c'*), and are used as the centres for describing the principal curves in the three parts of the trefoil and the outer chamfers, while the lines CD, EF, and the vertical diameter through 1,4, give the correct position for intersection of the chamfers.

(*d*) Shows another method of treatment of a window for a circular opening, in which the principal circle is spaced out into four smaller circular openings, thus forming a **quatrefoil.** The small sketch (*d'*) indicates the basis of the construction, while in (*d*) the centres for the various arcs of circles, and the respective radius lines, are shown.

Designs for Triangular Window Openings.—Fig. 190 (*a*) and (*b*) illustrate a method of treatment of a **triangular opening,** the design for which is built up as follows: Draw an equilateral triangle ABC, and with A, B, and C respectively as centres, describe the arcs BDA, AEC, etc. Bisect the angles ABC, ACB, etc., and draw the bisectors BE, CD, etc. Bisect the angles AEB, etc., *i.e.* the angle between the arc AE and the

line EB, and draw the bisectors EF, E2, etc. The points 1, 2, and 3 are the centres for the three circles. Join the points 1, 2, and 3 (*b*), making the equilateral triangle 1,2,3. Draw the inscribed circle, 4,5,6, and within this circle draw the inscribed equilateral triangle, part of the sides of which at 7, 8, and 9 give

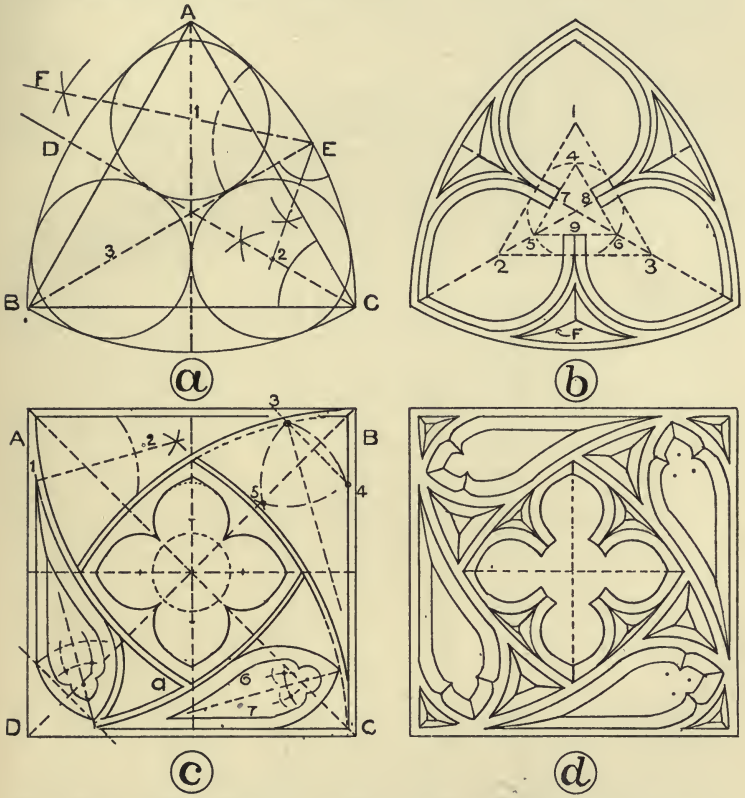


FIG. 190.—(*a*) Basis of construction of design for (*b*). (*b*) Design for tracery window to fill a triangular opening. (*c*) Basis of design for (*d*). (*d*) Square tracery window.

the correct line for finishing the straight parts of the stonework, as shown.

Designs for Square Window Openings.—Fig. 190 (*c*) and (*d*) show a more elaborate geometrical design applicable to a **square opening.** The pattern is built up as shown in (*c*). Draw the square ABCD, and within this square draw lines parallel to the sides of the square, and passing through the

points 1, 4, etc., thus forming a narrow parallel margin around the outer edge of the window. With centres A, B, C, and D respectively, and radius = B1—*i.e.* the length of side of original square less the width of the narrow margin—describe the four

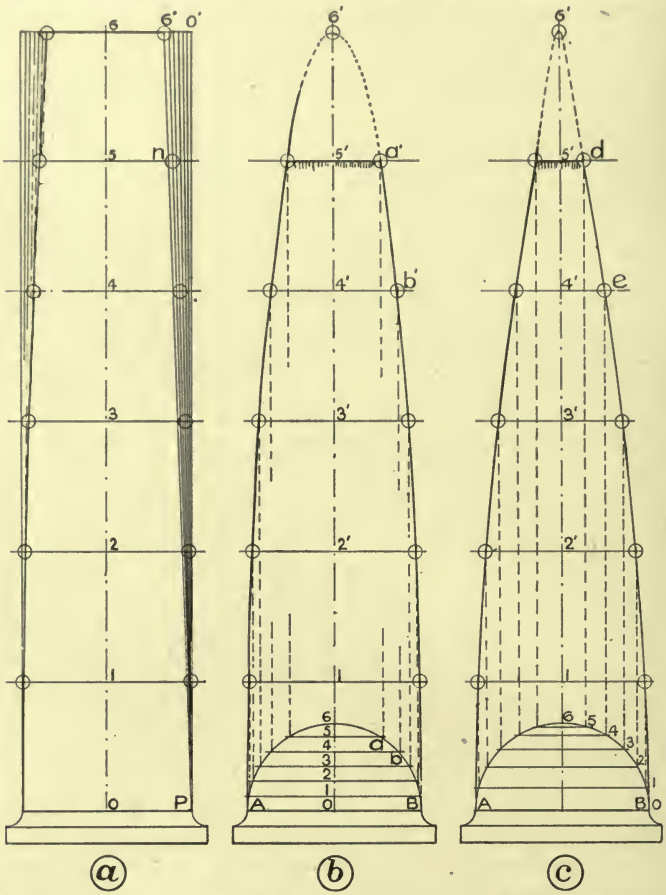


FIG. 191.—Shaping of columns. (*a*) Parabolic column. (*b*) Elliptical column. (*c*) Hyperbolic column.

pairs of curved lines, as at 1, *a*, etc. Within the space enclosed by these four pairs of lines set out the central portion of the window to a quatrefoil pattern, as shown. The successive steps in the ornamentation of the four corners of the square are shown at A, B, C, and D. The first step, shown at A, is to draw the

bisector 1,2. The second step (B) is to draw the arc 4,5, using as centre the darkened spot near 3, where the bisector 1,2 (A) intersects the dotted curve. With centres 4 and 5 respectively, describe the arcs 4,3 ; 5,3. This outline is added to in the manner shown at the corner C, and continued at D. The small triangular spaces are afterwards filled in, and lines drawn to represent the chamfers ; and the whole design completed in the manner shown in the sketch (*d*).

Shaping of Columns—Entasis.—Fig. 191 (*a*), (*b*), and (*c*) show various methods adopted for obtaining the correct **entasis**, or *shaping* of columns, in some of the well-known orders of Architecture. That shown at (*a*) is an example of a **Parabolic** column. The length of the column 0,6 ; and the diameter of the upper and lower ends having been fixed, the length of the column is divided into any number of equal parts, say six, at 1, 2, 3, 4, 5, and lines are drawn through these points parallel to the base of the column. The distance 0′6′ at the top of the column (*i.e.* the difference between the semi-diameter of the top of the column, and the semi-diameter at its base) is also divided into a number of equal parts, in this case, six, and from the respective points of division radiating lines are drawn to the exterior point, P, at the base of the column, as shown on the right-hand side of the illustration. The radiating line from the fifth point of division on 0′,6′, crosses the horizontal line through 5 at the point *n*, indicated by a small circle. Similarly, the intersection of the fourth radiating line and the horizontal line through 4, is also shown. These respective points of intersection are used, as shown on the left-hand side of the sketch, for giving the correct line for shaping the outline of the column. A *diminishing rule*, made *wider at the top* end by an amount equal to 0′6′, and having its *inner edge shaped* to the required curve, is used for testing the shape of the column as the shaping is proceeding.

Fig. 191 (*b*) is a longitudinal section through an **Elliptical** column, the correct outline for which is obtained as follows : On the diameter AB of the base of the column describe a semicircle A6B, and divide the radius, 0,6, of this semicircle into a number of equal parts (in this case six) at 1, 2, 3, etc. Through the points 1, 2, 3, etc., draw *horizontal lines* parallel to AB, as shown at 5*a*, 4*b*, etc. Divide the length, 0,5′, of the column into the *same number* of equal parts, and draw lines through these points and at *right angles* to the axis of the column. From the point *a* on the semicircle A6B draw a line *a*,*a*′, *parallel* to the axis of the column, and intersecting the horizontal line 5′*a*′ at *a*′. Similarly, from *b*, on the semicircle, draw *b*,*b*′ *parallel to the axis*

of the column and intersecting the line $4'b'$ through the fourth point of division at b'. Proceed in the same manner to obtain points on the horizontal lines $3'$, $2'$, and $1'$, and draw a fair curve through the points so found. Complete the left-hand side of the column in a similar manner. In the illustration the curved outline through $a'b'$ is produced to $6'$ by means of a dotted line, in order to show that the outline of the column, with the dotted portion $a'6'$ added, forms a **semi-ellipse.**

(*c*) is a longitudinal section through a **Hyperbolic** column. In this case the semicircle A6B on the diameter of the base of the column is drawn as in the last case; but in this case the *circumference of the semicircle* is divided into, say, six equal parts at 1, 2, 3, 4, 5. From these points lines $5d$, $4e$, are drawn *parallel to the axis* of the column, to meet the equally spaced horizontal lines $5d$ and $4e$ at d and e, respectively. The remaining points on the outline are obtained in a similar manner, and a fair curve is then drawn through the respective points d, e, etc., as shown. In this, as in the last case, the dotted portion between d and $6'$ is only added in order to complete the construction, the top end of the pillar being indicated by the shaded line $5'd$.

CHAPTER XI

BRICK AND MASONRY ARCHES

THE correct method of setting out the various types of arch in brickwork and masonry is based upon simple geometrical construction. The following examples will serve to illustrate some of the most important types.

(*a*) and (*b*), Fig. 192, illustrate a **Round, Roman**, or **Semi-circular** arch. The outline of the arch, as its name implies, is a semicircle, the centre of which is at the middle point 1 of the *springing line*. The joint lines 2,3; 4,5; etc., between the voussoirs and keystone, all radiate towards this centre, *i.e.* 1. Two alternative methods of treatment are illustrated in (*a*) and (*b*) respectively. At (*a*) the extrados, or outer edges of the voussoirs, are *stepped*, or cut in such a way as to course properly with the main part of the masonry. The length of joint line 2,3 having been determined, make 4,5 and all the remaining joint lines equal to 2,3. A semicircle, with centre 1 and radius 1,2, will cut all the joint lines to the proper length.

(*b*), Fig. 192, shows the same type of arch, but having *rebated* voussoirs and *curved* extrados. The joint line 2,4 from the intrados up to the rebate, is found by drawing a radial line to the centre of the springing line. The joint line 1,3 from the rebate out to the extrados is drawn parallel to the inner half of the joint line, as shown.

(*c*) and (*d*) are examples of so-called **Three-centre arches.** In (*c*) the span is divided into four equal parts at 2, 1, and 3. With centres 2 and 3 and radius 2,1 describe semicircles meeting at 1. With centre 1 and radius 1B describe the quadrant BD. From D draw D4 through point 3 on the springing line. With D as centre and radius D4 draw the curves to complete the top part of the arch. The joint lines for the lower part of the arch, *i.e.* from B to 4, are radial lines drawn from points 2 and 3 respectively, while the joint lines in the top portion of the arch are radial lines drawn from D.

(*d*) is another example of a three-centred arch. In this case the span 0,3 is divided into 3 equal parts at 1 and 2. With 1 and 2 as centres and radius 2,3 describe arcs 3,5 and 0,6. Using

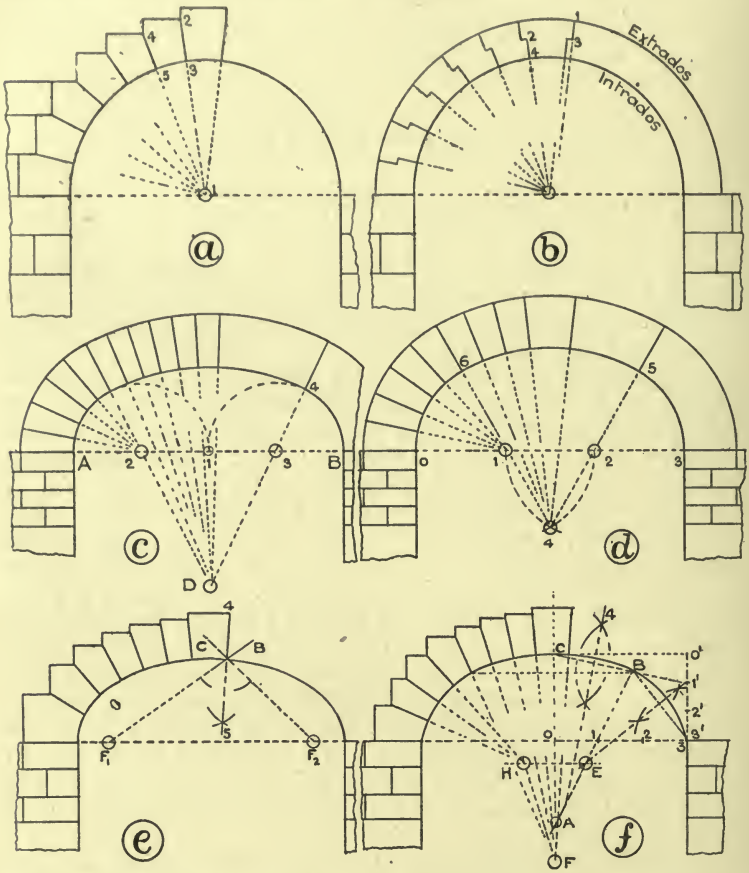


FIG. 192.—Brickwork and masonry arches. (*a*) and (*b*) Roman or semicircular arch. (*c*) and (*d*) Three-centred arches. (*e*) Elliptical arch. (*f*) False elliptical arch.

the line 1,2 as base construct an equilateral triangle 1,4,2. Produce the sides 4,1 and 4,2 of the equilateral triangle to intersect the arcs 3,5 and 0,6 at 5 and 6 respectively. With centre 4 and radius 4,5 draw the arc 5,6 which completes the top part of the arch. Points 1 and 2 are centres from which to draw radial lines for joints in the lower part of the arch. Similarly, point 4

is used for determining the joint lines in the upper part of the arch. Both these arches resemble a *true elliptical arch* and may be used as *substitutes* for the latter.

Compare the results obtained by (*c*) and (*d*). Note that (*d*) gives a *slightly greater rise* for any given *span* than the method illustrated in (*c*) produces. The observant student will notice that in each case, (*c*) and (*d*), the *span* of the arch is the only data given for working the problem. The *rise* is *determined by construction.* The advantage of such an arch is that *only two templates are required* for cutting the voussoirs : one for those in the lower parts 3,5 and 0,6 (*d*), and another template for the voussoirs from the keystone to 5 on the right and 6 on the left.

(*e*) shows a **true elliptic arch,** in masonry, with *stepped* extrados. The construction of the ellipse has already been fully explained (Chapter IX).

Given the *span* and *rise* of an **elliptic arch**, the curve is drawn by any of the methods already described. Note that the *span* of the arch is the *major axis* of the ellipse and the *rise* in the *semi-minor axis.*

The joint lines for such an arch are not as easily obtained as in the three-centred arches already described. Each joint line is a *normal* to the curve, see (Chapter IX). Hence it is necessary to obtain the **foci** F_1 and F_2. Draw the *focal lines* F_1B and F_2C and bisect the angle between the focal lines as shown at 4,5. Having obtained the correct joint lines for the keystone, decide how many voussoirs there are to be and divide the intrados into the required number of equal parts. At each point of division draw focal lines to F_1 and F_2 and bisect the angle so formed in order to obtain the normal at that point. The various joint lines are obtained in this way.

Compare this arch with (*c*) and (*d*). It has a *more graceful curve*, and is a *very strong* type of arch. Its drawback is that of *extra cost of labour*, as a separate template has to be made for each voussoir in one half of the arch. *No two voussoirs in the same half of the arch are shaped exactly alike. Experiment.*—Make a full-size drawing of any two voussoirs. Trace one of them, and by superimposing it on the drawing of the other voussoir verify the foregoing statements.

The so-called **bricklayer's ellipse** (*f*) is another method of obtaining a **false ellipse.** It resembles the true ellipse (*e*) more closely than either (*c*) or (*d*). This is a four-centred arch, the centres of which are obtained thus :—

Divide the half span 0,3 into three equal parts 1, 2, 3. At 3 erect a perpendicular and set out the required rise 3,0′. Divide the rise into three equal parts at 1′ and 2′. Join 1′ to C, the *crown*

of the arch. Find a point A at a distance below the *springing line* equal to the rise, *i.e.* make oA = oC. From A draw a line AB through point 1 on the springing line. This line meets the line C1′ in B. Join B3 and bisect the two chord lines CB and B3. The bisector at 4 produced below A gives the centre F. The line bisecting B3 cuts BA in E. Draw a horizontal line through E and obtain H equidistant from the centre line CF.

Although this is called a *four-centred* arch, only *three* centres are actually used for describing the curves, viz. H, E, and F.

With centres H and E and radius = EB describe the arcs forming the lower part of the arch. With centre F, radius FB, or FC, describe the arc which completes the upper part of the arch.

The joint lines in the lower part of the arch radiate from H and E respectively, and those in the upper part of the arch all radiate from F.

Double Doorway and Three-centred Arch.—(*a*) Fig. 193 affords a good exercise in setting out three-centred arches. A three-centred door frame provided with two three-centred door openings is shown.

Set out the span and rise as indicated by the dotted lines 1,3, and 3,2. Join 1, 2 and bisect the angles 3,2,1 and 3,1,2. These bisectors, 1,4 and 2,5, intersect at 6, making the angle 2,6,1. Bisect this angle and draw 6,10. Produce to meet centre line of doorway in 9. Find point 12 equidistant from centre line and on the opposite side to 10. Bisect the line 2,6 and draw bisector 7,8.

Then 10 and 12 are centres for striking the curves in the lower part of the doorway and frame; 9 is the centre for describing the curve for upper part of doorway; while 8 is the centre for describing all the flatter curves at the crown of the arch. The chamfered opening A is struck partly from centres 12 and 13, and partly from 8. The dimensions for *jambs*, etc., is a matter of judgment.

(*c*) is an example of a **Moorish arch.** Usually such arches are of a much more decorative character than that illustrated. Perhaps the best example in London is to be seen at the box office entrance to the **Alhambra Theatre** in Charing Cross Road.

This is a *segmental* arch, in which the segment is *greater than a semicircle.* The complete circle, of which the arch forms a part, is so arranged that it touches the level line 1,2 of the top of the *corbels* which support the arch. The distance 6,5 from the centre of the circle, 6, to the springing line 3,4 is one-half

the radius of the circle. The remainder of the construction is obvious.

Arabian Arch.—(*d*) is an **Arabian arch**, and is typical of various examples of constructional work to be found in Oriental

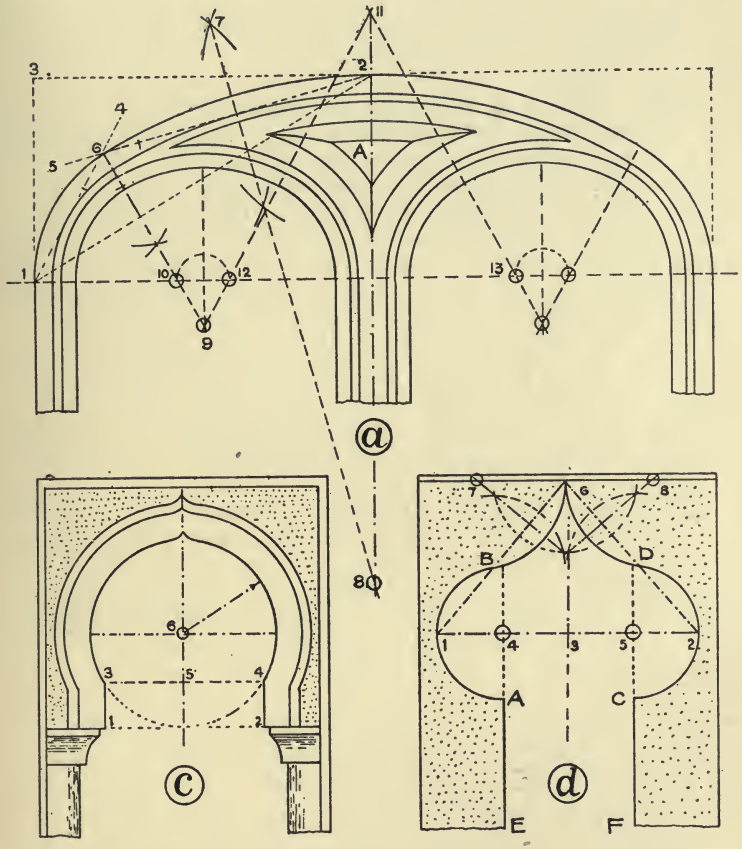


FIG. 193.—(*a*) Double doorway and three-centred arch. (*c*) Moorish arch. (*d*) Arabian arch.

countries, in *mosques* and other public buildings. The outline of the arch is made up of arcs of circles. The springing line 1,2 is divided into four equal parts at 4, 3, and 5. From 3 erect a perpendicular 3,6 to the desired height. (In the diagram the rise 3,6 is equal to half the span, *i.e.* 3,6 = 3,1.) Join 1 to 6

and 2 to 6, making the triangle 1,6,2. With centres 4 and 5 and radius 4,1, describe the arcs A1B and C2D.

Bisect the chords B6 and D6 and produce the bisectors to meet a horizontal line through 6 at 7 and 8. With centres 7 and 8 and radius 7,6, draw the arcs B6 and D6 to complete the outline of the arch proper. The width of the opening at the lower part of the arch is determined by drawing the vertical lines AE and CF in line with centres 4 and 5.

Gothic Arches in Masonry and Brickwork.—(*a*) Fig. 194 represents a **Gothic** or **equilateral** arch. An equilateral triangle 2,4,3 is constructed on the springing line 2,3 as base. The joint lines are found by dividing the intrados of the arch into the required number of spaces, according as the construction is in stone or brick. Radial lines are then drawn from each of these points to the centre 2 for joints on the right of the keystone, and to centre 3 for joints on the left side. This method of determining the joint lines serves very well in masonry. In brickwork, however, the method is not so satisfactory, as it requires the brick to be clipped at the top of the arch, as shown by the shaded wedge-shaped space at 4. If there is no weight on the top of an arch built in this way and the pressure on the sides is very great, *there is a tendency to push out the wedge-shaped bricks* at the top and thus cause the arch to collapse.

(*b*) indicates how this fault may be overcome. The arch is struck as shown in (*a*), using the centres 2 and 3 as before, for determining the outline of the arch. The points 4 and 5 are selected about three-fourths of the way up the extrados of the arch, and the lines 2,5 and 3,4 drawn. These lines intersect at 6. The joint lines on each side of the lower portion of the arch are found as in (*a*) by drawing radial lines from 2 and 3 respectively. The joint lines from 4 and 5 to the crown of the arch are found by drawing lines to radiate from point 6. By making use of this construction the top bricks near the crown of the arch are all uniform in size and shape and no faulty wedge-shaped pieces occur as in (*a*).

(*c*) illustrates another method of arranging the joint lines in an Equilateral Arch. Decide, first, how many voussoirs are required on each side of the keystone. In the diagram six are shown. Find the centre, 0, of the springing line, and divide the distance along the springing line between 0 and 6 into as many equal parts as there are to be voussoirs in the arch, say 6. Divide the soffit of the arch into the same number of equal parts, not counting the keystone. Join the lowest point, 6, on the soffit to the outside point 6, on the springing line. Join the other points

on the soffit to the respective points on the springing line, as shown. In the diagram only the alternate points 2, 4 6, are connected. This method of arrangement gives better-shaped voussoirs than if all the joints were made to radiate from one centre on either side.

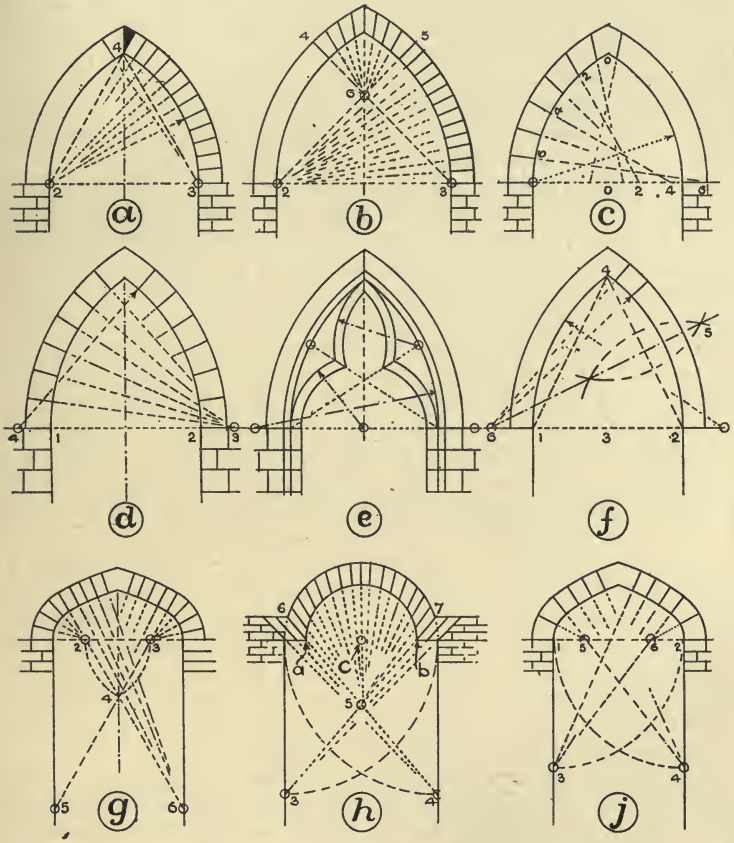


FIG. 194.—Brickwork arches. (*a*) Gothic or equilateral, showing ordinary method of obtaining joint lines. (*b*) and (*c*) Gothic or equilateral, showing alternative method of obtaining joint lines. (*d*), (*e*) and (*f*) Pointed or lancet. (*g*) Tudor. (*h*) Queen Anne. (*j*) Tudor (alternative method).

(*d*), (*e*) and (*f*) represent various forms of **Pointed or Lancet Arch.** In such cases the outline of the arch is invariably described with radii greater than the span, as shown at 3 and 4 (*d*). The joint lines are made to radiate from these centres. (*e*)

illustrates the same type of arch with added tracery. The centres for describing the arcs which make up the outline of the tracery are indicated.

(*f*) indicates the method adopted when a Pointed Arch has to be made to fulfil certain conditions as to span and height. A triangle 1,4,2 is constructed, 1,2 being the required span and 4 the height of the crown of the arch. Bisect the sides 1,4 and 2,4 of this triangle, as shown, and produce the bisector to meet the springing line in 6. This point, 6, and the corresponding point on the right-hand side, are the centres from which to describe the outline of the arch. The joint lines are made to radiate from these points.

(*g*) and (*j*) show two methods of setting out a **Tudor** Arch; so called because this type of arch is a distinguishing feature of the later period of Gothic architecture, particularly of ecclesiastical architecture executed during the **Tudor Period** in **English History.** In (*g*) the half spans are bisected at 2 and 3 respectively. With centres 2 and 3 and radius 2,3 describe arcs intersecting at 4. Draw the equilateral triangle 2,3,4. Produce 3,4 and 2,4 to meet the reveals at 5 and 6, respectively. Then points 2 and 3 are the centres for describing the quicker curves at the lower part of the arch, and 5 and 6 are centres for completing the curves in the upper part of the arch. All joint lines in the lower part of the arch on each side radiate from 2 and 3 respectively, while those in the upper part of the arch radiate from 5 and 6.

In (*j*) the points 5 and 6 are found in the same manner as points 2 and 3 (*g*). With the springing line 2,1 as radius and 2 and 1 respectively as centres, describe arcs 1,4 and 2,3, to intersect the line of reveal in 3 and 4. Centres 5 and 6 are used for striking the lower part of the arch and 3 and 4 for the upper part. Comparing (*j*) with (*g*) it is evident that the construction illustrated in (*j*) gives a somewhat *greater rise for any given span* than that given by the construction in (*g*).

(*h*) represents a **Queen Anne Arch** in brickwork. The soffit and extrados of the arch are *parts of semicircles* struck from the centre C as indicated. The soffit of the arch, however, instead of finishing *in line* with the *reveals* as in an ordinary segmental arch, finishes about two bricks' thickness *within* the reveals, as at *a* and *b*.

With the springing line as radius describe quadrants to cut the line of reveals at 3 and 4. Join 4,*a* and 3,*b* and produce to meet the extrados of arch at 6 and 7 respectively. The point 5, at which 4*a* and 3*b* intersect, is used as a centre for drawing all the joint lines in the circular part of the arch, while the points 3

and 4 are used for drawing the joint lines in the flat part of the arch on either side.

Several other types of arch are illustrated in Fig. 195, **Carpentry Arches.** Students who are specially interested in Brickwork and Masonry should study the construction shown in the various cases, and endeavour to work them out as *masonry* arches, adding the necessary joint lines.

Carpentry Arches (Fig. 195).—Oak door frames and window frames were a prominent feature of building work executed during the middle and later periods of Gothic architecture. (*a*), (*b*) and (*c*), Fig. 195, represent some of the *earlier* forms, while the remaining diagrams are examples of *later* work in buildings erected during the **Tudor Period.**

The **Equilateral** arched frame (*a*) is a common type frequently met with in Gothic architecture. It is described as follows. With length of span 2,3 as radius, and centres 2 and 3, construct an equilateral triangle as shown. The vertex of this triangle gives the *rise* of the arch. The outer edge of the curved head is described from the same centres, 2 and 3. The *haunch* joints, *h*, are placed about one-quarter the height of the arch above the springing, and radiate from centres 2 and 3 respectively.

The **Lancet** frame (*b*) is of a more pointed character than the equilateral arch. To find the centres p_1 and p_2 produce the springing line each way and describe semicircles on the extended line, as shown, using a radius equal to half the span.

The **Drop** arch (*c*) is similar in some respects to the equilateral arch (*a*), but is made in *varying degrees of depression according to the fancy of the designer.* Having given the lengths of span and rise respectively, construct a triangle as shown, using the springing line as a base. Bisect the sides of this triangle, and produce the bisectors to meet the springing line at *p* and *q*. These points are the required centres.

The **Tudor** arch (*d*) differs from each of the foregoing in that the head is struck from *two centres* instead of *one.* To find these centres the span is divided into *four equal parts*, and *two semicircles* described as shown, having a radius equal to *one-quarter of the span.* Using the length of the span, *rp*, as a radius, and centres *r* and *p*, describe the arcs pp_1 and rp_1. Then p_1 and *q* are the two centres required for describing one half of the arch. The line drawn from p_1 through *q* and produced gives the correct position for the haunch joint.

The **Cyma Reversa, Ogee,** or **Wave Arch** (*e*), is described as follows. Using the springing line 2,3 as a base, construct an equilateral triangle (60°), as shown. Bisect the sides of this

triangle, as at 4, and through the point of bisection 4, draw a line 5,5 parallel to the springing line. With points 3 and 4 as

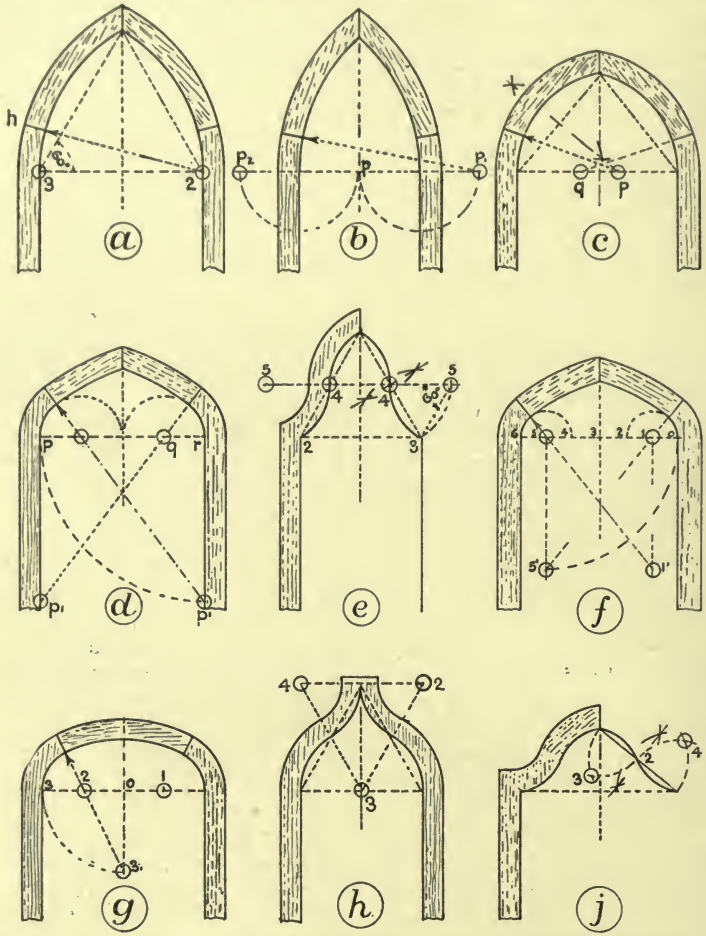


FIG. 195.—Carpentry arches and door-frame heads. (*a*) Equilateral arched frame. (*b*) Lancet-headed frame. (*c*) Drop-arched door-frame head. (*d*) Tudor-headed frame. (*e*) Ogee frame (cyma reversa). (*f*) Four-centred arch. (*g*) Three-centred arch. (*h*) Ogee frame (cyma recta). (*j*) Ogee frame (cyma reversa).

centres, and radius equal to 3,4, describe arcs cutting the horizontal line 5,5, as shown. The points 5, 5 are the centres for striking the lower half of the cyma reversa curve, while the

points 4, 4 are the centres for striking the curves in the upper half of the arch.

The **Four-Centred** arch (*f*) should be compared with the **Tudor** arch (*d*). In this case the span is divided into *six* equal parts, as shown. From points 1 and 5 draw vertical lines. Taking the length of span as radius and centres 0 and 6, draw quadrant as at 0,5, cutting the vertical line 5,5′, at 5′, as shown. The points 1 and 5 on the springing line are the centres from which to describe the arcs in the lower portion of the arch; 1′ and 5′ are the centres for describing the upper portion of the arch.

The **Three-Centred** arch (*g*) is of simple construction. The curve when complete resembles a semi-ellipse, and is therefore frequently used as a substitute for a true elliptical arch (see note, *re* Brickwork Arches, p. 163). The half span 0,3 is bisected at 2, which is the centre for describing the *quick* curve in the part of the arch near the springing line. With centre 0, and radius 0,3, describe arc 3,3′, cutting the vertical centre line at 3′. This is the centre for describing the *flat* curves at the crown of the arch.

The **Cyma Recta** or **Ogee** arch (*h*) is based upon the equilateral triangle (compare with (*e*)).

An equilateral triangle is built on the springing line as a base. Another equilateral triangle, *inverted*, is placed as shown at 2, 4, 3, so that its base passes through the vertex of the first triangle. The point 3 is the centre for the curves on both sides of the lower part of the arch. Points 2 and 4 are centres for the right and left sides respectively of the upper part of the arch. This form of arch is frequently used for window frames, and, less frequently, for pavilion roofs.

The **Reversed Ogee** (*j*) is similar to that shown in (*e*), but the *rise of the arch is less* as compared with the span. The relative amount of rise is purely arbitrary. In the illustration the rise is roughly one-third of the span.

Having given the span and rise, describe a triangle on the springing line as base, and the "rise" as altitude. Bisect the sides of the triangle as at 2. Points 3 and 4, from which the two parts of the ogee curve are struck, are found in the same manner as that already described. See Cyma Reversa (*e*) above.

CHAPTER XII

SOLID GEOMETRY

Theory of Projection.—The term *projection* as used in this connexion has a specific meaning. A view is said to be *projected* when it is **thrown forward** upon a screen, as the view on a lantern slide is projected upon a white opaque surface. If a rectangular object O, Fig. 196, is so placed that rays of light from

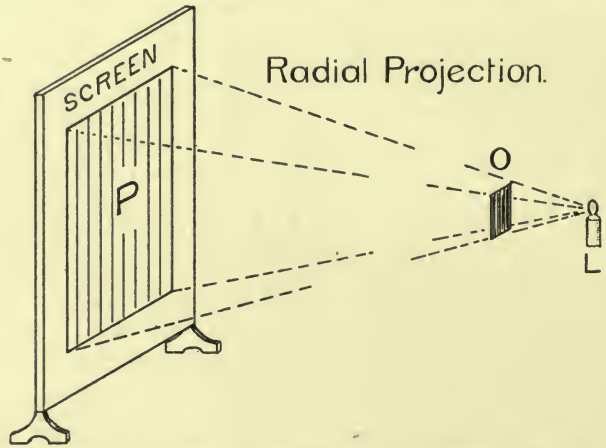


FIG. 196.—L : Light. O : Object, P : "Projection" of object on screen.

L fall upon it, the rays continue in straight lines from every point on the object O and diverge. If an opaque screen is placed in the path of these rays, an enlarged image or projection of the object O is thrown upon the screen as shown at P. Where the rays of light are divergent, as in this case, and the projection is an **enlargement** of the original, it is called a *radial projection.*

A thin sheet of card O, Fig. 197, is held in a vertical position and rays of light R,R from a distant source, *e.g.* the sun, fall upon the card. These rays are really divergent, but *the divergence is so slight* that for all practical purposes they may be taken as

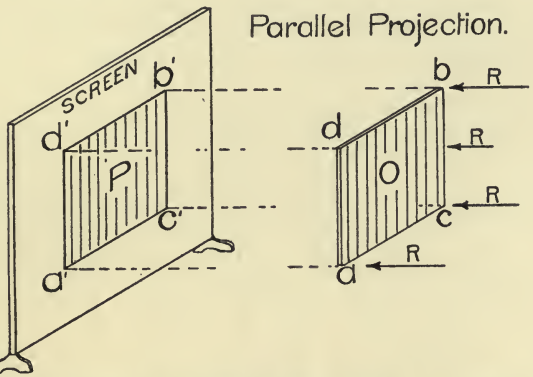


FIG. 197.—R, R, R: Parallel rays of light. O: Object. P: Projection of object on screen.

parallel rays. The rays of light which pass through the corners *a, c, b, d* of the card proceed in parallel straight lines and the image P projected on the screen is the **same size** as the original.

This is an example of *parallel projection.*

PLANS AND ELEVATIONS

Orthographic Projection.—When a plan and elevation of an object are to be drawn, it is assumed that the image of the object is *projected* forward as a result of the *parallel rays* which pass through every point on the object. The point at which the rays of light from any point meets the screen is said to be the **projection** of that point on the screen. The imaginary line from the actual point to its projection on the screen is called a **projector.** Thus the lines *aa', bb'*, etc., in Fig. 197, are *projectors*, and *a', b'*, etc., are the *projections* of the points *a, b*, etc., on the given plane (the screen). It is evident that these projectors might be parallel projectors, and yet meet the screen obliquely. In drawing plans and elevations, the parallel projectors are assumed to be *right* projectors, *i.e.* they meet the screen at right angles. In such a case, the drawing, *i.e.* plan, elevation, etc., is said to be an **orthographic projection** (*orthos* = *right*).

Horizontal Plane and Vertical Plane.—In order to make a drawing which shall indicate the position of an object in space,

two *plane surfaces* (A, Fig. 198) are assumed. These two *planes* are at right angles to each other, one being vertical and the other horizontal. It is usual to denote these by using the letters VP (**Vertical Plane**) and HP (**Horizontal Plane**).

Ground Line.—The intersection of these two surfaces is a straight line GL, and may be likened to the line in which the floor of a room meets the wall. This is known as the **Ground Line.**

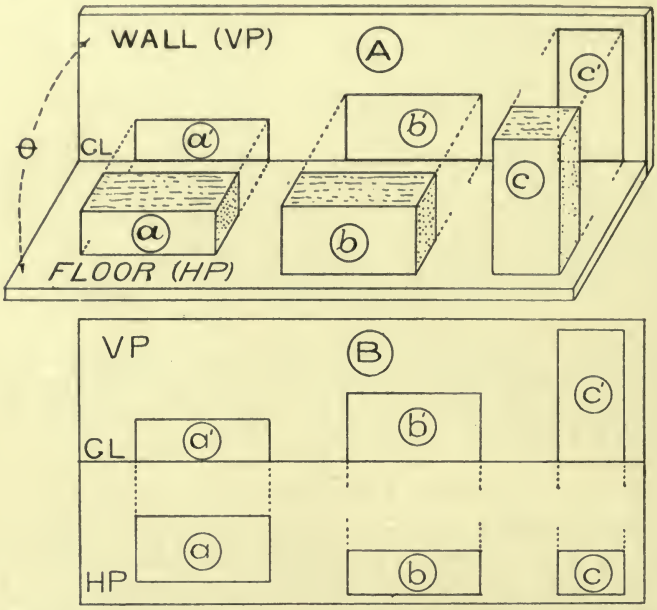


FIG. 198.—(*a*) Pictorial projection of brick in three different positions in relation to the vertical plane and horizontal plane. (*b*) Orthographic projection of brick in three different positions in relation to the vertical plane and horizontal plane.

Dihedral Angle.—The angle θ between these planes—and in fact the *real* angle between any two plane surfaces—is termed a **Dihedral Angle.**

Fig. 198 (A) is a pictorial representation of an ordinary building brick placed in three simple positions (*a*), (*b*), and (*c*), in each case with one face, or end, resting upon the ground (HP). If *projectors* are drawn at right angles to the wall (VP), as shown pictorially at (A), and the *projections* of the four corners of the rectangle are joined, an elevation of the object is obtained, as shown at (*a'*), (*b'*), (*c'*).

These elevations are the **orthographic projections** of the brick as they would appear if *projected* forward upon the VP.

This pictorial view (A) will help the student to understand the theory of the ordinary orthographic projection which is the basis of all plans, elevations, etc.

When a plan and elevation are to be drawn on paper, the face of the paper is *all in one plane*, and it is somewhat difficult at first to combine mentally the two drawings on the flat sheet of paper in such a way as to gain an idea of the position of the solid in space. Fig. 198 (B) illustrates this. The drawing represents a sheet of paper on which a ground line GL is drawn. If the sheet of paper is dented along the line GL, and folded until the part above GL is at right angles to the part of the paper

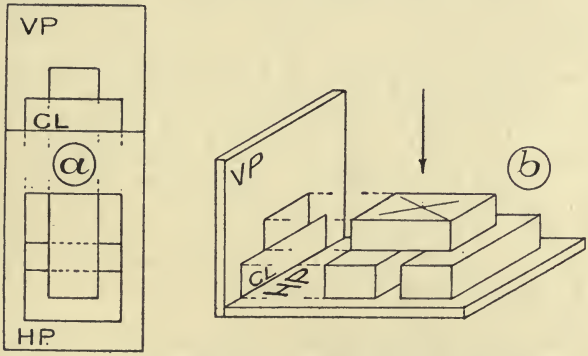


FIG. 199.—(*a*) Orthographic projection (plan and elevation) of three bricks. (*b*) Pictorial projection of bricks shown in (*a*).

below GL, the plane surface of the original sheet has now become two planes, the HP and the VP, similar to (A) in the illustration. Whenever plans and elevations are drawn with reference to any given ground line, the student must endeavour to realise that GL represents the line along which the *hinge* would occur if the paper were *folded* so as to make the two **Planes of Projection**. Referring to (B), Fig. 198, the distance between the rectangle *a* and the ground line indicates how far the inner edge of the brick is from the VP. (See *aa'* in pictorial sketch A.) Again, since the 9 ins. × 4½ ins. face of the brick rests upon the ground the elevation (*a'* in Fig. 198 (B)) will rest upon the ground line as shown. In *bb'* (A and B) the brick rests with one of its 9 ins. × 3 ins. faces on the ground. Hence the plan is a 9 ins. × 3 ins. rectangle and the elevation a 9 ins. × 4½ ins. rectangle. In *cc'* (A and B) the brick rests with one end 4½ ins.

× 3 ins. upon the ground. Hence the full length (9 ins.) of the brick appears in the elevation and the plan is a $4\frac{1}{2}$ ins. × 3 ins. rectangle. The pictorial drawings (A) and the orthographic projections (B) should be studied and carefully compared before proceeding further.

Unseen Edges.—Fig. 199 (*b*) represents pictorially three building bricks, the bottom pair baving their 9 ins. × $4\frac{1}{2}$ ins. faces resting on the HP, and their 9 ins. × 3 ins. faces parallel to the VP. The top brick has one of its ends $4\frac{1}{2}$ ins. × 3 ins. prarallel to the VP. Hence the elevation (*a*), Fig. 199, would be shown as two rectangles, the lower one 9 ins. × 3 ins. and the upper one $4\frac{1}{2}$ ins. × 3 ins. The dotted or *broken* lines in the plan (*a*) indicate that parts of the edges of the lower bricks are hidden by the top brick. It is usual to represent **unseen edges** in this way.

Square Based Pyramid.—(*a*) Fig. 200 represents a stone cap resting upon a brickwork pier. The base of the cap is square and four inclined surfaces meet at the top point or *vertex*. The cap is a practical example of a **square based pyramid**. In the elevation (*b*) the horizontal and vertical joint lines on the side farthest away from the VP are projected. In the plan (*b*) the four vertical faces of the pyramid *project* as solid lines, making a square, and the four vertical faces of the brickwork, which are *hidden* by the stone cap if observed from above, are represented by dotted lines.

To draw the plan and elevation (*b*). Draw any Ground Line GL. Set out the dotted square in plan to scale to represent an 18 ins. × 18 ins. brick pier, leaving a space between GL and the dotted square equal to the distance that the pier is supposed to stand in front of the VP. Allow a distance of 3 ins., to scale, and draw another square to represent the plan of the upright faces of the stone cap. Draw the diagonals of the square. These are the plans of the four *slant edges* of the pyramidal cap. The point at which these diagonals meet is the plan of the top point or vertex of the cap. Having completed the plan, draw dotted projectors at right angles to GL from both squares in the plan. The projectors from the dotted square in plan give the position of the vertical edges of the brickwork in the elevation. Seven courses of brickwork are shown. The joints between these courses as seen in elevation are represented by lines parallel to GL and 3 ins. apart (to scale). The vertical joints between bricks, half-bricks, and *closers* are then fitted in as follows. Bottom course, two whole bricks or stretchers each 9 ins., making 18 ins. Second course, commencing at the angles, or *quoins*, two

headers, showing $4\frac{1}{2}$ ins. on the face of work ; next, a closer on either side, showing $2\frac{1}{4}$ ins. wide on the face. The remaining space in the centre, $4\frac{1}{2}$ ins. wide, is made up by fitting in a header, making a total of 18 ins., *i.e.* three headers at $4\frac{1}{2}$ ins. $= 13\frac{1}{2}$ ins. + two closers at $2\frac{1}{4}$ ins. $= 4\frac{1}{2}$ ins., $13\frac{1}{2}$ ins. + $4\frac{1}{2}$ ins. $= 18$ ins.

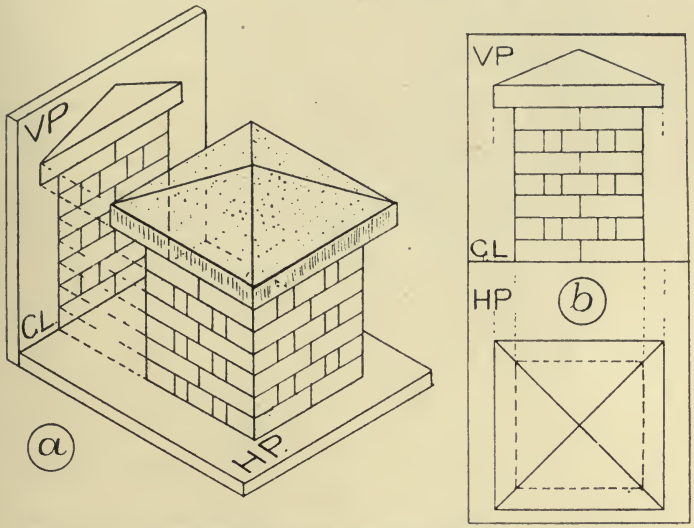


FIG. 200.—(*a*) Pictorial projection of brickwork pier. (*b*) Orthographic projection of pier shown in (*a*).

Projectors from the outer square in plan, drawn at right angles to the GL, and produced upwards, give the position of the vertical edges of the cap in elevation. Set up the height of these edges, 3 ins. Draw a projector from the intersection of the diagonals of the square in plan, and produce it upwards in order to fix the position of the vertex of the pyramid in elevation, making the difference in height between the square part of cap and the vertex $4\frac{1}{2}$ ins. to scale.

Cylinder.—The pictorial diagram (*a*), (*a'*), Fig. 201, represents a white lead keg or drum standing at the top of a flight of three stone steps. The keg is an example of the geometrical solid known as a **cylinder.** The pictorial sketch (*a'*) indicates what the character of the elevation of such a solid is when projectors are drawn at right angles to the VP. To make the orthographic projection *b*,*b'*, draw any ground line GL. Set out a rectangle in plan to represent the over-all length and width of the flight of

steps. Draw lines square across the rectangle to represent the plan of the front edges of the steps. Draw a circle equal to the diameter of the keg in the position shown. This is the plan of a cylinder with its *axis* or centre line vertical. From each of the

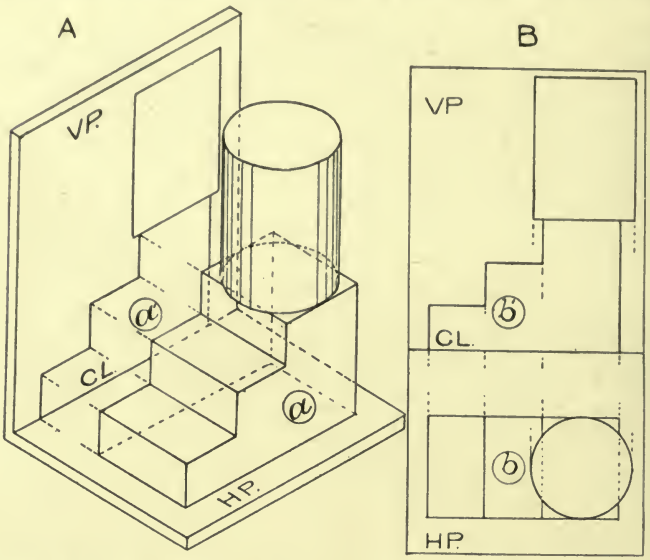


FIG. 201.—(*a*) Pictorial projection of steps and cylinder. (*b*) Orthographic projection of steps and cylinder.

points in plan draw projectors square with the GL. Set up the height of the *rise* for each successive step, and complete the outline of the three steps. Draw projectors from the circle in plan in order to determine the position of the long sides of rectangle which, in the elevation, represents the cylindrical drum.

Cone.—The inset pictorial sketch (*a*) Fig. 202 represents a conical sheet metal vessel much used in olden times for warming beer over the fire. This is an example of an *inverted* cone. If the shaded part of the cone near the vertex is removed in such a way as to make a funnel, a *frustum* of a cone is obtained.

The plan and elevation (*b*) Fig. 202, are made as follows: Draw a circle, in plan, equal to the diameter of the base of the inverted cone. Mark the centre *v* of this circle. This is the plan of the cone. By projecting from the centre of the circle *v* in plan the position of the vertex *v'* of the cone on GL is obtained. In the same manner, by drawing projectors from

the outer circle in plan, determine the position of the ends of the base of the triangle which is the elevation of the conical vessel. Plan and elevation of the handle are then drawn as shown. Note that any concentric circles parallel to base, such as *aa* and *bb*, appear as *straight lines* in elevation and *circles* in plan.

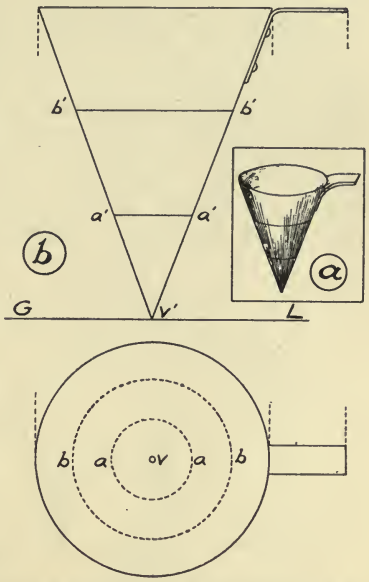


FIG. 202.—(*a*) Sketch of beer warmer as example of inverted cone. (*b*) Plan and elevation of beer warmer.

Frustum of Cone.—Fig. 203 (*a*) and (*b*) represents, pictorially and orthographically, a pail resting upon two bricks. The pail is an example of a *frustum* of a cone, to which reference has already been made. Referring to the pictorial sketch (*a*), note that the bricks are standing *on edge*, so that the elevation as projected on the VP is a 9 ins. × 4½ ins. rectangle. Both bricks are exactly behind each other as seen by an observer looking from the front, and towards the VP. Hence only *one* rectangle is needed as the elevation of the *two* bricks. Note that since the circular top and bottom of the pail are both *parallel* to the HP, the elevation of each of these ends is a *straight* line *parallel to* the GL.

Referring to (*b*), draw to scale the two circles which in plan represent the frustum of a cone (the pail), diameters of ends, 8 ins. and 12 ins. respectively. Draw the rectangles—dotted—9 ins. × 3 ins. which represent the plan of the bricks on edge, the view of which, as observed from above, is shut off by the larger end of the pail. Projectors drawn from the ends of these dotted rectangles in plan help to fix the position of the 9 ins. × 4½ ins. rectangle in elevation which represents the view obtained by an observer looking towards the VP. Projectors from the circumference of the small circle in plan help to fix the position of the lower end of the frustum in the elevation. Similarly, projectors from the larger circle in plan are used to determine the position of the points on the elevation of the wide end of the frustum, the difference in height between the

horizontal lines which represent the elevation of the two ends being 10 ins.

EXAMPLE.—Copy the orthographic projection (*b*): scale $\frac{1}{4}$ full size.

EXAMPLE (FIG. 204).—*Draw the plan and elevation of a conical spire 21 ft. high, diameter of base 16 ft., resting upon a square tower of 23 ft. side, showing the lead laid on with horizontal joints, five successive widths of lead being used to make up the slant height of the spire.*

The plan of the square tower is represented by the square *abcd*, the elevation of the top of the tower being as shown at *a'b'c'd'*, etc. The plan of the base of the spire is a circle *kl*, centre *v*, radius *vk* = 8. From *k* and *l* in plan draw projectors square with GL to meet the line

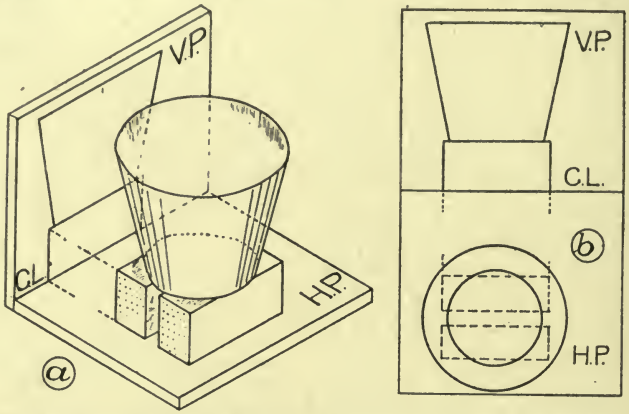


FIG. 203.—(*a*) Pictorial projection of frustum of cone. (*b*) Orthographic projection of frustum of cone. (A pail resting upon two bricks.)

e'h' in the elevation, and *k'* and *l'*. From *v* in plan draw a projector and set off the height of *v'* above *e'h'* = 21 ft. to scale. Join *v'k'*, *v'l'*. The triangle *k'v'l'* is the elevation of the spire. Divide the slant height *k'v'* into five equal spaces and draw *horizontal* lines 1', 2', 3', etc. These lines are the elevations of the joints in the lead. With centre *v*, in plan, and radii *o*1', *p*2', etc., taken from the elevation, describe the four circles which in the plan represent the joints 1', 2', 3', etc., in the elevation.

EXAMPLE.—Draw the plan and elevation of the tower and spire, scale $\frac{1}{4}$ in. = 1 ft., and add lines to the plan and elevation to represent (*a*) a 3-in. lap at each horizontal joint in the sheets of lead; (*b*) each successive width of lead made up of four lengths to make up the total circumference of the spire at that level, *e.g.* show the sheet of lead between *k'* and *l'* (elevation) made up of four pieces, each equal and similar in shape, instead of cutting this width of lead in one piece as in Fig. 204.

Sphere.—Fig. 205 represents a gateway pillar P supporting

a square stone cap A,A′, surmounted by a spherical block of stone, C,C′, such as is sometimes seen at the entrance to old-fashioned mansions. The plan and elevation of a ball or *sphere* is invariably *a circle*, equal in diameter to that of the sphere. This is a *great circle* of the sphere. Imagine, for example, that a line *a′a′* is drawn or chased on the ball midway between the topmost point and the point at which the ball rests upon the cap. The elevation of this line, which corresponds to the equator line

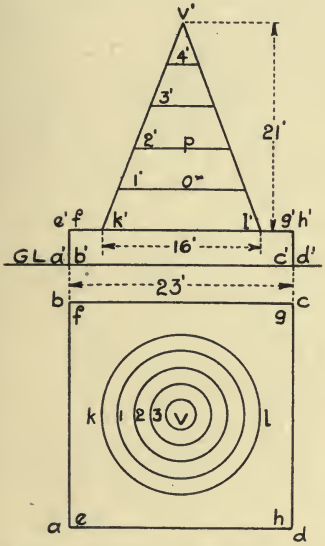


FIG. 204.—Plan and elevation of conical spire resting upon a square base. (The lines 1, 2, 3, etc., and 1′, 2′ 3′, etc., represent joint lines in the sheet lead.)

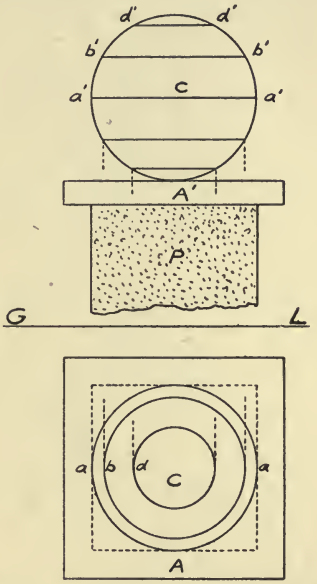


FIG. 205.—Plan and elevation of square gateway pillar surmounted by a square cap and spherical stone block.

on the earth, is a *straight* line *a′a′*, *parallel* to GL. The plan of the same line *a,a* is a great circle of the sphere. If other lines are chased on the ball above and below the midway line *a′a′* and parallel to it, the elevation in each case will be a horizontal line, *b′b′* ; *d′d′*, and the plans of these lines will be *circles* radii C*b* and C*d*, smaller than the great circle of the sphere. When the surface of a sphere is thus divided up into spaces by means of a series of circular belt lines, as in this case, the spaces between the belt lines are termed **zones.** The method of finding the true shape of the various pieces of sheet lead or copper for covering a

spherical surface such as this is dealt with fully in Chap. XVI (Development of Surfaces).

The pictorial sketch (*a*), Fig. 206, is introduced in order to illustrate an important point in connexion with the pyramid. The sketch represents a square turret, the slant surfaces of which are to be covered with sheet lead. Assuming that the turret is 10 ft. high, then the slant surfaces of the turret are too large to allow of the lead being cut in one piece to cover each slant surface, and horizontal joints would occur as shown at *a* and *b* in (*a*), where each successive layer of lead overlaps the layer next

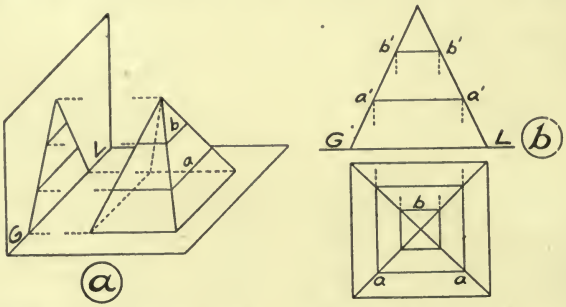


FIG. 206.—(*a*) Pictorial sketch. (*b*) Plan and elevation of square based turret. (The lines *aa*, *bb*, etc., represent joint lines in the sheet material used for covering the turret.)

below it. The dotted lines which would indicate the lap, or part of the surface of one sheet covered by the next sheet above, are omitted in this small sketch for the sake of clearness.

Comparing (*a*) with (*b*) note that these joint lines are all *horizontal* and *parallel to the HP.* Hence, in the elevation the joints appear as *horizontal lines a'a'* and *b'b'* in (*b*), while in plan they appear as lines *parallel to the square base* of the turret.

Octagonal Pyramid.

PROBLEM (*Fig.* 207).—*Draw the plan and elevation of an octagonal based turret* 11 *ft.* 6 *ins. high, the octagonal base to be the largest it is possible to make from a* 9*-ft. square; the lead to be laid on in three widths to make up the total length of each of the slant surfaces of the turret.*

Draw a ground line GL, and, in order to simplify the subsequent drawing of the elevation, arrange the 9-ft. square for the plan with one of its sides parallel to GL. Draw the diagonals of this square (not shown in the diagram). Then the distance from O (one of the corners of the square) to V, the point at which the diagonals intersect, gives the correct distance for marking off 0,7; 0,2, etc. Join 1,8, etc., and thus obtain the octagon for plan of base of turret. Draw the lines 1,5;

2,6; 3,7; etc. These are the plans of the *slant edges* of the turret, meeting at V. Since all the points 1, 2, 3, etc., on the base of the turret are assumed to be *resting upon* the HP, the elevation of each of these points 1 . . . . 8 will *occur on the* GL, and may be obtained by drawing projectors from the various points in plan at right angles to the GL. From V the plan of the top point, or vertex, of the turret, draw a *right* projector and continue it above the GL, marking it off to a length of 11 ft. 6 ins., to scale, to obtain V′, the elevation of the vertex of

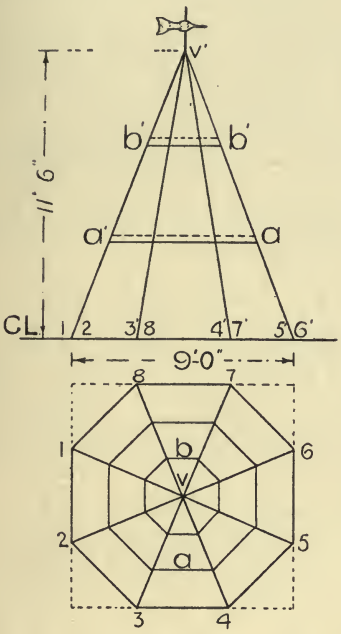


FIG. 207.—Plan and elevation of octagonal turret.

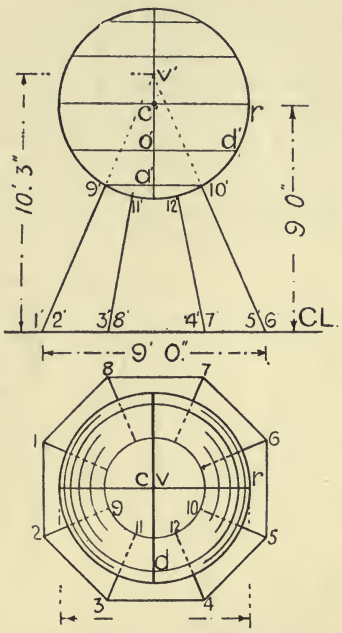


FIG. 208.—Plan and elevation of octagonal turret and cupola.

the pyramid. Join 1′V, 3′V, 4′V, etc. These are the elevations of the slant edges of the turret. Divide the slant height 1′V′ in the elevation into three equal parts, and draw lines *a′a′*, *b′b′*, parallel to GL, to represent the joint lines of the lead. The dotted lines in elevation indicate the *lap* of the covering material at the joints *a′a′* and *b′b′*.

Spherical Cupola.

PROBLEM (*Fig.* 208).—*Draw the plan and elevation of an octagonal turret* (*diameter of base* 9 *ft., height* 10 *ft.* 3 *ins.*) *showing part of the top of the turret removed to accommodate a cupola* (*or sphere*) 7 *ft.* 6 *ins. diameter; the centre of the cupola to be* 9 *ft. above the base of the turret. The cupola*

is to be covered with copper laid on as **zones** *of the sphere, each zone is to be made up of four separate pieces of copper. Scale* $\frac{1}{2}$ *in. to* 1 *ft.*

Draw the plan and elevation of the turret, as in the previous problem. Find a point c', 9 ft. above GL, and in line with the axis or centre line of the pyramid. With radius $c'r' = 3$ ft. 9 ins. describe a circle. Divide the circumference of the circle into twelve equal parts, *i.e.*—each quadrant divided into four parts—and draw lines $c'r'$, $o'd'$, $a'10'$, etc., to represent the horizontal joint lines between the copper zones. Through c' draw a vertical line $a'v'$. This is the elevation of the joints at the front and back of the cupola between two parts of the same zone; while the circle which represents the elevation of the cupola is also the elevation of the joints between two parts of the same zone, on the left and right of the cupola. To draw the plan of the cupola: as c, the centre of the cupola, is on the axis of the pyramid, its plan will coincide with V, the plan of the vertex. With c as centre, $cr = c'r'$ (3 ft. 9 ins.) as radius, describe the great circle which is the plan of the spherical cupola. Again, with centre c, and radii $c'r'$, $o'd'$, $a'10'$, taken from the elevation, describe three concentric circles. These circles are the plans of the horizontal joints $c'r'$, $o'd'$, etc., shown in the elevation. The plan of each slant edge, $1v$, $2v$, $3v$, etc., is interrupted at 9, 11, etc., these points being the plans of the points $9'$, $11'$, where the elevation of the slant edges $1',9'$; $3'11'$; etc., meet the horizontal joint line $9'$, $10'$. The arcs of circles shown in plan are introduced to give an idea of sphericity, and not as the plans of any lines on the sphere.

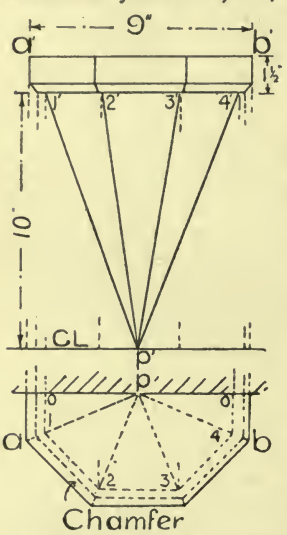


FIG. 209.—Plan and elevation of stone corbel.

Stone Corbel and Cap.

EXAMPLE (*Fig.* 209).—*The drawings given are the plan and elevation of a stone corbel such as might be used for supporting a cylindrical shaft or pillar in a church. It is made up of a slab,* 9 *ins. long* × $1\frac{1}{2}$ *ins. thick, the plan of which is half a regular octagon, supported by another block of stone,* 10 *ins. long, made by cutting an octagonal pyramid vertically from the vertex to the centre of the base. Draw the plan and elevation to a scale of* 3 *ins.* = 1 *ft.*

Note that the pyramid is *inverted*, and that looking down in plan, the stone slab would hide the view of the pyramid. Hence the lines $p2$, $p3$, etc., in plan are shown dotted. Note also that the inner edge of the chamfer, which is shown in elevation as a solid line, is not seen in looking down from above and is therefore represented by a dotted line in plan, as indicated. The inner dotted lines, 1,2; 2,3; etc., in plan, represent the line of intersection of the pyramid and the slab.

Change of Ground Line

In the foregoing examples the objects were arranged in simple positions with regard to the two planes of projection, and one plan and elevation was sufficient to show all the necessary details as to measurement and shape of the object.

Reasons for changing Ground Line.—It is, however, sometimes necessary or desirable to make a new elevation or a new plan of the object in order to bring out more clearly some detail which is not shown on the first simple plan and elevation. In such cases it is necessary to assume that a new plane of projection is arranged in a suitable position, and to make a new projection on this plane.

New Elevation of Cylinder.—Fig. 210 (*a*) and (*b*) illustrate this point more clearly. A cylindrical pipe (*a*) is arranged in

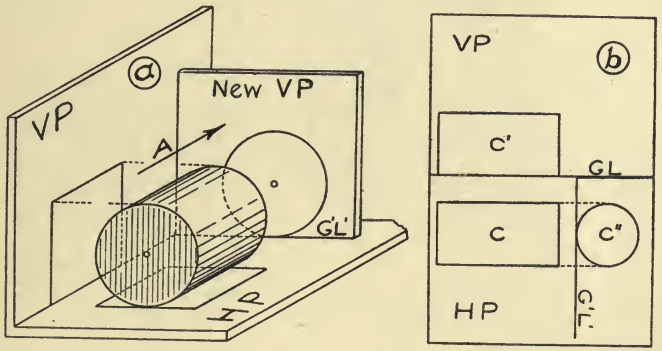


Fig. 210.—Change of ground line as applied to **new elevations.** (*a*) Pictorial sketch of new plane in position. (*b*) Orthographic drawing showing new elevation at C″.

such a way that its axis is parallel to the HP and VP. In such a position both its plan and elevation are *rectangles* C,C′ (*b*). Thus the plan and elevation do not convey any adequate idea as to the *cylindrical* character of the object, and might equally well be the plan and elevation of a square prism. In order to give some idea as to the true shape of the pipe a **new vertical plane** is assumed, (*a*) Fig. 210. A new view of the pipe, looking in the direction indicated by the arrow, A, would, if projected upon the new VP, appear as a *circle*, and would serve to supplement the incomplete details of the original plan and elevation.

The pictorial sketch (*a*) shows the original HP and VP folded up into position. The new VP intersects the original planes of projection in straight lines such as G′L′. Hence, in order to represent the position of the new plane of projection upon a flat sheet of paper (*b*), a **new ground line** G′L′ is drawn at right angles to the original ground line. Projectors drawn from the plan of the pipe C, at right angles to the new ground line G′L′, determine the size and position of the circle C″ which is the *new elevation* of the pipe. The fact that the rectangle C′ rests upon the GL, indicates that the curved surface of the pipe is touching the ground. Hence the circle C″, which is the new elevation of the pipe, is shown resting upon, or *tangential* to, the new ground line, G′L′.

New Elevation of Hexagonal Nut.—The pictorial sketch, (*a*) Fig. 211, still further illustrates this point. A hexagonal nut is arranged in such a position that its two hexagonal faces

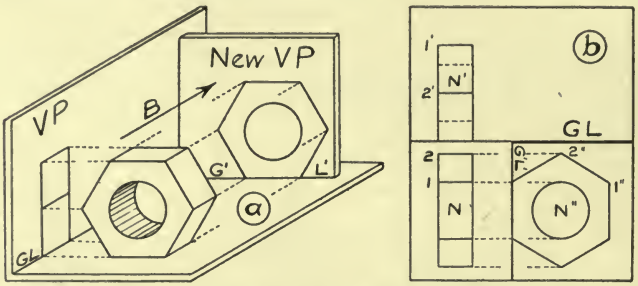


FIG. 211.—(*a*) Pictorial sketch of hexagonal nut with new plane in position. (*b*) Orthographic projection of nut, etc., shown in (*a*).

are square with the VP and HP. The plan and elevation of the nut in this position (see N,N′ in (*b*)) would not give any adequate idea as to the real shape of the object. If a new VP is assumed, as shown in (*a*), and a view is made showing what an observer would see if looking in the direction of the arrow, B, the new view would give a fuller idea of the correct shape of the nut. In making an orthographic drawing of the nut (*b*) the position of the new VP can only be indicated by drawing a new ground line, G′L′, at right angles to the original ground line, as shown. Projectors from each point on the plan are drawn at right angles to the new ground line, and the height of any point 1, 2, etc., above the original GL measured and transferred to the new elevation. Thus, in (*b*) Fig. 211, to make the new elevation N″, projectors from each point, 1, 2, etc., on the plan N are drawn

square with the new ground line. The height of 2′ above the original GL is measured at N′ and is used to give the correct distance of 2″ away from the new G′L′, etc.

A much better idea of this will be gained by creasing the drawing paper along the lines GL and G′L′ in (*b*) of the last two cases, and after cutting through the paper along the thickened part of GL, fold up the original VP and HP at right angles to each other. The new VP should then be folded up in such a way as to be at right angles to the original planes of projection, and a small cardboard cylinder used to illustrate the model.

New Elevation of Dovetail Joint.—The two views of the dovetail halved joint, (*a*) and (*a′*) Fig. 212, serve as another useful illustration of this principle. Neither of these views gives any adequate idea as to the shape of the dovetail. Hence, a new ground line G′L′ is drawn. This is assumed to be the line of intersection of a new VP at right angles to the original VP, in the same relative position as that shown in the two previous cases. A new projection of the dovetail is made upon this plane, showing what an observer would see, looking in the direction indicated by the arrow D. The new elevation *a″* shows this new projection, when the new VP has been folded out flat, using G′L′ as a hinge. The method of obtaining the new elevation is precisely the same as that already described ; *e.g.* the height of any point 1″, 2″ in (*a″*) is found by measuring the heights of the corresponding points 1′, 2′ above the original ground line GL, and transferring these heights—measured from G′L′—along the projectors coming from the points 1 and 2 in the plan (*a*).

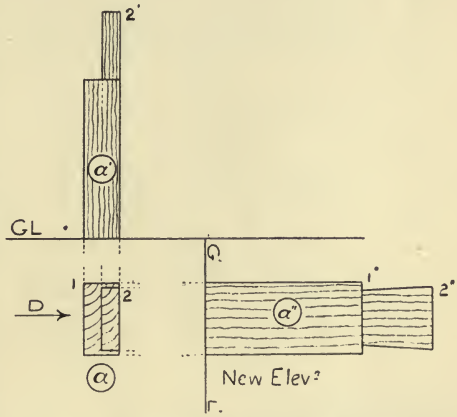


FIG. 212.—Plan, elevation and new elevation of dovetail halved joint.

EXAMPLE.—Copy these drawings, taking the following as the principal dimensions : length 6 ins., width 2 ins., thickness $\frac{7}{8}$ in. Dovetail pin diminishing from $1\frac{3}{4}$ ins at the wide end to $1\frac{3}{8}$ ins. at the narrow end. Scale one-half full size.

Application of Change of Ground Line to Building Work.—*An end elevation of a* **sleeper** *wall, such as is used for supporting the joists of a ground floor, is shown at* (*a*) *Fig.* 213. *Draw a new elevation, looking in the direction indicated by the arrow* A.

Draw horizontal projectors in line with the bed joints in brickwork, the top surface of the plate, joist, and floor boards. Draw a vertical line to represent the quoin at Q, and space out the distances along the horizontal joint lines in the brickwork to determine the position of the vertical joints. Note that bricks which show as *stretchers* (S) in the elevation (*a*) will appear as

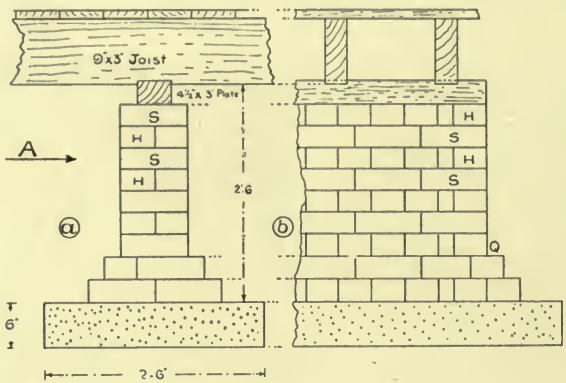


FIG. 213.—New elevation of sleeper wall.

headers (H) in the new elevation (*b*). Similarly, headers (*h*) in (*a*) will appear as stretchers (*s*) in (*b*). A closer should be inserted in each course next to the first header. The joist, which shows its 9 ins. face in (*a*), will show its 9 ins. × 3 ins. end in (*b*), and floorboards, the ends of which show at (*a*) will appear lengthwise at (*b*).

EXAMPLE.—*From the elevation* (*a*) *project a plan, using the measurements on the new elevation* (*b*), *to determine the position of the respective parts of the construction, and omitting the floor boards in order to avoid the use of dotted lines to represent the sleeper wall plate, joists, etc., which would otherwise be hidden by the floor boards.*

Iron Roof Truss Shoe.—*Plan and elevation* (*a*) *and* (*b*), *Fig.* 214, *of an iron shoe at the junction of the rod and principal rafter in an iron roof truss are shown. Make a new elevation* (*c*) *looking in the direction indicated by the arrow* A.

The method of procedure is the same as that already given in the foregoing problems, with a slight variation of the method of obtaining the heights of points on the new elevation. The heights of the respective points 1, 2, 3, 4, etc., on the elevation (*a*) are levelled across to a vertical line *ab* which is on G′L′ produced. With *b* as centre and radii *ab*, etc., describe the arcs *ad*, etc., and

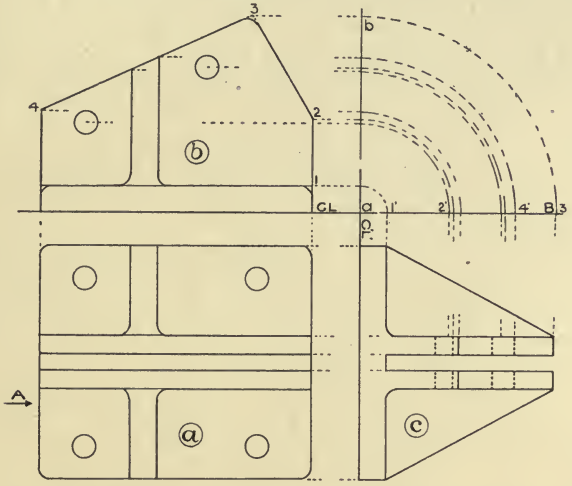


FIG. 214.—New elevation of iron shoe for roof truss.

so determine the position of the points 1′, 2′, 3′, 4′ on *bd*. From these points draw projectors parallel to the new ground line G′L′ to determine the position of the respective points on the projectors drawn from the original plan.

New Elevation to show both Front and Side on One View.—In the foregoing cases, the new elevation was made with the object of showing what a new face of the object would look like. It is sometimes an advantage to show on one drawing how the front and side (or end) of an object would appear if viewed from some new point of view. This is illustrated in Fig. 215. The plan and elevation of a stone cross resting upon a square stone slab are drawn with reference to a given ground line GL. In order to obtain this elevation, it is assumed that the observer's line of vision is at right angles to the original VP. Suppose that the observer *changes his point of view* so that he now looks along the direction indicated by the arrow O, at 30° to the original GL

The heights of all points on the object *remain exactly as before.* It is the *observer* who has changed his position. He will now see part of the *left-hand ends* of the cross and the slab as well as the *front* of each part. In order to obtain a new elevation showing what he would see in the new position, a new ground line G′L′ is drawn. This line represents the intersection of a *new* VP and the *original* HP, and is drawn at right angles to the arrow mark O, *i.e.* at 60° to the original GL. Draw projectors from all points *a*, *b*, *c*, *d*, etc., on the slab, and 1, 2, 3 . . . 8, on the cross, at right angles to the new ground line G′L′. Remember that the

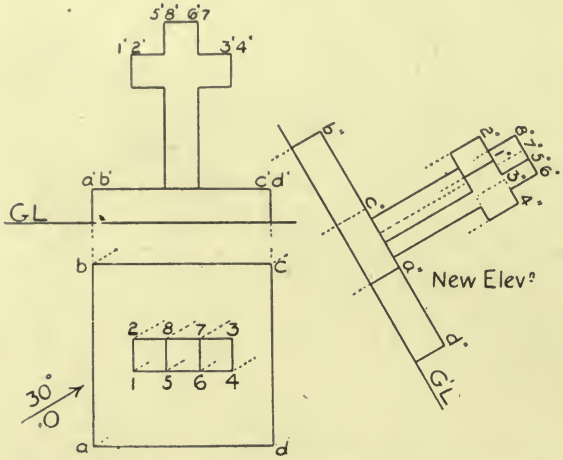


FIG. 215.—Plan, elevation, and new elevation of stone cross.

heights of all points on the object *remain unchanged.* Hence, measure the heights of any points such as *a′*, *b′* in the elevation, and transfer these heights, measured from G′L′ along the projectors coming from *a* and *b* in the plan. Proceed in the same way to obtain the new elevation of the various points on the cross, measuring heights such as 1′, 5′, 3′, etc., above GL, and transferring these heights to the projectors coming from 1, 5, 3, etc., in plan, so that the distance from G′L′ to 1″ is equal to that from GL to 1′ on the original elevation. Note that certain edges of the slab and cross would be out of sight when viewed from O. These edges are represented by broken lines on the new elevation. The vertical edge of the square slab at *c*; the vertical edge of the upright part of the cross at 7; and the vertical edge of the horizontal arm of the cross at 3 would each be

out of sight and so appear as *dotted lines* on the new elevation. The drawings should be copied twice the size given in the illustration.

New Elevation of Brickwork Pier.—*Fig.* 216 *shows the plan and elevation of a* 9 *ins. square brick pier, with two courses of footings, and resting upon a concrete foundation. Make a new*

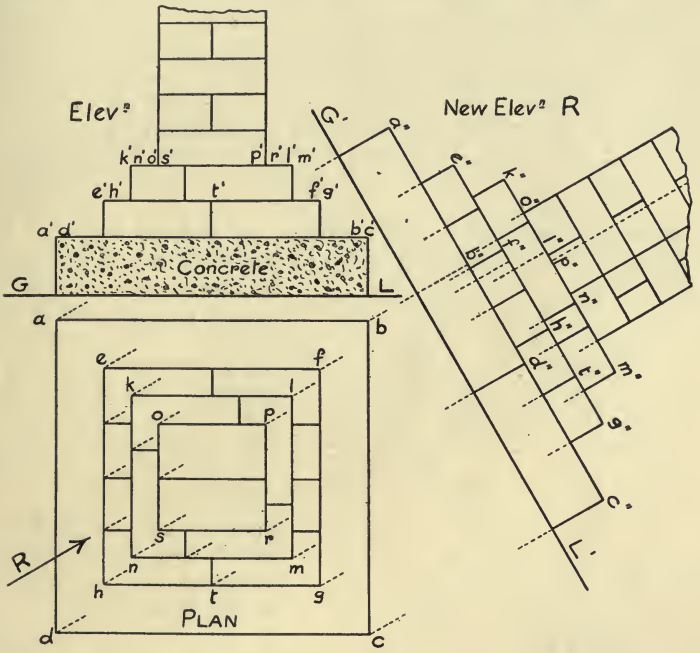


FIG. 216.—Plan, elevation, and new elevation of brickwork pier and concrete.

elevation of this pier as seen from the direction indicated by the arrow (R).

Draw G′L′ at right angles to arrow line R, and from each point on the plan of the pier, etc., draw projectors at right angles to G′L′. Measure the heights of the respective points above GL and transfer to the new elevation. Thus the position of the concrete foundation in the new elevation is obtained by drawing projectors square with G′L′ from points *abcd* in the plans, measuring the height of *a′* and *b′* above GL, and making the heights of *a″*, *b″*, *c″*, *d″* equal respectively to the heights taken from the original elevation. The position of the bottom course

of footings is obtained similarly by projecting from *e*, *f*, *g*, and *h* in the plan; making the height of *e''f''*, etc., above G'L' equal to the height of *e'f'* above GL. The new elevation of the upper course of footings is projected from *k*, *l*, *m*, *n*, in the plan; heights measured from *k'l'm'n'* and transferred to *k''l''*, etc., while projectors drawn from *o*, *p*, *r*, *s* in the plan give the correct position of the pier in the new elevation.

Three upright edges of the concrete slab and of each course of footings are seen and one is unseen, the latter being represented by broken lines *b''*, *f''*, and *l''*. Similarly, one quoin or upright edge of the pier itself is unseen and is also represented by a broken line at *p''*. The position of the various joint lines in the brickwork on the new elevation is obtained by projecting from the *plans* of the joints as at *t*, etc.

New Elevation of Danger Post.—(*a*) and (*b*) Fig. 217 still further illustrate this principle. (*a*) is the elevation of triangular framework supported upon a post which is fixed near dangerous hills as a warning to motorists. A driver coming along a road from the direction indicated by the arrow mark P would see the danger signal as shown at (*a*).

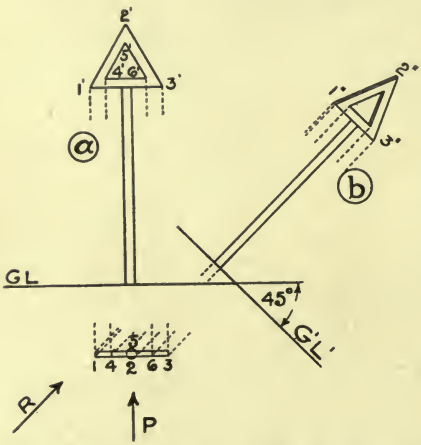


FIG. 217.—Plan, elevation, and new elevation (*b*) of danger post.

If the driver approached the danger signal from the direction R, his view of the triangular frame would be *foreshortened* as shown at (*b*).

To obtain this view (*b*) draw a new ground line G'L' at right angles to the direction R (say 45°). Make the elevation (*a*) first and draw projectors from the three points 1', 2', 3' in elevation, in order to fix the position of the plans 1, 2, 3 of the corners of the larger triangle. Similarly, by projecting from 4', 5', 6', find the plans of these points 4, 5, 6. From the points 1, 2, 3; 4, 5, 6 in the plan, draw projectors parallel to R or at right angles to the new ground line G'L'. Measure the heights of the various points above the original ground line and transfer

these heights along the respective projectors. Thus the height of 1′ above GL is measured and stepped off from G′L′ on the projector coming from 1 in the plan, etc. Note that since the points 2′ and 5′ are vertically above each other these two points *have one point* for their *common plan.* Hence the projector drawn from 2 in the plan is used for marking off both the new heights 2″ and 5″ on the new elevation.

New Elevations of Surfaces involving Curved Lines.—(*a*) *and* (*b*), *Fig.* 218, *represent the plan and elevation of a clock face, the rectangle abcd in plan and the dotted square in the*

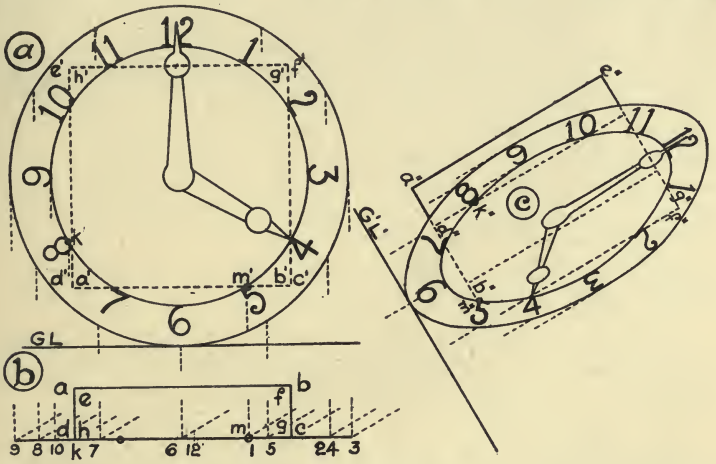


FIG. 218.—Change of ground line. New elevation (*c*) of a clock face (*b*) as an illustration of the special method of treatment of curved lines.

elevation representing the case containing the works. Make a new elevation of the clock on the ground line G′L′.

Divide the circles in the elevation into any convenient number of equal parts, say twelve, as shown. Imagine the clock face to be made from thin sheet material. Then its *plan* would appear as a *straight line* 9,3. On this line obtain the plans of the points 1″, 2′, 3′, etc., on the outer circle in the elevation by drawing projectors at right angles to the original ground line GL. From the points 1, 2, 3, etc., in plan draw projectors at right angles to the new ground line G′L′, and, having measured the heights of the respective points above the original ground line, transfer these heights to the corresponding projectors, measuring all distances from the new ground line G′L′. A fair curve through the points thus obtained is an ellipse, as shown,

and this is the new elevation of the outer edge of the clock face. The new elevation of the inner circles of the clock face is obtained in a similar manner, having first obtained the plans of the twelve points, such as *kk'*, on the elevation of the inner circle of the clock face. The plans of all these twelve points will also occur upon the straight line 9,3 in plan, since they are all included in the plane surface of the face of the clock. From these points *k*, *m*, etc., draw projectors square with the new ground line, and having measured the heights of *k'*, *m'*, etc., above the original ground line, obtain the new elevations of these points by measuring these distances above G'L' on the projectors coming from *k*, *m*, etc., respectively, in the plan. Note that the new elevation of the inner circle on the clock face is an ellipse, but it is *not parallel* at all points to the outer ellipse. The reason for this is dealt with fully in Chap. IX (Plane Curves Ellipse). The points *a''*, *e''*, *d''*, etc., which represent the new elevation of the box behind the clock face are obtained in the same manner from *a*, *b*, *c*, *d*, etc., in plan, the height of any point, *a''* measured from G'L', being the same as the height of *a'* above the original ground line GL.

EXAMPLE.—*Copy the given drawings twice the size shown in the illustration.*

(*a*) and (*b*), Fig. 219, are the plan and elevation of a passage between two walls, the passage having a semicircular *soffit* or top. *Make a new elevation of the passage* (*c*) *as seen from the direction indicated by the arrow* D.

The method to be adopted in this case is similar to that employed in the previous problem. The span *a'b'* of the arch *a'c'b'* is divided into any convenient number of equal parts, as shown, 1', 2' 3', etc. *Ordinates* are drawn from these points, at right angles to *a'b'*, to meet the semicircle at *o'*, *p'*, *c'*, etc. Consider all these points as the elevations of imaginary points on the outer edge of the front face of the entrance to the passage. The plans of all such points 1, 2, 3, 4 in (*a*) are included in the line *ab*, which is the plan of the front face of the passage, and are obtained in the usual way, by means of projectors drawn at right angles to GL. Draw a new ground line G'L' at right angles to the arrow D, and from all the points *a*, 1, 2, 3, 4, etc., in plan draw projectors at right angles to the new ground line G'L'. Measure the height of the springing line *a'b'* above GL, and transfer this height to the projectors from *a* and *b* in the plan, measuring from G'L'. Join *a''b''*, and by drawing projectors from the points 1, 2, 3, etc., in plan, obtain the new elevations of these points, as at 1'', 2'', etc. Take off the lengths

of the respective ordinates $1',o'$; $2',p'$; etc., from (b), measuring from the *springing line* $a'b'$, and transfer these heights to the new elevation (c), measuring from $1''$, $2''$, etc., already found. A curve through these points, o'', p'', c'', etc., is the new elevation of the outer or *extrados* line of the front of the arch. Projecting from points 2 and 8 in the plan, obtain the position of the vertical edges or quoins n'', $2''$, etc., at the entrance to the passage. Points such as d'' on the inner or *intrados*, line of the arch are obtained by measuring the height of d' above the

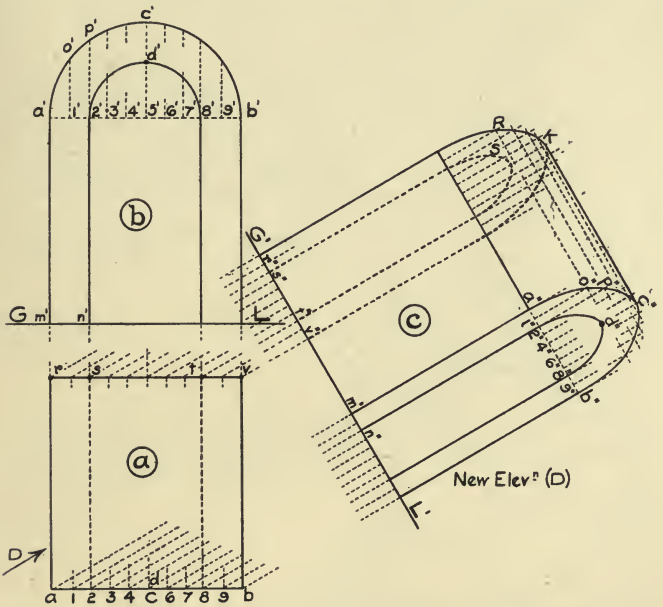


FIG. 219.—(a) Plan of archway. (b) Elevation of archway. (c) New elevation of archway as seen from D.

springing line $a'b'$ in the original elevation, and using this for the height of d'' above $a''b''$ in the new elevation.

The new elevation of the rear end of the passage r'', s'', t'', v'' is obtained in the same way, by drawing projectors from r, s, t, v on the line rv, which is the plan of all points included in the vertical plane surface at the end of the passage farthest from the observer. The two curves R and S, representing the exterior and interior edges respectively of the arch at the rear end of the passage, may be obtained by a method similar to that used for the curves at the front end of the passage ; or, as an alternative,

lines such as C″K, O″R may be drawn, parallel to G′L′, and the length of the passage *m″r″* marked off upon these lines, *i.e.* C″K = *m″r″*, *o″*R = *m″r″*, etc.

New Elevation of Frustum of Cone.—A further example of the method of obtaining new projections of curved surfaces is given in Fig. 220.

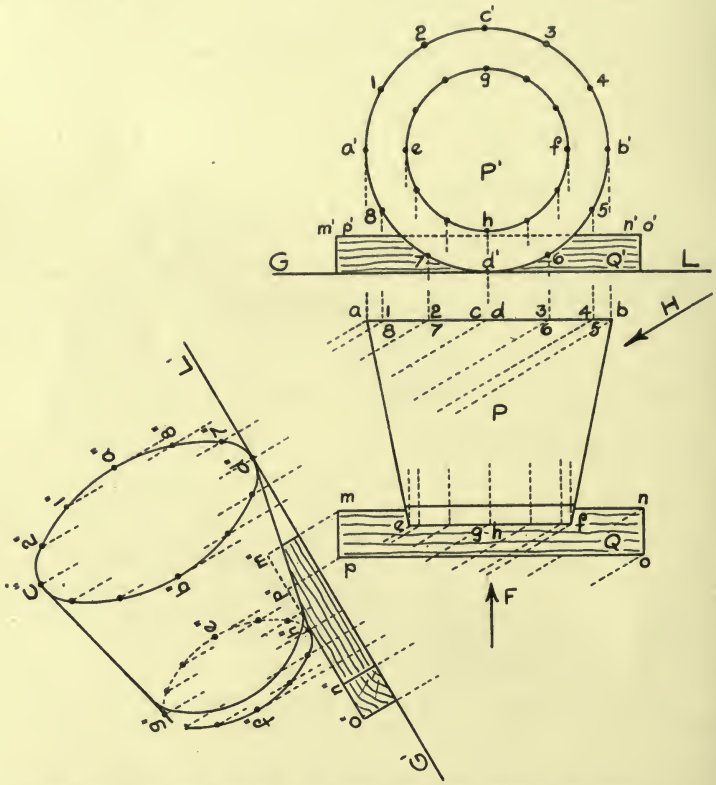


FIG. 220.—P, Plan of pail. P′, Elevation of pail as seen from direction F. New elevation on G′L′ indicates what an observer would see if looking from the direction H.

A pail P, resting with its small end upon a piece of quartering Q, is shown in plan and elevation. Make a new elevation of the pail and quartering looking in the direction H.

Some idea as to the character of the new elevation may be gained by considering the change which is supposed to take place as regards the observer's *point of view*. In order to obtain

the elevation P′ the observer must look towards the VP in a direction parallel to F. From this position he sees the small or closed end of the pail. In the new position H, however, he would see into the open or wide end of the pail. This is made clear on the new elevation. The method of obtaining points on

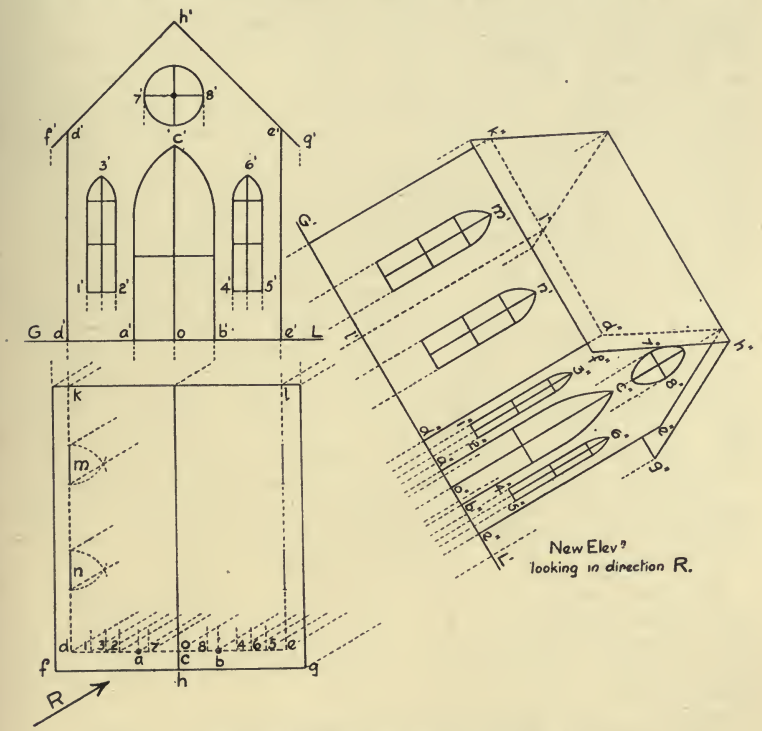


FIG. 221.—New elevation of building.

the new elevation is the same as that already described in detail, and there should be no need to add any further explanation if the lettering and numbering of the plan, elevation, and new elevation are carefully compared.

New Elevation of House.—A skeleton line drawing of a small building, Fig. 221, furnishes another practical example of the same type of work. The eaves at the south end of the building project over the face of the work as indicated in plan by the dotted line *dce*—the distance between this line and the solid line *fhg* representing the distance that the roof projects beyond

the face of the building ; *m* and *n* are the plans of two windows on the western side of the building. *Make a new elevation, showing what an observer would see of the south* **and** *west sides of the building, looking towards the building from the direction* R.

Draw G′L′ at right angles to the line R. Draw projectors from *d*, *e*, *l*, *k* in the plan at right angles to G′L′, and make the heights *d″d″*, *e″e″*, etc., on the new elevation equal to *d′d* on the original elevation. These lines, *d″d″*, etc., represent the four corners of the building, three of which are seen, and one (*l″l″*) unseen. Join *d″k″* and *e″l″* to represent the line of intersection of wall and roof. Obtain the new elevation *f″*, *g″*, and *h″* by means of projectors from *f*, *g*, and *h* in the plan, making *f″*, *g″*, *h″* the same height above G′L′ as the corresponding points *f′*, *g′*, and *h′* are above GL. The position of the doorway on the new elevation is obtained from points *a*, *b*, and *c* in the plan, the height of the top point *c″* above G′L′ being equal to that of *c′* above GL. Similarly, the position of the front windows is obtained from points 1, 3, 2 ; 4, 6, 5 in the plan, while that of the circular window is obtained from 7, 8, *c* in the plan. The side windows are shown in plan at *m* and *n*, on the dotted line K*d*, which is the plan of the side of the building, and the new elevation of these windows, *m′* and *n′*, is projected from these lines (*m* and *n*) in plan, the height of the top points *m′* and *n′* above the springing line of the window being determined by means of an *auxiliary construction* shown at *n*. The two side windows on the eastern side of the building are omitted from the new elevation in order to avoid confusion.

Example.—Make a plan and elevation of a building similar to that shown in the illustration, 32 ft. × 26 ft. × 25 ft. high. Doorway 10 ft. wide; windows 4 ft. wide; eaves at *h* projecting 2 ft. over the front wall of building. Height of ridge *h′* above ground 38 ft. Scale $\frac{1}{8}$ in. to a foot.

New Plans.—The method of obtaining a new elevation, having given a plan and one elevation, has been explained at some length. The method of drawing a *new plan* is similar to that for drawing a new elevation ; a *new plane of projection* is assumed as in making a new elevation, but instead of assuming a new vertical plane, as in making a new elevation, a *new ground plane* is assumed. Again, in making a new elevation, the projection was obtained by drawing lines from the *given plan.* In making a *new plan* the process is reversed, and the new view is obtained by drawing projectors from every point on the *given elevation.*

New Plan of Square-based Pyramid.—An example of this

is shown, Fig. 222. (*a*) and (*b*) are the plan and elevation of two square slabs of stone, resting one on the other, surmounted by another block of stone in the form of a square-based pyramid. *Draw the given plan and elevation, and make a new plan, showing what an observer would see of the blocks of stone, looking along the direction indicated by the arrow* (P).

The *original* plan (*a*) is the view obtained by an observer situated *vertically above* the original ground plane, the front edge

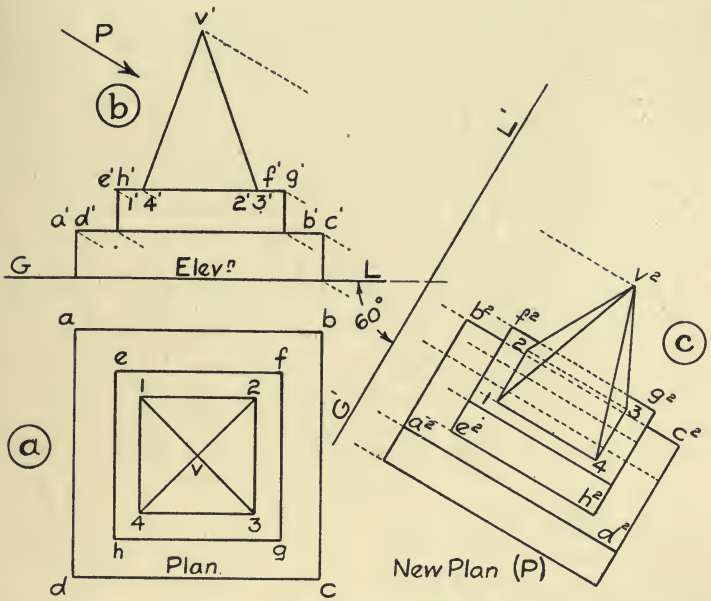


FIG. 222.—Change of ground line as applied to **new plans.** (*a*) Plan of square-based turret resting upon square slabs. (*b*) Elevation of same. (*c*) New plan looking in direction P.

of which is represented by GL. Imagine that this ground plane is *tilted up* until it is at right angles to the arrow line (P) produced. Then G'L' would be an *edge view* of the ground plane in the new position. The blocks of stone, meanwhile, have not altered their position with regard to the vertical plane. The change is a change from one point of view to another *on the part of the observer*, and not a change of position *of the object.* This point is very important, and the student is advised to use a small model to illustrate the changing of the position of the ground plane, and the consequent effect upon the view obtained by the

observer as he looks down at right angles to this new ground plane, represented by G′L′.

The line of direction P is taken at 30° to GL. Hence the new G′L′ will make an angle of 60° with GL if drawn at right angles to the direction P. Draw G′L′, and from every point on the elevation of the object draw projectors *at right angles to the new ground line* G′L′. In drawing a plan, the question of the heights of the various points is not considered at all. A *plan* shows only the *lateral distance* between the various points on the object, and the position of each point *with regard to the VP*. If, therefore, it is remembered that the slabs and pyramid remain in their original position, the reason for the following method of working will be obvious. Measure the distance between the inner vertical face of the bottom slab and the VP, *i.e.* the distance between *ab* and GL. This face is parallel to the VP, hence, using the projectors which are drawn from *a′* and *b′* in the original elevation, make the distance of a^2, b^2 away from G′L′ equal to that between *ab* and GL in (*a*). Proceed in the same way to obtain the plans of all points on the object. Thus, on the projector coming from *v′* in the elevation, the position of the new plan of *v* is obtained by making the distance of v^2 from G′L′ equal to the distance of *v* from GL on the original plan. Similarly the distance between the line 1,2 and G′L′ on the new plan is equal to the distance between 1,2, and GL on the original plan. Having obtained the new plan of each point on the object by the same method, it remains necessary only to exercise care in joining the points together in such a way as to convey an impression of what the observer would see at P; *e.g.* the slant edges of the pyramid $v^2$1, $v^2$4, would appear *longer* than the edges $v^2$2, $v^2$3 on the new plan, while one bottom edge, 2,3, of the pyramid, and one edge of each slab would be unseen, and are therefore represented in each case by dotted lines.

EXAMPLE.—*Make a new plan of the same objects, showing what the observer would see if looking from a point in line with v3′, produced*, i.e. *looking along the slant surface of the pyramid. Note, that from this point of view the new plan of the vertex v will fall upon the new plan of the base edge* 2,3.

New Plan of Mortise Gauge.—Plan and elevation of a mortise gauge, Fig. 223, are shown. *Make a new plan of the object looking along the direction* T.

Draw G′L′ at right angles to the line of direction T, and from all points on the elevation draw projectors at right angles to the new ground line G′L′. Measure the distance between the various points on plan and GL, and transfer these distances to the projectors for the new plan, measuring from G′L′. Thus, $o^2p^2r^2s^2$

is the new plan of the top end of the stem of the gauge, the distances of o^2, p^2, etc., from G′L′ = distance of o and p from GL in the given plan. Similarly $a^2b^2c^2d^2$, $e^2f^2g^2h^2$, is the new plan of the stock of the gauge, the distance of a, etc., from the given ground line GL being measured and transferred to the new plan. Note that one edge of the stock, f^2g^2, is unseen and is

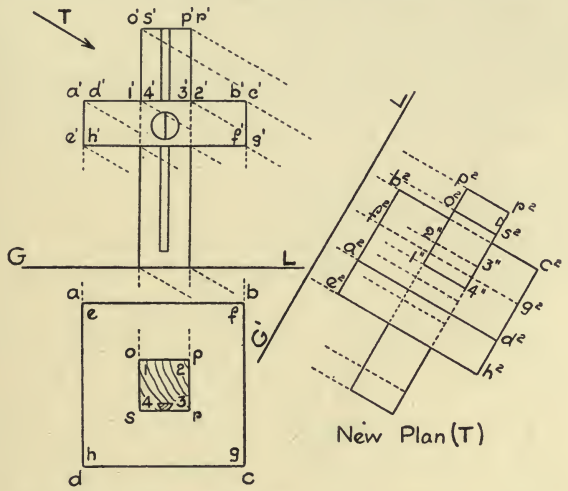


FIG. 223.—Plan, elevation, and new plan (T) of a mortise gauge as seen from T.

shown as a dotted line. For the same reason, the edge 2″,3″, where the stem meets the upper surface of the stock, is shown as a dotted line.

EXAMPLE.—*Copy the given drawings twice the size shown in the illustration.*

New Plan of Circular Bath.—Plan and elevation of a circular bath are given, Fig. 224, the bottom of the bath resting on the ground. *Make a new plan of the bath looking in the direction indicated by the arrow mark* D.

Divide the two circles, 1,2,3,4, etc., and *acbd*, which are the plans of the top edge and bottom surface of the bath, into any convenient number of equal parts, say twelve, as shown. All the points 1, 2, 3 . . . 12, are at the same height above GL on the upper edge of the bath, and consequently the elevation of all such points will occur *on the horizontal line* 1′ . . . 7′ in elevation.

Fix the position of the new ground line G′L′ at right angles to D, and from 1′,7′ in the elevation draw projectors square with

the new ground line G′L′. In the given plan 1 and 7 are at equal distances from GL. These distances should therefore be transferred to the new plan, so that 1″ and 7″ are at the same distance away from G′L′. On the projector coming from 4′,10′, in the given elevation, fix the positions of 4″ and 10″, the distance of these points from G′L′, being equal to the distance of 4 and 10 away from GL in the given plan. Proceed in the same way to fix the position of the remaining points on the new plan of the top edge of the bath 5″,6″; 8″,9″; 11″,12″; 2″,3″; a fair curve through these points gives the ellipse which makes part of the new plan.

The points *a*, *c*, *b*, *d* are on the circumference of the *bottom*

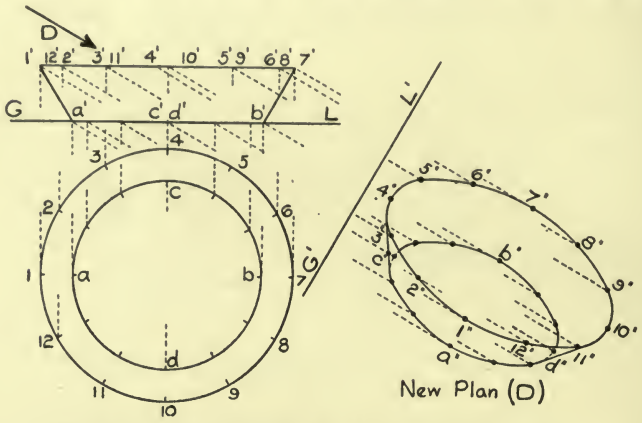


FIG. 224.—Plan, elevation, and new plan (D) of foot-bath.

surface of the bath. Hence the elevation of these points will occur *upon* GL in the original elevation. From each of these twelve points draw projectors at right angles to G′L′ and make the distances of the points *a*″, *c*″, *b*″, *d*″, and intermediate points (not lettered on illustration) from G′L′ equal to the respective distances of *a*, *c*, *b*, and *d* from GL. The ellipse *a*″*c*″*b*″*d*″ is the plan of the circle in which the sides of the bath intersects the bottom. The new plan is completed by drawing two straight lines, *c*″3″ and *d*″11″, tangent to the two ellipses as shown, to represent the sloping sides of the bath. Note that part of the small ellipse near the ends of the major axis at *c*″ and *d*″ should be dotted to indicate that the view of this part of the bottom of the bath is cut off by the upper edge of the bath, 2″, 1″, 12″, etc.

New Plan of Sheerlegs.—Plan and elevation of a sheerlegs are given, Fig. 225, the lines 2,4; 3,4 representing the plans of

the legs, and the line 1,4 representing the plan of the back stay.

Draw the plan and elevation of the sheerlegs to the given dimensions (scale $\frac{1}{16}$ in. to 1 ft.), *and make a new plan of the sheerlegs as seen from the direction indicated by the arrow mark* R.

Make the elevation first. With centre 1′ and radius 60 ft. (to scale), describe an arc cutting a horizontal line 23 ft. above GL at 4′. Join 1′, 4′; this is the elevation of the *stay*. With centre 4′ and radius 40 ft. (to scale) cut GL at 2′. Join 4′, 2′. This *single* line is the elevation of the *two* legs 4′,2′; 4′,3′: the feet of which are both on a projector, 2,3, in the plan.

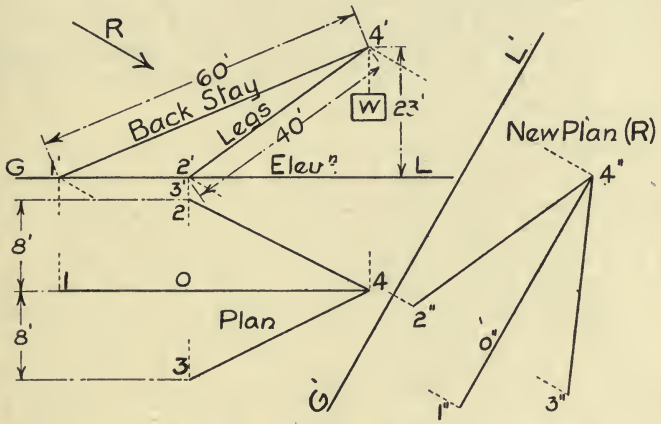


FIG. 225.—Plan, elevation, and new plan of sheerlegs as seen from R.

Draw the plan of the stay, 1,4, parallel to GL and between the projectors coming from 1′, 4′ in the elevation. Draw a projector from 2′ in the elevation and make the distances 0,2; 0,3 on this projector = 8 ft. (to scale). Join 4, 2; 4, 3. This completes the plan.

The new plan is obtained by employing the same method as that already described in detail in the foregoing problems. Thus the distance of 4″, measured from G′L′, on the projector coming from 4′ in the original elevation, is the same as the distance of 4 from GL on the given plan. In the same way the distance between G′L′ and 2″ = the distance between GL and 2; the distance G′L′ . . . 3″ = distance GL . . . 3, etc. Hence it follows that the distances 0″,2″; 0″3″ on the new plan will be equal to the distances 0,2; 0,3 on the original plan, *i.e.* 8 ft. on each side of the new plan of the stay 1″,4″.

EXAMPLE.—*Make a new plan of the sheerlegs on a ground line parallel to the given elevation of the back stay* 1',4'.

True Shape of Surfaces inclined to the Planes of Projection.—It is often necessary for practical purposes to determine the true shape of surfaces inclined either to the ground or the vertical plane. The following problems illustrate the principles which must be understood before it is possible to understand the method of determining the correct bevels for cutting the various roof timbers, etc., and it is of the utmost importance that the problems should not be merely copied, but that the principles are completely mastered. Wherever possible, a model of some sort should be used to illustrate the problem. The meaning of the various lines used in the construction will then be more obvious, The 45° and 60° set-squares are used as practical examples. because they are ready to hand and because the student is familiar with their shape and properties.

Rabatment of Surfaces.—*abc*, Fig. 226 (*a*), is the plan of a 45° set-square resting with its longest edge (the hypotenuse) *ab on the ground.* The right-angled corner *c* is *tilted up* off the ground. *Determine the real shape of the set-square and the inclination,* θ, *of the plane of the face of the set-square to the H.P.*

Use an open book to illustrate the planes of projection. Rest the set-square flat upon the horizontal page. In this position the plan of the set-square gives its true shape *a*C*b*. Keep the edge *ab* on the horizontal page and tilt up the other corner *c*, using *ab* as a hinge. Place a pencil, held vertically, against the corner *c* as the set-square is swung (or *rabated*) upwards. As *c* moves upwards the pencil moves along a line C*c* at right angles to the hinge *ab*. Mark one of the successive positions of the lower end of the pencil on the page, as at *c*, and join *ca*, *cb*. This is the plan of the set-square in an inclined position. Note that the angle *acb* is not now a right angle, although it is the *plan* of a right angle. In the same way each of the angles *cab* and *cba* is *not* 45°, although these angles are the *plans* of corners each of which contains an angle of 45°. The inference to be drawn from this is that when a plane surface is inclined to any plane of projection its true shape does not appear in the projection of the surface made on that plane, but the true shape may always be found by *rabatting* (or swinging) the surface into parallelism with the plane of projection.

Hold the horizontal page of the book on a level with the eye and rotate the set-square as before about *ab* as a hinge. Viewed from this position, the corner C appears to travel in a circular path. Taking GL, Fig. 226, to represent the intersection of the

horizontal and vertical pages of the book, $C'c'$ would represent the circular path of the corner of the set-square as projected upon a vertical plane. Draw a projector from c in the plan to intersect the curve $C'c'$ at c'. From a draw a projector to determine the position of the point $a'b'$ on the GL, which is the elevation of the long edge of the set-square. Join $a'c'$. Then the whole surface of the set-square is represented by a *single line* $a'c'$ in elevation, and the angle θ is the inclination of the

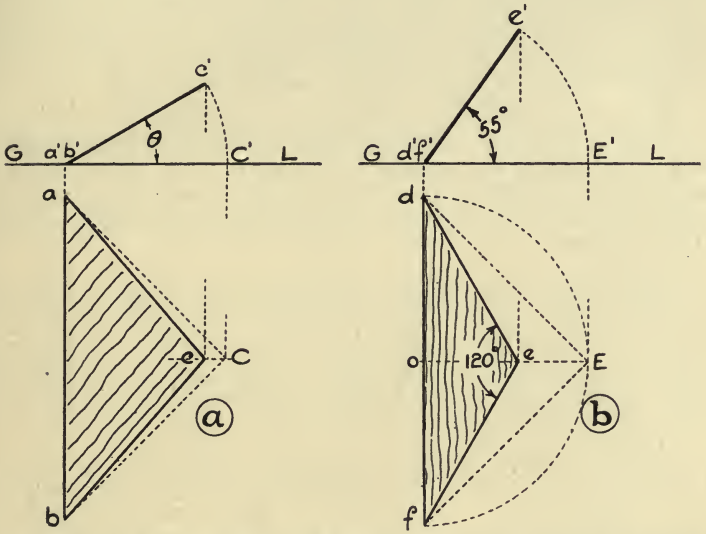


FIG. 226.—45° set-square used to illustrate the rabatment or turning of a surface upon a hinge. The dotted triangles in (*a*) and (*b*) indicate the real shape of the set-square, while the shaded triangle is the plan of set-square after rabatment.

plane of the set-square to the horizontal when its plan is represented by the shaded triangle *acb*.

Plan of Set-Square in inclined Position.—In Fig. 226 (*b*) *def* is the *true shape* of a 45° set-square. The set-square is *rotated* about *df* as an axis until the elevation of the set-square $d'f'e'$ makes an angle of 55° with GL. *Draw the plan of the set-square when in this position, and measure the angle def* (i.e. *the plan of the right-angled corner*) *in degrees.*

Draw any line *df* at right angles to GL to represent the hypotenuse of the set-square. Bisect *df* at *o*, and with *o* as centre describe the semicircle *d*E*f*. Draw *o*E at right angles to *df*, and join *d*E, *f*E. Then the angle at E will be a right angle,

because it is *the angle in a semicircle*, and the triangle *d*E*f* represents the set-square lying on the ground. Draw a projector from E to obtain E′. With *d′* as centre and *d′*E′ as radius describe the arc E′*e*, *i.e.* the circular path of the corner E as viewed from the front. From *e′* draw a projector *e′e* to intersect *o*E in plan. Then since E*e* is the plan of the path of E as it moves upwards along the circular path E′*e′*, *e* must necessarily be the plan of the corner E when it is on a plane which makes 55° with the ground. Join *de*, *fe* and so complete the triangle *def* which is the plan of the set-square when its surface is tilted up at 55° to the ground. In this position the angle *def*, although it is the *plan of a right angle*, has a *magnitude of* 120°.

From the foregoing problems, the following important principle is deduced:—**When a plane surface is rotated about one of its edges as a hinge the paths of all points on the surface appear either as circular arcs (on one plane of projection) or as lines square with the hinge (on the other plane of projection).**

Rabatment of Set-Square with reference to the Horizontal Plane.—(*a*) Fig. 227 is a further example of a surface inclined to the Horizontal Plane; *kmn* is the plan of a 60° set-square resting with its *shortest* edge, *kn*, on the ground, and the corner at *m* *raised above the ground* so that the *plan* of the 60° corner, at *n*, is an angle of 50°.

Find the height of m above the ground, and the inclination of the plane of the set-square to the horizontal. Imagine the set-square lying on the ground. In this position its true shape, *k*M*n*, would appear as shown. Produce *km* to M and set out *n*M at 60° to *kn*. The elevation of the set-square, lying upon the ground, is shown by the straight line *k′*M′, on the ground line. From *m* draw a projector *mm′* at right angles to GL. With *k′* as centre and *k′m′* as radius describe an arc M′*m* intersecting *mm′* at *m′*. Then *m′* is the elevation of the corner *m* of the set-square when it is raised to such a position that its plan appears at *m*. Join *k′m′*. This line is the elevation of the set-square in the required inclined position. The height of the raised corner *m* is shown at *om*, and the inclination of the plane of the set-square to the horizontal is 47°.

Rabatment with reference to the Vertical Plane.—(*b*) Fig. 227, illustrates the *converse* of the foregoing problems. The shaded triangle *o′p′r′* is the elevation of a 60° set-square, which rests with its shortest edge, *or*, on the ground, the plane of the face of the set-square being vertical. *How far has the*

corner o been swung out from the position of parallelism with regard to the vertical plane?

Consider *p'r'* as the *axis of rotation* of the set-square as the corner *o* is moved outwards or inwards with regard to the vertical plane. As the set-square is rotated about *p'r'*, the edge *p'r' remains constantly in the same position.* The plan of this edge, or vertical axis, is represented in plan by the *single point pr.* While the corner *o* moves around, swinging about the axis *pr*, it is always *touching the ground.* Hence the *plan* of the path of the point *o* is a *circular arc*, *o*O, while the *elevation* of its path

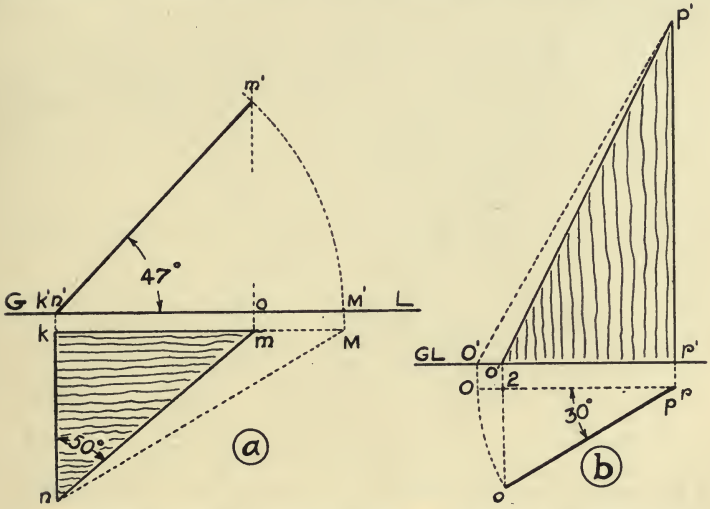


FIG. 227.—Rabatment of set square with reference to (*a*) Horizontal plane; (*b*) Vertical plane.

is a *horizontal line*, *o'*O'. Draw the elevation, O'*p'r*, of the set-square when it is parallel to the VP, and its true shape is shown in elevation. In this position the plan of the set-square is a *straight line*, O*r*. With *r* as centre, and *r*O as radius, describe the arc O*o*. This is the plan of the path of *o* as it moves around on the axis *pr*. From *o'* in elevation, draw a projector *o'o* at right angles to GL. Then *o*, the point of intersection of this projector and the circular arc O*o*, is the plan of the corner *o* in the position represented in elevation by *o'p'r'*. In this position the plane of the face of the set-square makes an angle of 30° with the VP, and the distance of the point *o* in front of an imaginary vertical plane through *or* is measured at *o*2.

Selection of Suitable Axis of Rotation or Rabatment.—In each of the four previous problems, one of the edges of the set-square has been used as an axis of rotation. It is sometimes necessary, however, to assume an arbitrary axis of rotation. In (*a*) Fig. 228 A'B'C' is the elevation, and AB the plan of a 60° set-square, when the *plane of its face is vertical* and *parallel to the* VP, and *one* corner, *c*, *rests upon the* HP. *Draw the plan and elevation of the set-square when it has been rotated through an angular space of* 30°, *keeping c meanwhile on the ground, and the corners at* A *and* B *constantly at the same height above the* HP.

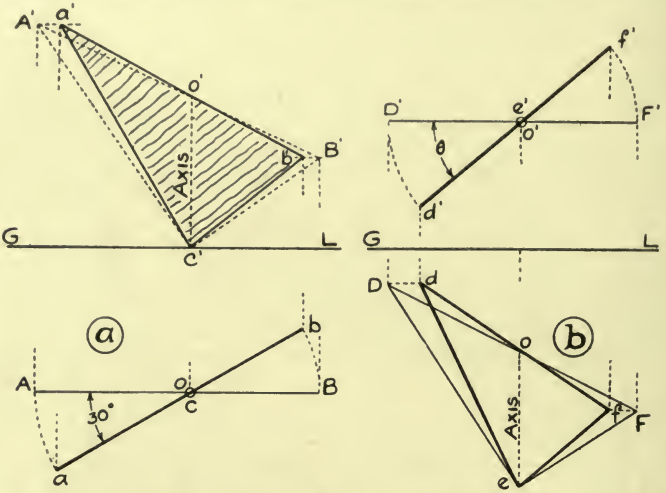


FIG. 228.—Rotation or rabatment of set-square about an arbitrary axis of rotation.

Hold a set-square vertically in the position indicated, and with a pencil held against the set-square vertically above *c'*, as shown by the dotted line *c'o'*, rotate the set-square, using the pencil as an axis of rotation.

Looking down on the set-square, the plan of the path of A and B respectively is circular, and two arcs, A*a* and B*b*, struck with *o* as centre, are the plans of the paths of the corners A and B. Note that while A moves *away from* the VP, B moves *towards* it.

Hold the set-square level with the eye, and note that A and B appear to move along *horizontal lines* as the set-square is rotated about *co'*. The horizontal lines, A'*a'* and B'*b'*, are the elevations of the paths of A and B respectively.

Set out a line *ab* in plan making 30° with AB. This line

cuts the circular arcs, A*a* and B*b*, at *a* and *b* respectively. Then *acb* is the *plan* of the set-square in the required position. From *a* and *b* in plan draw projectors *aa'*, *bb'* to meet the horizontal lines A'*a'* and B*b'* in elevation at *a'* and *b'*. Join *a'c'*, *b'c'*, *a'b'*. Then the triangle *a'b'c'* is the elevation of the set-square when it has been rotated through an angular space of 30°.

In (*b*) Fig. 228 the triangle DF*e* is the plan, and the horizontal line D'*e'*F' is the elevation of a 60° set-square with the plane of its face parallel to the HP. Draw its plan and elevation when it has been rotated through an angular distance θ, using *oe* as an axis of rotation. Note that the elevation of the line *oe* is a *single* point, *o'e'*. From D and F in plan, draw D*d* and F*f* parallel to the ground line GL. With *o'* as centre, and *o'*D', *o'*F' as radii, describe the arcs D'*d'*, F*f'*. Then D*d*, F*f* are plans, and D'*d'*, F'*f'* are the elevations respectively of the paths of the corners D and F as the set-square is rotated about the horizontal axis *oe*. The corner D moves *downwards*, while the corner F moves *upwards*, from the horizontal position. Hence the plans of D and F (*d* and *f* in the new position) will be *nearer* the plan of the axis *oe*. Set out a line *d'f'* in the elevation making the required angle θ with D'F', and intersecting the arcs D'*d'* and F'*f'* at *d'* and *f'* respectively. From *d'* and *f'* fix the position of the plans of the corners D and F by means of projectors *d'd*, *f'f*, drawn at right angles to GL. Join *def*. This is the plan, and the line *d'f'* is the elevation of the set-square in the required position.

Surfaces inclined to the Horizontal and Vertical Planes.—In (*a*) Fig. 229 the figure *a'b'c'd'* is the elevation, and the line *ac* is the plan of a piece of cardboard, which rests with the edge *ad* on the ground, the plane of the face of the cardboard being vertical.

Find (1) *the true shape of the cardboard*
(2) *the inclination of the top edge of the cardboard, bc, to the* HP *and* VP *respectively.*

(*b*) Fig. 229 is a pictorial projection, showing the card in the position indicated, and should be studied in connexion with (*a*) in order to make the meaning clearer.

The cardboard is inclined to the VP, and consequently its true shape *will not appear in the elevation a'b'c'd'*. The *real* shape of the card may be determined in either of two ways.

1st method.—Consider the card to be rotated about the edge *ad* as an axis until the card lies upon the HP. Then *a*,5,6,*c* is the plan of the card lying flat upon the HP, and shows the true shape of the card. To obtain this view, set out *c*6, *a*5 at right

angles to the plan *ac*, and make $a5 = a'b'$, and $c6 = d'c'$, taken from the elevation. Produce 6,5, *ac* to meet at O. Then the angle θ, *i.e.* $ao5$, is the actual inclination of the top edge of the cardboard to the ground. The rabatment of the cardboard into the HP is illustrated in the pictorial drawing (*b*).

2nd method.—Select the vertical edge $c'd'$ of the cardboard as an *arbitrary axis*, and rotate the card about this axis until its face is parallel to the VP. This operation can be represented by striking an arc, *o*O in plan, with *c* as centre. From b' in elevation draw b^1b^2 parallel to GL. These two lines, $a2$, b^1b^2, are the plan and elevation respectively of the path of *b* as the cardboard is rotated about the edge *cd*. Join b^2c', and draw a perpendicular

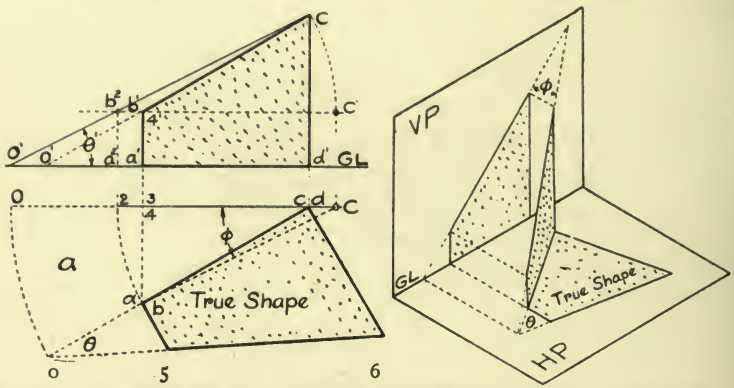


FIG. 229.—Plan, elevation, and pictorial sketch illustrating method of finding true shape of inclined surfaces.

b^2a^2. Then $a^2b^2c'a'$ is the **true shape** of the piece of card. Produce $c'b^2$ to meet the GL at O′. Then the **true inclination** of the top edge *bc* of the card to the horizontal is shown at θ', *i.e.* the angle between c'O′ and the ground line GL.

Meaning of Terms "Theta" and "Phi."—The **real angle** between the top edge, *bc*, of the card and the vertical plane is found as follows. Select an imaginary horizontal line $b4$ (in plan) as an axis of rotation. This line is *horizontal* and *at right angles* to the VP; therefore its elevation is a **single point** $b'4'$. Keeping this axis, $b4$, horizontal, rotate the imaginary triangle, $4ca$, about it until the point *c* is brought into a horizontal plane through $b'4'$. As this operation is carried out, the path of *c* would appear in elevation as an arc of a circle, centre b', radius $b'c'$. The intersection C′ of this arc and a horizontal line through $b'4'$

is the elevation of the point *c* when the plane of the imaginary triangle 4*cb* is horizontal. The plan of the path of *c* is shown by producing 4*c*, in plan, to C vertically under the point C′ in elevation. The triangle 4C*b* is the real shape of the imaginary triangle which fits between the cardboard and the vertical plane, and the angle ϕ (Phi) between the lines 4C and *b*C in plan is the inclination of the top edge *bc* of the card to the VP. This angle is indicated on the pictorial sketch (*b*) at ϕ. It should be noted, however, that in this position the angle ϕ cannot show its real magnitude in plan, *i.e.* the angle 4*cb* does not measure the actual inclination of the top edge of the cardboard to the VP, because the imaginary triangular surface 4*cb* is inclined to the HP. Hence the need for rabatting it into parallelism with the HP in the manner already described.

The symbols Theta (θ) and Phi (ϕ) are used respectively to denote inclination to the Horizontal and Vertical Plane.

PROBLEM. — A postcard A*b*C*d* has a piece of wire attached to it along the line of a diagonal *bd* as shown, Fig. 230. The card is then rotated about the wire as a horizontal axis until the *plans* of each of the two opposite corners A and C are angles of 120°. *Draw the plan and elevation of the postcard in this position.*

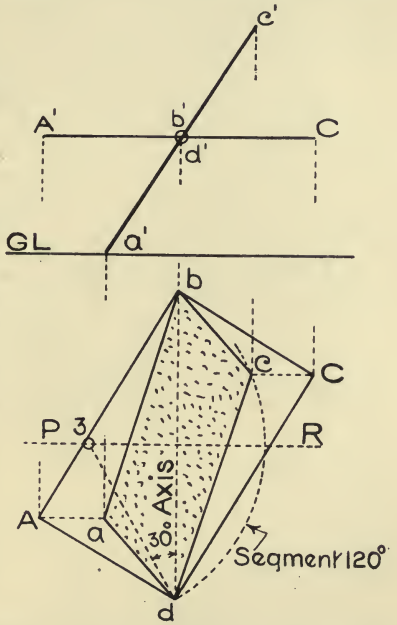


FIG. 230.—A*b*C*d* True shape of postcard. *abcd* Plan of postcard after rotation about the axis *bd*.

Draw a rectangle A*b*C*d* $5\frac{1}{4} \times 3\frac{1}{4}$ ins. to represent the *true shape* of the postcard, arranging one diagonal *bd* at right angles to GL for convenience in rotation. On *bd* describe a segment *bcd* to contain an angle of 120° (as shown in Chapter VI, Angle Properties of Circles). The centre of this segment, 3, is obtained by drawing PR, the bisector of *bd*, and setting out at *d* an angle *bd*3 of 30° (*i.e.* 120° − 90°). Draw C*c* and A*a* at right angles to the axis of rotation *bd* and meeting the segment at *c* and *a* respectively. The lines C*c* and A*a* are the plans of the paths of the opposite corners of the postcard as it is rotated about *bd*. The corner C travels *upwards* while A travels *downwards*.

Draw A′C′ to represent the elevation of the postcard in a horizontal position. The point $b'd'$ on this line is the elevation of the line *bd*, the axis, since this line is horizontal and at right angles to the VP. With b' as centre and radius $b'C'$ (or $b'A'$) describe the arcs $C'c'$, $A'a'$. Draw projectors from *a* and *c* in plan at right angles to GL, to intersect the arcs $C'c'$, $A'a'$ at c' and a' respectively. Draw $a'c'$ to represent the elevation of the postcard in the required position. Join *ab*, *bc*, *cd*, *da* in plan to represent the plan of the postcard after rotation has taken place.

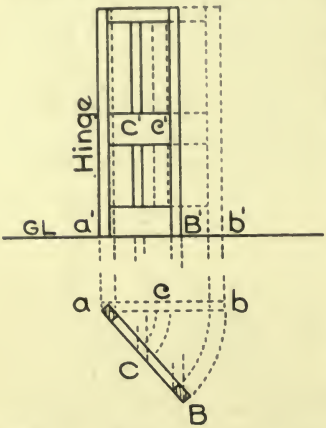


FIG. 231.—Plan and elevation (solid lines) of door after having been swung on its hinges from position indicated by dotted lines.

New Plan of Door.

PROBLEM.—The plan and elevation of a door are shown by dotted lines, *ab*, *a′b′*, Fig. 231. *Draw a new plan and elevation of the door after it has swung through an angular space of 45°.*

Draw the plan and elevation to scale to represent a 6 ft. 9 ins. × 2 ft. 9 ins. door with stiles *a* and *b* and muntin *c* $4\frac{1}{4}$ ins. wide. Set out a line *a*B in plan making 45° with the original plan *ab*, and with *a* as centre describe the circular arcs *b*B, *c*C, to represent the plans of the paths of the stiles and muntins as the door swings on its hinges. Draw projectors from C and B in plan to meet the horizontal edges of the three rails in elevation as at B′ and C′. Join the various points so obtained, and thus complete the elevation of the door, showing (by solid lines) how it would appear after having swung through an angle of 45°.

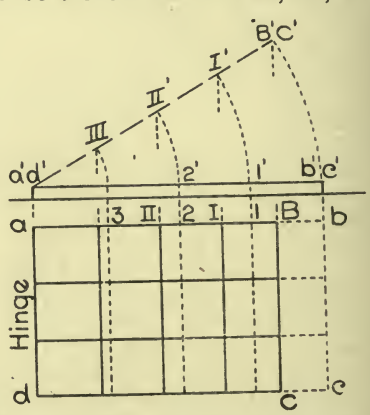


FIG. 232.—Plan of skylight after rotation upon a hinge.

New Plan of Skylight.

PROBLEM.—The rectangle *abcd*, Fig. 232, with the dotted lines at 1, 2, and 3, is the *plan* of a skylight in a *horizontal* position. *Make a new plan showing how it would appear after it had been opened to a position indicated by the line a′B′ in elevation.*

The procedure in this case is very much the same as that already described fully in the foregoing problems, and need not be given in

detail. The arcs $b'B'$, $1'I'$, etc., represent the elevation of the paths of the various corners of the panes of glass as the skylight is rotated about the hinges at *ad*. Each of these points in plan appears to move *square with the hinge ad*; *b* moving to B, *c* to C, etc. Then the rectangle *a*BC*d*, with the solid lines at I, II, and III, is the plan of the skylight in the open position required.

Openings in Roofs.—A thorough grasp of the principles of rabatment or rotation already described will enable the student to understand many practical problems of a geometrical character. The determination of the true shape of holes cut in roof surfaces, the method of finding the bevels for various oblique cuts in carpentry and joinery work, and for roof timbers, etc., all fall within this category.

True Shape of Openings in Inclined Surfaces.

PROBLEM.—The triangle *hkm*, Fig. 233, is the *plan* of a piece of wood (a triangular prism) which penetrates through a roof surface shown in elevation at $h'k'$, where the *shaded part* (a') represents the part of the piece of wood which projects *above* the roof surface. *Determine the correct shape of the hole which must be cut in the roof to allow this piece of wood to pass through.*

Draw any ground line GL and produce the line $h'k'$, which represents the slope of the roof, to meet the GL at *o*. Set out a line *od* at right angles to GL, to represent the imaginary *hinge* or *axis of rotation*. The elevation of the three corners of the triangular hole are shown at h', m', and k'. Imagine the roof surface, with the triangular hole, to swing upon *od* as a hinge until it lies upon a horizontal plane GL. The arcs $h'H'$, $k'K'$ are the elevations of the paths of the three corners of the hole as the roof surface is being rabatted downwards. Similarly, *h*H, *k*K, and *m*M, in plan, *drawn at right angles to the hinge op*, are the plans of the paths of the three points *h*, *k*, and *m* during rotation. Projectors from H′ and K′ on the GL to meet the paths *h*H, etc., at H, K, and M respectively, determine the position of the three corners of the triangular hole after rabatment has taken place, and the triangle HKM is the true shape of the hole which must be cut in the roof.

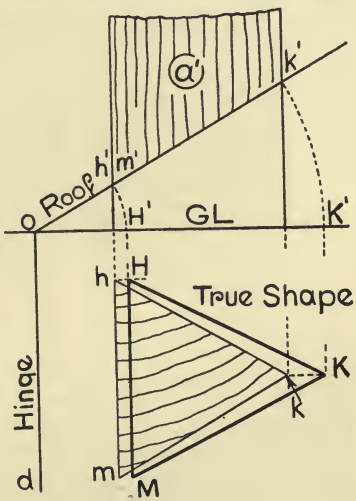


FIG. 233.—Method of finding the true shape of triangular opening in roof.

Roof penetrated by Octagonal Shaft.

PROBLEM.—A roof surface is penetrated by an octagonal shaft, Fig. 234. *Determine the true shape of the hole which must be cut in the roof covering material so that it may fit tightly around the shaft when in place.*

The method of construction here is similar to that already described and should be easily followed. Draw a line $a' \ldots e'$ to represent the roof slope, and draw an octagon *abcdefhk* to represent

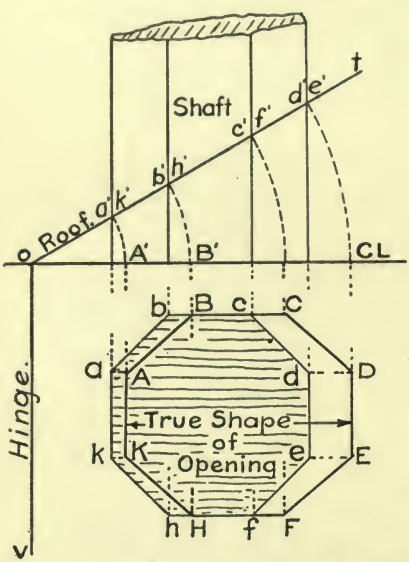


FIG. 234.—True shape of opening in roof for octagonal shaft.

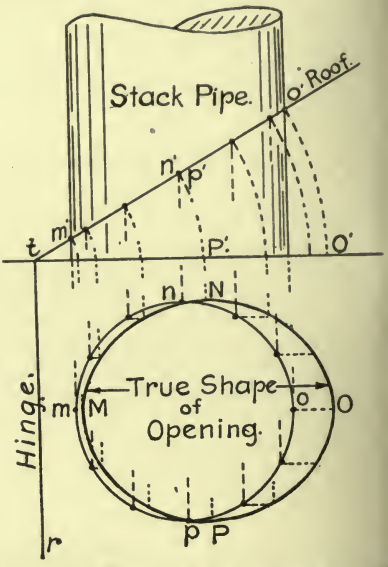


FIG. 235.—True shape of opening in roof to fit around stack-pipe.

the plan of the shaft. From each of these points draw vertical projectors and complete the elevation of the shaft. With centre *o* and radii *oa'*, *ob'*, etc., describe the arcs *a'*A', *b'*B', etc. From A', B', etc., draw projectors *at right angles to* GL to meet the lines *b*B, *c*C, etc., in plan. Join AB, BC, CD, etc. Then ABCDEFHK is the true shape of the required hole.

Roof penetrated by Cylindrical Stack-Pipe.

PROBLEM.—A stack-pipe of circular section, Fig. 235, is to be carried up through a roof as shown. *Determine the correct shape of a hole to be cut in a piece of sheet lead which is to lie on the roof surface and fit tightly around the stack-pipe when in position.*

Draw the circle *mnop*, making the diameter *mo* equal to the diameter of the cross section of the stack-pipe, and by drawing

vertical projectors from *m* and *o*, determine the elevation of the stack-pipe. Divide the circle, in plan, into any convenient number of parts as shown (twelve in this case), and find the elevation of these points on the plane of the roof as shown at *m'*, *n'*, *p'*, *o'*, etc. With *t* as centre draw the arcs *p'*P', *o'*O', etc., to meet GL. From P', O', etc., draw projectors at right angles to GL to determine the position of O, P, M, N, etc., the lines *o*O, *p*P, etc., in plan having been drawn *at right angles to the imaginary hinge, tr.* Draw a fair curve through the twelve points O . . . P . . . M . . . N. The resultant ellipse is the correct shape for cutting the hole in the sheet lead.

True Shape of Opening in Wall.

PROBLEM.—(*a*) is the plan and (*a'*), Fig. 236, is the elevation of a wall with an opening *a'b'c'd'* cut through it. This opening appears in elevation as a circle. *Determine the true shape of the opening.*

This problem is the converse of the foregoing problems, and the student should realise that since the wall is *inclined away from* the vertical plane, as shown at *ac* in plan, the circle *a'b'c'd'* in elevation is *not the true shape* of the opening. In order to determine its true shape select an arbitrary line PL parallel to the ground line. Divide the circle in elevation into, say, twelve equal parts and obtain the plans of these points at *abdc*, etc. Using *a* (on PL) as centre describe the circular arcs *b*B, *c*C, etc., to represent the plans of the paths of the various points as the plane of the face of the wall is rabatted into parallelism with the VP at PL. Draw the horizontal lines *b'*B', *c'*C', etc., in elevation, and by drawing vertical projectors from B, C, etc. (on PL), in plan, determine the points B', C', etc., in elevation. An ellipse through the twelve points *a'* . . B' . . C' . . D', etc., in elevation gives the true shape of the outline of the opening.

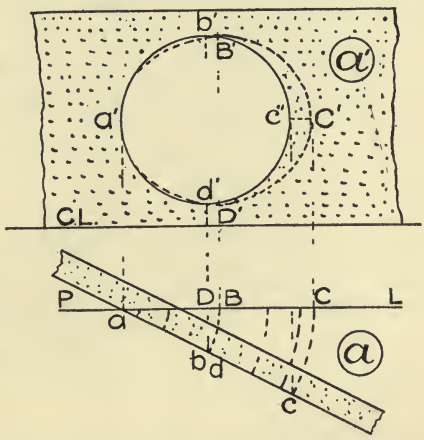


FIG. 236.—True shape of opening in wall; the elevation of the opening being a circle.

True Shape of Slant Side of Turret.

PROBLEM.—The plan T, Fig. 237, and the elevation T', of a square turret are shown. The lines 1, 2, in elevation represent the horizontal joints between the pieces of lead, which is laid in three sheets *a'*, *b'*, and *c'*. *Determine the* **true shape** *of one of the slant surfaces of the*

turret showing the true shape of each of the three successive pieces of lead a, b, and c.

Draw the square o,3,4,o and its diagonals, o*v*4, o*v*3. This is the plan, and the triangle 3′*v*′o′, is the elevation of the turret. Divide the slant height o′*v*′ into three equal lengths at 1 and 2, and from these points draw projectors to meet the diagonals of the square in plan as at 5 and 6. From 5, 6, etc., draw lines 5,7, 6,8, etc., parallel to the square to represent the joints in the lead. Using oo as a horizontal axis of rotation, rabat the face o*v*o around until it lies upon the ground. To represent this operation, proceed in the manner already described, by combining the two views of the paths of the points *v*, 6, 8, 5, 7, as the face o*v*o is rotated about oo.

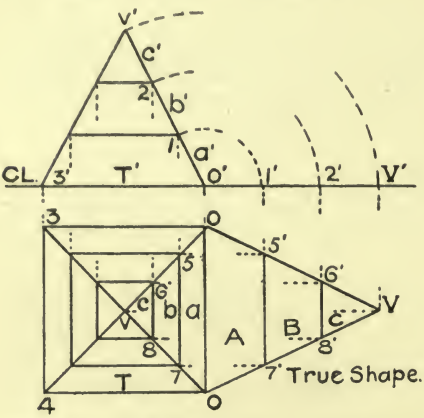


FIG. 237.—True shape of one sloping side of a square-based turret.

With o′ as centre, radii o,1, o,2, o*v*′, respectively, describe arcs 1,1′, 2,2′, *v*′,V′, and from points 5, 6, *v*, 8, and 7 in plan, draw projectors, 5,5′, 6,6′, etc. Draw vertical projectors from 1′, 2′, V′, on the GL, to determine the position of points 5′, 6′, V, etc., on the lines 5,5′, 6,6′, etc. Then oVo is the *true shape* of one of the *slant faces* of the turret, and the spaces lettered A, B, and C, respectively, represent the **true shape** of each of the successive pieces of *sheet lead* required for covering this surface.

True Shape of Face Template for Hopper.

PROBLEM.—The plan and elevation of a hopper made from sheet metal are shown at H, H′, Fig 238. *Determine the correct shape of the* **face template** *for marking out one of the sloping sides of the hopper, assuming that it is made of four pieces of sheet metal of the same shape and size, soldered along the four corners*, 2,6, 3,7, 4,8, 1,5.

Draw the square 1,2,3,4 to represent the plan of the upper edge of the hopper. Draw a level line 1′,4′, at a height above GL equal to the height of the hopper, and obtain the elevation of the various points 1, 2, 3, 4, on this line by projecting from 2 and 3 in plan. Draw lines 2′,6′, 4′,8′, in elevation to indicate the slope which the hopper is required to have. Draw the diagonals 1,3, 2,4 of the square in plan, and by drawing vertical projectors from 7′, 6′ in elevation, obtain the plans of the four lower corners 6, 7, 8, 5 on the diagonals 1,3, 2,4; join 6,7, 7,8, 8,5, 5,6 to represent the plan of the bottom edges of the hopper. Assuming 7,8 to be a hinge, rabat the surface

7,3,4,8 down to the ground. Thus, with centre 7′ (elevation), and radius 7′,3′, describe the arc, 3′,III′; draw 3,III, and 4,IV, in plan, at right angles to the hinge 7,8, and draw a projector from III′ on the GL to meet 3,III at III and 4,IV at IV. Join 7,III, III,IV, IV,8, and thus obtain the true shape of one face 7,III, IV,8.

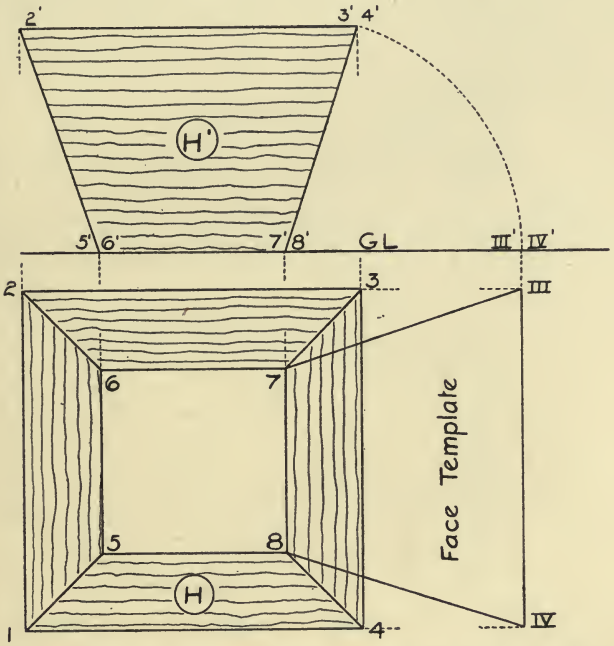


FIG. 238.—Plan and elevation of "hopper," showing method of finding true shape of one side of hopper.

True Length of Strut obtained by Rabatment.

PROBLEM.—(*a*) and (*a′*), Fig. 239, are the plan and elevation of a wall. A strut AB is to be cut to fit up into the corner in such a position that its *plan* is the line AB, and its *inclination to the floor* of the room is 30°. The foot of the strut A is *not at the same distance from the* VP as B. Hence the *true inclination* of the strut to the horizontal *would not appear in the elevation* at A′. To obtain this inclination *the strut must be rotated* about an imaginary axis B′C′ (the corner of the room) *until* A *is at the same distance from the* VP (or front wall) *as* B.

The arc A*a*, struck from B as centre, is the plan of the path of the foot of the strut as the rotation is performed. Meanwhile the foot of the strut has been constantly in contact with the floor. Hence the *elevation* of the path of A during rotation is represented by a *horizontal*

line A′*a*′, on the ground line. Having obtained the elevation of the point *a*′ on the ground line, by projecting from *a* in plan, set out the true inclination (30°) of the strut to the horizontal and so draw *a*′B′. *This is the elevation of the strut when placed against the front wall.* During rotation B′ remains *stationary*, being on the axis, so that when the foot of the strut is swung around from *a* to A, A′ is the elevation of the foot and B′ is the elevation of the top end. Join A′B′ to represent

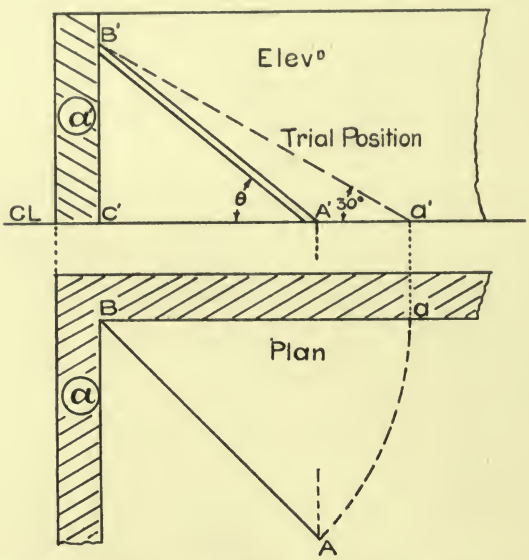


FIG. 239.—Finding true length of strut AB by rabatment.

the elevation of the strut in the required position. Note particularly that the angle θ, which is *greater* than 30°, *is the elevation of the angle of* 30° which the strut makes with the horizontal. If an open book is stood vertically upon the drawing board to represent the walls and a pencil fitted into the angle between the two parts of the book to represent the strut, a very much clearer idea of this important principle may be obtained.

True Length of Diagonal of Rectangular Room.

PROBLEM.—A room, Fig. 240, is 19 ft. 6 ins. × 15 ft. 6 ins. × 11 ft. 6 ins. high. A cord TT′ is stretched tightly from a point *a on the floor* to the *opposite* corner *b* of the *ceiling*. *Find the true length of the cord.*

Draw the plan and elevation of the floor to any suitable scale. Then *ab* is the plan, and *a*′*b*′ is the elevation of the cord; *the true length of which does not appear in plan*, because the cord is *not parallel*

to the ground, nor does it appear in the elevation because it is *inclined away from* the observer and is *not parallel to the* VP. The inset pictorial sketch will make the matter clearer. Comparing this with the plan and elevation it is evident that the cord is not equal in length to a diagonal *ab* of the floor, 19 ft. 6 ins. × 15 ft. 6 ins., nor is it equal in length to the diagonal, *a'b'*, of one of the walls 19 ft. 6 ins. × 11 ft. 6 ins. A cord stretched across the room as shown in the inset sketch would be longer than the diagonal of 19 ft. 6 ins. × 15 ft. 6 ins. rectangle, since this diagonal is the *plan* of the cord and one end of the cord is *higher* than the other. To obtain the true length proceed as follows. Consider the right-angled triangle *aob*, made up of the plan of the line as a base, *ob* as a perpendicular, and the cord itself, *ab*, as the hypotenuse. Keeping the lower end *a* of the cord stationary, rotate the triangle *aob* into parallelism with the VP. With *a* as centre, radius *ab* in plan, describe the arc *b*B. Through *b'* in elevation draw a horizontal line *b'*B'. From B in plan draw a projector at right angles to GL to determine the elevation B' of the top end of the cord when the triangle *aob* has been rotated about *a*. Thus *a'*B'O' is the real shape of the triangle *a'b'o'*, and the real length of the cord is found by measuring off the length of *a'*B' to the same scale to which the plan and elevation have been drawn.

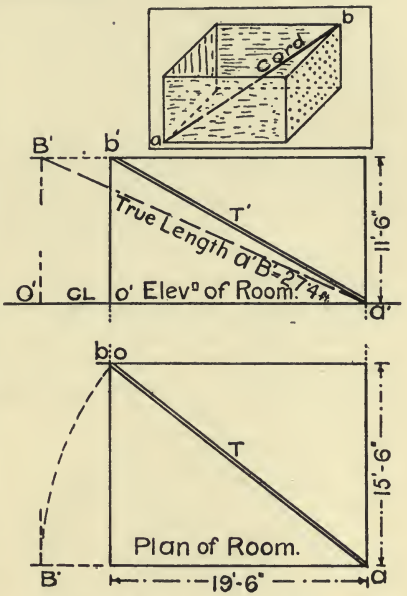


FIG. 240.—Finding length of diagonal of rectangular room.

CHAPTER XIII

SECTIONS OF SOLIDS

Sectional views are of great importance to the technical draughtsman. By means of a sectional view it is often possible to make clear some detail of construction which would not otherwise appear clear from an inspection of either the plan or elevation of the object. The term **section** implies that the object has

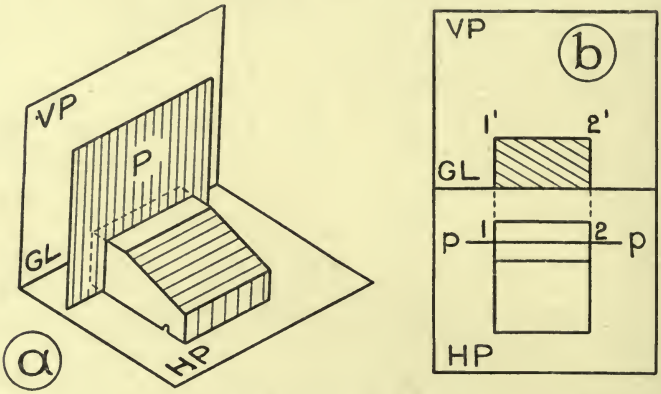


FIG. 241.—(*a*) Pictorial sketch illustrating meaning of "vertical section" made by plane P. (*b*) Orthographic projection of same showing VP and HP laid out flat.

been *cut through* at some suitable point, the sectional view showing the shape of the *cut surface* after one part of the object has been removed. In order to obtain this sectional cut through the object, it is usual to assume that a plane surface P, passes through the object, as shown in (*a*), Fig. 241. This **plane of section** meets one or other of the co-ordinate planes (HP or VP) in a *straight line.* This line of intersection is termed the **trace** of the plane. Thus, referring to (*a*), Fig. 241, a piece

of window sill is shown standing upon the HP. A vertical plane of section P penetrates through the entire thickness of the sill along a line parallel to the front edge of the sill. The sketch (*a*) is introduced here merely to give a correct idea as to the meaning of the term *section*. The usual drawing made would be a plan and elevation of the sill as shown at (*b*), Fig. 241, where the line *pp* represents the line of *intersection* or **trace** of the plane P and the HP. The points 1 and 2 indicate the limits of the sectional surface, since the section plane enters the sill at 1 and emerges at 2. Thus, projectors from 1 and 2 in plan, upwards to GL, give the distance apart of 1′ and 2′; the height of 1′2′ above GL

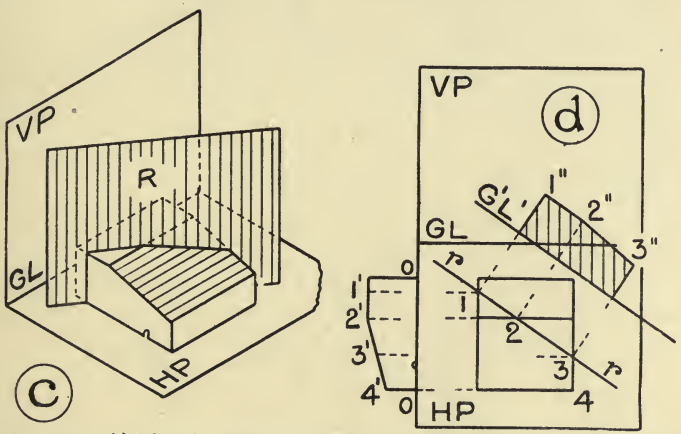


FIG. 242.—(*c*) Pictorial sketch and (*d*) orthographic projection of vertical section made by a plane R inclined to VP.

representing the thickness of sill through which the plane of section passes.

The pictorial sketch (*c*), Fig. 242, shows a *vertical plane of section* R, which differs from P in the previous case in that it is *not parallel* to the VP, the cut being made obliquely across the sill. Referring to the plan and elevation at (*d*) the line *rr* is the plan of the section plane R as viewed from above, while the points 1 and 3 are the plans of the points at which the vertical section plane enters and emerges from the sill, while the point 2 is the plan of the point of intersection of the section plane and the sharp arris on the sill. In order to show the true shape of the cut surface it is necessary to assume a *new plane* standing vertically upon the new ground line G′L′, which is made parallel to the section line *rr*. Projectors are then drawn, as shown, from 1, 2, 3 in plan, and at right angles to G′L′. The height of 1″ is

obviously equal to the thickness of the sill, and is obtained from the auxiliary drawing (an end elevation of the sill), shown at the left-hand side of (*d*), and using 00 as a ground line. The height of 2″ on the sectional view is obtained in the same way from 2′ on the end elevation. It should be noted, however, that the height of 3″, which is the elevation of the point 3 in plan, *i.e.*

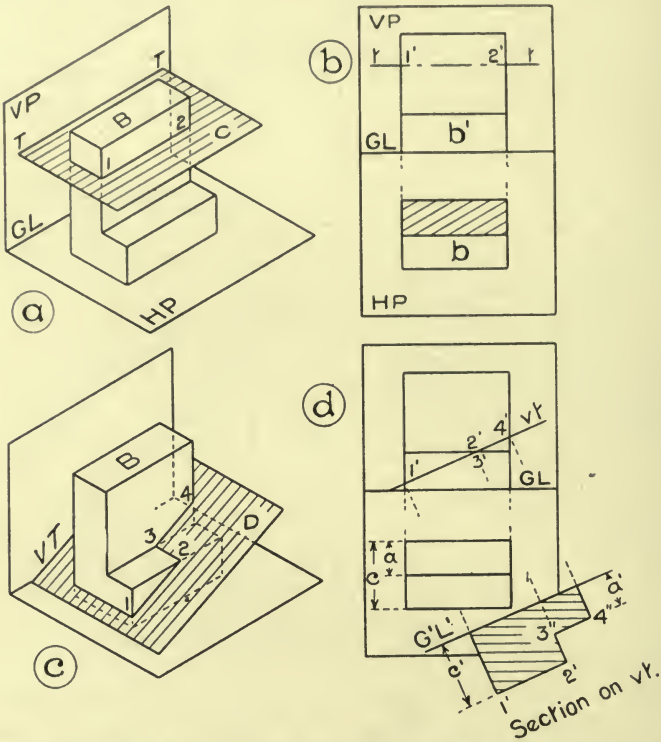


FIG. 243.—(*a*) and (*b*) Pictorial sketch and orthographic projection of section made by horizontal plane C. (*c*) and (*d*) Section made by inclined plane D.

the point at which the section plane emerges from the sill, is not exactly equal to the thickness of the thinner edge of the sill, 04, on the end elevation, but is obtained by drawing a projector from 3 in plan, at right angles to 00, to meet the sloping line 2′4′ at 3′ on the end elevation. The height of 3″ above G′L′ in the sectional view is then made equal to the height of 3′ above 00 in the end elevation.

Note that in both the foregoing cases (*a*) and (*b*) the section plane is *vertical*. Hence the lines *pp* in (*b*) and *rr* in (*d*) appear on the plan of the object. These cases should be carefully compared with the cases which immediately follow, in which the plane of section is *not* vertical.

Referring to (*a*), Fig. 243, a pictorial sketch of an L-shaped block of wood B is shown. A horizontal plane of section C cuts completely through this block and meets the VP in the line TT. On the orthographic drawing of the block, shown at (*b*), the section plane is represented by one straight line *tt* on the elevation *b'*, and the fact that the section plane passes through the upright arm of the letter L and misses the lower part, is indicated by the shaded part in plan, *b*.

The sketch (*c*), Fig. 243, represents a similar block B to that shown in (*a*), but the plane of section D is *inclined* to the ground. Note that there are four points at which the section plane intersects the front edges of the block, the points 3 and 4 being on the upright part of the block, and the points 1 and 2 being on the lower and thicker portion of the letter L. The inclined plane meets the VP in a line VT (vertical trace). Hence, in order to represent the plane of section on the orthographic drawing (*d*) it is only necessary to draw a single line *vt*, which is, in effect, a front-edge view of the inclined plane, while 1′ and 4′ indicate the limits of the sectional surface. The points 2 and 3 being both at the same height above the ground, and exactly behind each other, appear as one point in the elevation, marked 2′,3′. To obtain the true shape of this section draw a new ground line G′L *parallel to the section line vt*, and draw projectors 1′1″, 2′2″, etc., from points 1′, 2′, 3′, 4′, in elevation at right angles to G′L′. The distance of 1″ and 2″ away from G′L′, *i.e. c'*, is equal to the full width, *c*, taken from the plan; while the distance of 3″ and 4″ from G′L′, *i.e. a'*, is equal to the part width, *a*, taken from the plan.

(*a*), Fig. 244, is another simple illustration of the same type. In this case a building brick *abcd* is cut by a vertical section plane, the *edge view* of which, looking down from above, is represented by the line RS. In this case, *the section line* RS *itself is used as a ground line* on which to draw the sectional view 1PR2, in which the distance 1P (or 2R) is made equal to the thickness of the brick 1′1″ measured from the elevation. The conventional method of indicating that a sectional view has been obtained is to *cross-hatch* the outline of the section as shown in the preceding diagrams.

(*b*), Fig. 244, shows the plan and elevation of a square stone cap for a gate pillar which is cut by a vertical section plane on

the line AB. *Make a view showing the true shape of the sectional cut.* Note first that all faces of the stone cap are plane surfaces, and that *when a plane surface intersects another plane surface their intersection is a straight line.* Hence we conclude that the outline of the sectional view in this case will be made up of **straight lines.** Draw a new line G′L′ parallel to AB, and from the points 5, 6, 7, 8 at which the section line AB cuts the plan of the various edges of the cap, draw projectors 5,5″ ; 6,6″, etc., at right angles to G′L′. Referring again to the plan and elevation, note that at 5 (in plan) the section plane has to pass through a

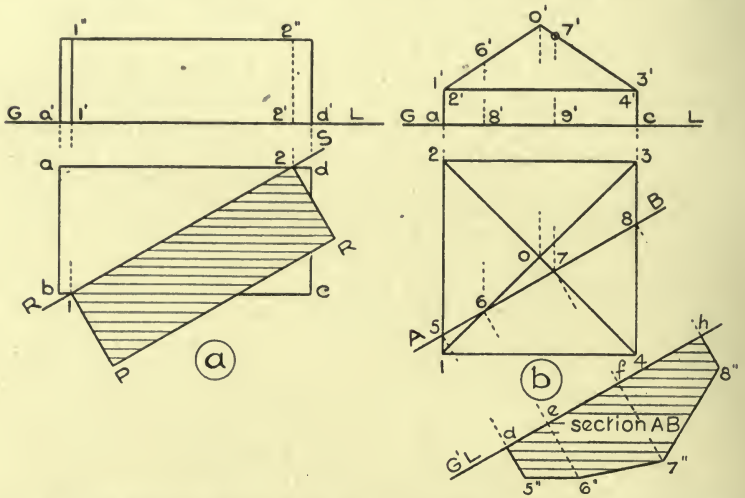


FIG. 244.—(*a*) Vertical section through brick cap. (*b*) Vertical section through stone

thickness equal to the height *a*1′, taken from the elevation. Similarly, at 8 in plan, the section plane has to cut through a thickness equal to *c*3′, taken from the elevation. Hence, the distances *d*5″ and *h*8″ on the sectional view are made equal to *a*1′ or *c*3′ on the front elevation of the cap. Referring again to the plan, note that at the point 6 the section plane cuts through the sloping edge 1,0, of the cap. The height of this point, 6, above the base of the cap is given at 8′6′ in the elevation. Hence the distance *e*6″ in the sectional view is made equal to 8′6″ in the elevation. In the same way, the distance *f*7″ on the sectional view is made equal to 9′7′ in the elevation. Joining the points *d*, 5″, 6″, 7″, 8″, *h* by straight lines, the true shape of the sectional view on AB is obtained.

Sections of Solids with Curved Surfaces.—It has already been noted that when a plane surface intersects another plane surface, the line of intersection is *straight.* The two cases which immediately follow, viz. (*a*) and (*b*), Fig. 245, represent the intersection of a **plane** surface and a **curved** surface, the line of intersection in each case being a *curved* one.

(*a*), Fig. 245, represents a semi-cylindrical blue Staffordshire brick, such as is used for *coping* on an external brick wall, such as a boundary wall. The length of the brick is shown at *ab* in plan, the width of base of the brick at *a'b'*, and the height at *p'*3' in the elevation. This half-cylinder (the brick) is cut by a

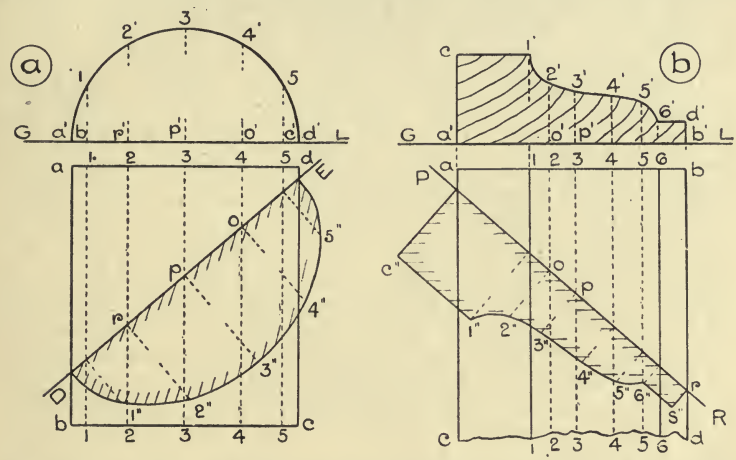


FIG. 245.—(*a*) Vertical section through semi-cylindrical coping brick. (*b*) Vertical section through architrave.

vertical section plane, the edge of which, as seen from above, is represented by the line DE. *Make a view showing the true shape of the section on* DE.

Divide the semicircle in elevation into any convenient number of equal parts at 1', 2', 3', 4', 5'. The 60° set-square may be conveniently employed in all such cases in which a circle or semicircle is to be divided into equal parts, the 30° angle of the set-square being used to determine the position of 1' and 5' (*i.e.* the angle *a'p'*1' is 30°), and the 60° angle of the set-square being used to determine the points 2' and 4'. Draw projectors from 1', 2', 3', etc., in the elevation to determine the position of the lines 1,1, 2,2, 3,3, etc , in the plan. These lines are the plans of imaginary lines or *stripes* running along the entire length of the curved surface of the half-cylinder. From

the various points, *r*, *p*, *o*, etc., at which the section line DE crosses the various stripes 1,1, 2,2, etc., set off projectors *r*2″, *p*3″, etc., making the distances *r*″2, *p*3″, etc., respectively equal to the heights *r*′2′, *p*′3′, etc., taken from the elevation. Through the points so found draw a free curve, D1″,2″,3″,4″5″E. This curve is a *semi-ellipse*, and is the true shape of the section of the semi-cylinder. Where the true shape of a sectional cut is shown in such a way as to overlap another drawing, as in this case, it is not advisable to cross-hatch the whole of the sectional view. Neatly drawn hatched lines to indicate the limits of the section, as indicated, are quite sufficient.

(*b*), Fig. 245, represents the plan and end elevation of a piece of architrave. *Find the correct shape of a cross section of the moulding along a line* PR.

Using the section line PR as a new ground line, set-off projectors *ac*″, *rs*″, etc., at right angles to PR, making the distances $ac'' = a'c'$, $rs'' = b'd'$, etc. Select any number of points, 2′, 3′, 4′, 5′, etc., on the elevation, and from these points draw projectors at right angles to GL, to determine the position of the parallel stripes 2,2, 3,3, 4,4, etc., in plan. Set off the distances *o*2″, *p*3″, etc., equal to the heights *o*′2′, *p*′3′, etc., measured from the elevation. Note that this moulding is an illustration of a case in which the section plane cuts through another *plane surface*, *i.e.* the flat portions of the moulding, at some parts, and at other parts cuts through a *curved surface*, *i.e.* the OG curved part of the moulding. Hence the sectional view is made up partly of *straight lines*, as at *ac*″1″, and *rs*″6″, while the outline of the section at 1″ . . 3″ . . 5″, etc., is a *curved line*. The sectional view in this case represents the true shape of the end surface of the moulding if it is cut to mitre along the line PR.

Sections of Curved Surfaces which Produce a Straight Line Outline.—The pictorial sketch (*c*), Fig. 246, represents a frustum of a cone P (a pail) supported on two bricks BB, and intersected by two plane surfaces (*a*) and (*b*). *Determine the true shape of the section in each case.*

Taking the horizontal section made by the plane (*b*) first: the plan and elevation of the pail and brick are shown at (*d*), the line *bb* representing the front edge view of the horizontal section plane. Note that the section plane passes through the pail, but does not cut through the bricks. **All horizontal sections of a cone are circular sections.** Hence the true shape of the section on the line *bb* is obtained by drawing projectors from 3′ and 4′, *i.e.* the points at which the edge view of the plane *bb* intersects

the elevation of the outer edge of the pail, and using 3,4 as diameter, describe the circle 3,5,4, centre *o*, which is the true shape of the horizontal section of the pail at *bb*.

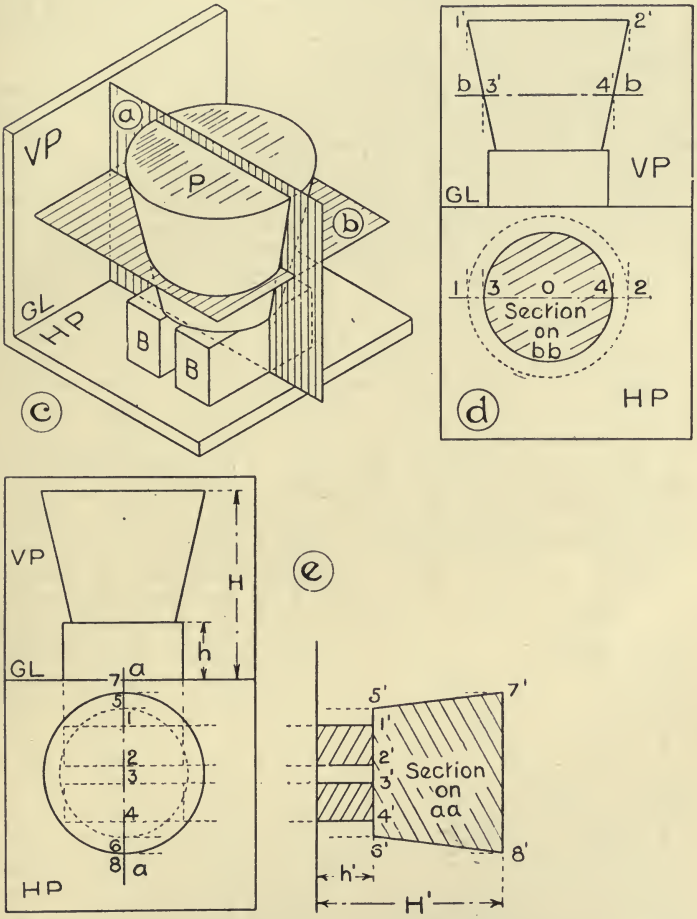


FIG. 246.—(*c*) Pictorial sketch of vertical section plane (*a*) and horizontal section plane (*b*) cutting through a frustum (P) and two bricks BB. (*d*) Plan, elevation, and horizontal section. (*e*) Plan, elevation and vertical section.

(*e*), Fig. 246, represents the same objects, *i.e.* the pail and bricks, the two circles being the plans of the larger and smaller ends of the pail, while the dotted rectangles represent the plans

of the bricks. Determine the true shape of a vertical section made by a plane *aa*.

Draw a new ground line parallel to *aa*. From points 1, 2, 3, 4, at which the section line *aa* crosses the plans of the bricks draw projectors at right angles to the new ground line, and make the heights of 1′, 2′, 3′, 4′ equal to the height of the brick *h*, measured from the elevation.

Referring again to the plan, the section line crosses the outer circle at 7 and 8. These are the plans of two points on the larger end of the pail, which stands at a height H above the GL. Hence the position of 7′ and 8′ on the sectional view is found by drawing a level line, 7′8′, and making the distance H′ equal H on the elevation. Then from 7 and 8 in plan draw projectors parallel to GL to meet the line 7′8′ at 7 and 8. In the same manner determine the position of the points 5′ and 6′ on the sectional view. The points 5 and 6 in plan represent points which occur on the smaller circular end of the pail at a height *h* above GL. Hence 5′ and 6′ on the sectional view will be found by drawing projectors from 5 and 6 in plan to meet a line at level *h*′ above the new ground line. Join 5′7′8′6′. This is the true shape of the section of the pail. Note that the outline of section is made up entirely of *straight lines*, notwithstanding the fact that we are dealing with the intersection of a *plane surface* and a *curved surface*. This is an exception to the general rule which applies to such cases, which is, as already stated, *when a plane surface intersects a curved surface the line of intersection is a*

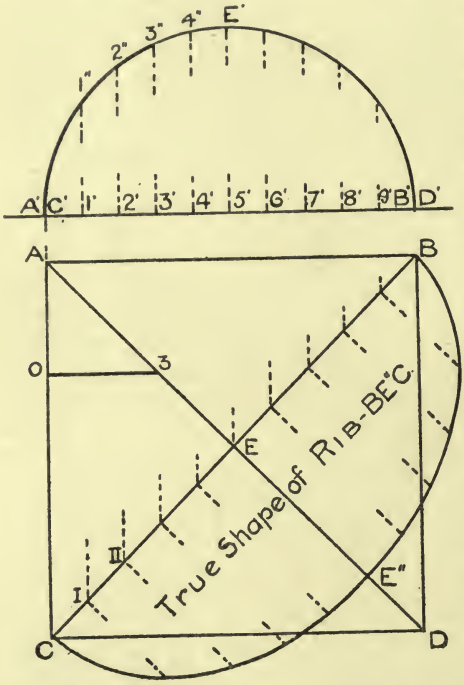


FIG. 247.—Finding true shape of rib BC for lantern light over well hole ABCD.

curved line. The foregoing cases, which illustrate the general rule, and exceptions to the rule, should be carefully studied and compared.

Several practical illustrations are given as further exemplifying these principles.

Practical Examples of Sections.—Fig. 247 represents the plan ABCD of a well hole over which a lantern light is to be placed. The elevation of the lantern light is to be a semicircle, A′E′D′. *Determine the correct shape of the outline of the ribs* AE, BE, DE, *and* CE, *to which the sash bars, such as* O3 *are to be fitted.*

Divide the semicircle in elevation into any convenient number of points, at 1″, 2″, 3″, etc., and draw projectors at right angles to A′D′ to meet the plan CEB of the rib at I, II, etc. Set out projectors I,I′; II,II′, etc., at right angles to CB, and make the distances I,I′; II,II′; EE″, etc., respectively equal to 1′,1″; 2′,2″; 5′E′, etc., in the elevation. Draw the free curve through the points C, I′, II′, E″, B, etc. This curve, which is an ellipse, is the correct shape of the upper edge of either of the ribs CEB or AED.

This is a further illustration of the fact that an oblique section through a cylindrical object produces an elliptical section. Note that the true shape of the various sash bars such as O3 in each of the four bays are parts of circles, *e.g.* A′3″ is the true shape of the outline of the bar O3 shown in plan.

Fig. 248 is the plan *efhk* and the elevation *e′f′h′k′* of a square stone slab, through which a circular hole is pierced. *Find the true shape of the section made by a vertical plane* RS.

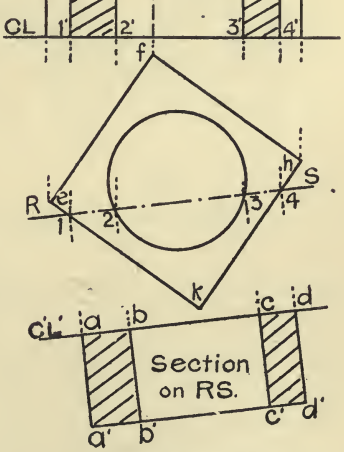


FIG. 248.—Vertical section through square stone slab.

Draw a new ground line G′L′ parallel to RS, and draw a line *a′d′* parallel to G′L′, the distance between G′L′ and *a′d′* being equal to the height of *e′h′* above GL in the elevation. From the points 1, 2, 3, and 4 in plan draw projectors 1*a*, 2*b*, 3*c*, 4*d*, and set out the solid lines *aa′*, *bb′*, *cc′*, *dd′* on

the sectional view. The parts *abb'a'*, and *cdd'c'*, are *shaded* to represent the **solid** part of the stone through which the section plane passes, while the portion *bcc'b'* is left *unshaded* to represent the distance 2,3 on plan at which the section plane is passing through the open circular hole in the slab.

The shaded portions of the elevation also indicate the parts in section but *do not give the correct shape*, because the vertical section plane RS is *not parallel* to GL.

In Fig. 249 a cylindrical column C, C', is shown in plan and elevation. An inclined cut is to be made through the column, as indicated by the line RS, the top portion of the column, *i.e.* above RS, is to be removed, and a piece of sheet lead is to be cut to shape to fit the sloping cut surface of the column. *Find the correct shape of this piece of lead.*

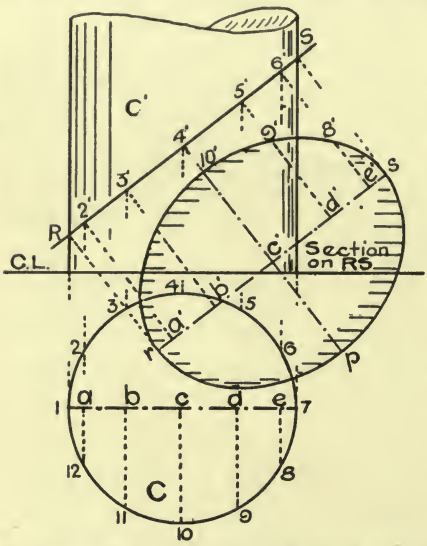


FIG. 249.—Section through a cylindrical column made by an inclined plane RS.

Divide the circle, in plan, into any convenient number of parts, say twelve, since the 60° set-square can then be used to obtain the various points 2, 3, 4, ... 12 in plan. Draw the diameter 1,7 in plan, and from 8, 9, 10, 11, 12 draw projectors *8e*, *9d*, etc., at right angles to 1,7. From points 2, 3, 4, etc., in plan draw projectors at right angles to GL, to cut the section line RS at points 2', 3', 4', etc. Before commencing the sectional view it is advisable to use a cylindrical object, such as a roller, or a cocoa tin, to illustrate the following very important principle. Standing the model upright, as indicated at C', imagine a hole bored right through the cylinder from front to back and on a level with 4' in the elevation. The distance through which the gimlet passes from the time it *enters the front* of the cylinder until it *comes out at the back* is the distance 4,10 on the plan, *i.e.* the **full diameter of the circular end of the cylinder.** Then if an axis *rs* is drawn, parallel to RS and equal in length to RS on the section line, this axis gives the full length of the section

of cylinder required. From 4′ in elevation draw a line 4′p at right angles to rs and make $c'p$ and $c'10'$ each equal to $c10$ in plan. Thus the whole length $p10'$, which is equal to 4,10 in plan, is the length of hole bored by the gimlet, and is, therefore, the fullest width of section.

Reasoning along similar lines it should not be difficult to see that, if a hole is bored right through the cylinder at the level of 3′ or 5′ in the elevation, the gimlet only passes through a thickness of wood equal to 3,11, or 5,9, on plan. Hence the ordinate $d'9'$ on the sectional view is made equal to $d9$ on the plan. Thus, by using the various lengths $a12$, $b11$, $c10$, $d9$, and $e8$, and setting off these lengths along the respective ordinates or lines drawn at right angles to the axis rs, a free curve (*an ellipse*) can be drawn through these points and will be the correct shape of the outline of the section on RS.

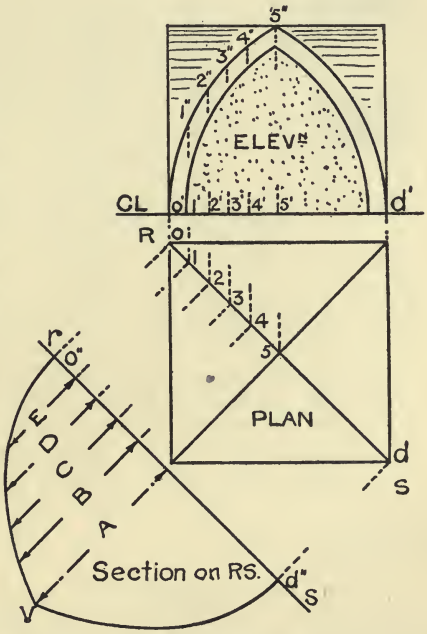


FIG. 250.—Plan, elevation, and section of Gothic cap.

Fig. 250 shows the plan and elevation of a Gothic cap for a gateway pillar. *Find the correct shape of a vertical section taken through the cap, along the line* RS.

The method of working here should not be difficult to follow after having studied the foregoing examples. The total height A on the sectional view is equal to 5′5″ on the elevation; B = 4′4″; C = 3′3″ etc.

The examples which immediately follow are practical examples such as may be found in most technical works on building construction, carpentry and joinery, etc. They are introduced at this point in order that the student may see how the foregoing geometrical principles may be applied to such cases.

(*a*), (*b*) and (*c*), Fig. 251, are *front elevation, longitudinal section,* and *transverse section* of a 4-in. lead soil pipe, showing details

of what is known as a plumber's wiped joint. If a section is taken along the line RS in (*a*) we have to consider what is, in effect, a section parallel to the axis of a hollow cylinder. This would appear as shown in (*c*), while (*b*) represents a section taken across the pipe at AB. The cut at AB passes through the upper length of pipe but does not cut through any part of the wiped joint. Hence a simple sectional view at this point is represented by the **shaded** portion in (*b*). If, however, the outer circle in (*b*) is added, in order to show some part of the object—the wiped joint—which is not included in the plane of section, we have what is termed a **sectional plan**, *i.e.* partly a section, and partly a plan. It is sometimes an advantage to make a "combination" drawing such as this in order to make some detail of construction clearer. Thus a **sectional elevation** would show not only the part of the object *cut through*, but any other parts of the object which happen to fall *behind the plane of section*, but which are seen by an observer looking at the front, or side, of the object.

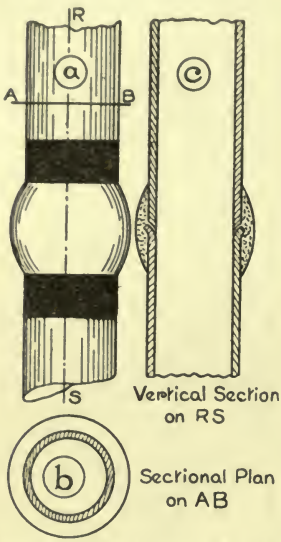


FIG. 251.—(*a*) Elevation of lead soil pipe. (*b*) Horizontal section of pipe on line AB. (*c*) Vertical section of pipe on line RS.

Fig. 252 shows the plan, front elevation and side elevation of a square brickwork pier surmounted by a stone cap. *Make a sectional view on the line* RS.

The width of the section through the brickwork is obtained by drawing projectors from *a* and *b* in plan, while the position of the joints 3′, 4′, etc., in the sectional view of the brickwork is obtained by means of projectors from points 3, 4, 5, 7, etc., in plan. The extreme width of the section through the cap is shown at 1,2 in plan, and by drawing projectors from these points, the position of points 1″, 2″ on the sectional view is obtained. Similarly the position of the points 6″ and 7″ is found by projecting from points 6 and 7 in plan, making the distances 0,6″, 0,7″ in the sectional view equal to 0,6′, 0,7′ in the elevation.

Fig. 253 (*a*) and (*b*) shows the plan and elevation respectively of the joint between a trimmer and trimming joist in a wooden floor, while (*c*) is a vertical section on the line AB. (*b*) and (*c*) should be carefully compared in order that the dotted

and solid lines on a workshop drawing may have their full significance.

Sections through Panelled Door Framing.—Fig. 254 is another illustration of the practical use of sectional views in order to elucidate some details of construction that would otherwise be more or less obscure.

The top left-hand drawing is a front elevation of the joint between the bottom rail and style of an ordinary panelled door. The drawings at (*a*) are *vertical sections* at ST and PR respectively; while those at (*b*) are *horizontal sections* at AB, CD, and EF respectively. The vertical plane of section at ST has to cut through the thickness of the style (say 2 ins.) throughout the entire length ST. The section plane cuts through the double tenons of the bottom rail at points 1, 2, 3, and 4. Hence projectors from these points, drawn at right angles to ST, determine the position of the points 1′, 2′, 3′, 4′ in the tenons on the sectional

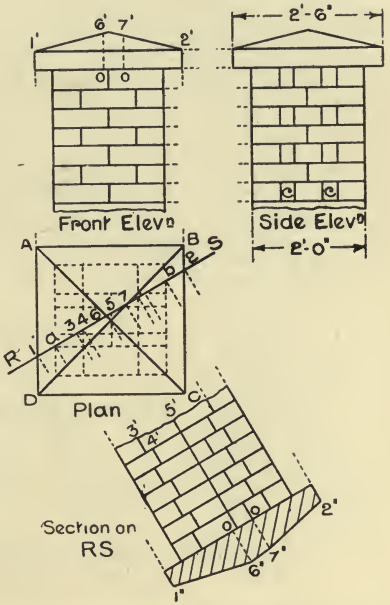


FIG. 252.—Plan, elevation, and section of pillar and stone cap on line RS.

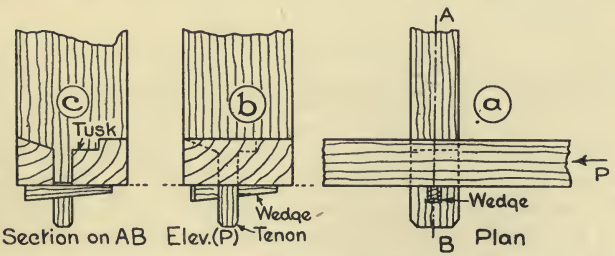


FIG. 253.—(*a*) and (*b*) Plan and elevation of tusk tenon joint. (*c*) Section of joint on line AB.

view (*a*), as shown. The vertical section PR cuts through the bottom rail and panel. The point 5, on the front elevation, is

the point at which the section plane cuts through the lowest point of the plough-groove in the bottom rail, and a projector from 5 at right angles to PR determines the position of 5′ on the sectional view, *i.e.* the point at which the edge of the panel rests upon the bottom edge of the plough-groove.

In a similar manner, it is obvious that a horizontal section at AB will pass through the intersection of the style and panel as

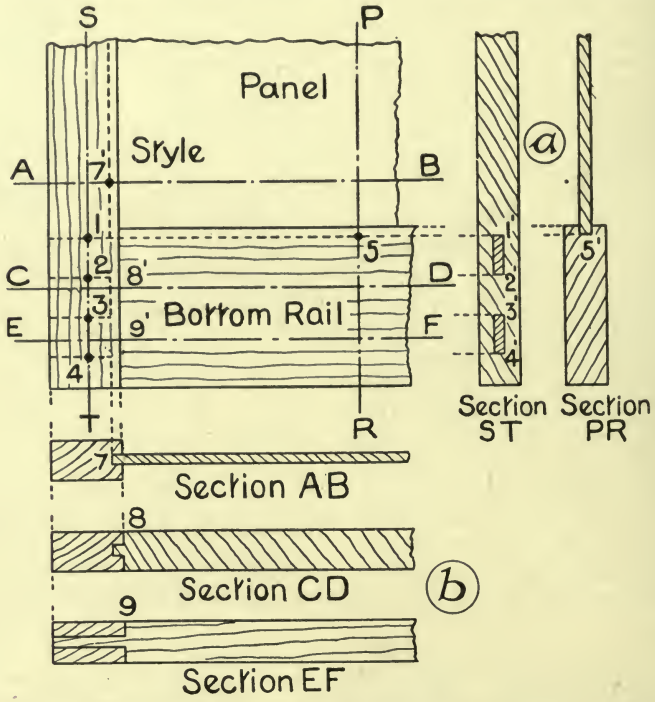


FIG. 254.—Elevation of joint between bottom rail and style of panelled door. (*a*) Vertical sections. (*b*) Horizontal sections.

shown at 7 in (*b*). On the other hand, a horizontal section at CD would not cut through the panel, but would pass through the bottom rail and style as shown at (*b*) 8, and only the haunched part of the tenon, say ½ in. long, would be shown on the section, as at 8 in (*b*).

The section EF, however, would pass completely through the length of the tenon between 9′ and E, and would appear in (*b*) as a pair of parallel lines running the whole width of style from 9.

Conic Sections

Sections of a Cone.—The pictorial sketches (*a*), (*b*) and (*c*), Fig. 255, show three sections of a cone, the respective sections differing entirely in character according to the position of the plane of section. In (*a*) the plane of section RS enters the cone at a point S on one of the generators VB, and passes out of the cone at a point R on a generator VA, on the opposite side of the cone from VB.

When a cone is cut by any such section plane, the outline of the sectional surface is an **ellipse.** If the plane of section entered VB at a point *intermediate* between V and S and came out at a point intermediate between R and A, the section would still have an *elliptical* outline, but the major axis of the ellipse would be

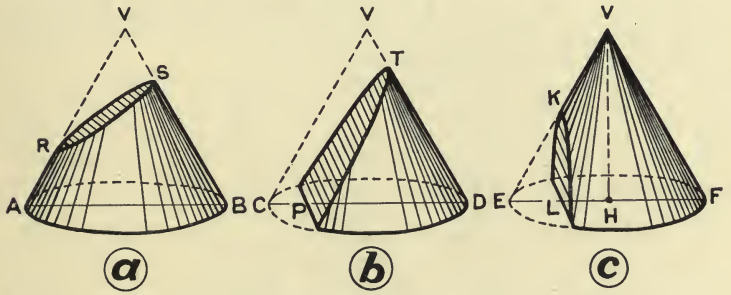


Fig. 255.—Pictorial sketches of various sections of a cone. (*a*) Elliptical section. (*b*) Parabolic section. (*c*) Hyperbolic section.

longer in proportion to the length of the minor axis. On the other hand, as the plane of section approaches the horizontal—*i.e.* is more nearly parallel to the base of the cone—the major axis of the elliptical section is *shorter* in proportion to the length of its minor axis, while the true shape of **a horizontal section of the cone at any level parallel to the base is a circle.**

In (*b*) the plane of section TP enters a generator VD at a point T and passes through the cone in a direction parallel to a generator, VC, on the opposite side of the cone. It is obvious that such a section would never cut through the generator VC, and the outline would be a **Parabola** with its vertex at T and terminating in a straight line P, where the plane of section cuts through the base of the cone. If the position of the plane of section PT is nearer CV or further from it the section is still a parabolic curve, provided that the plane of section is parallel to a generator such as CV, and all such sections produce *single-lipped* curves, *i.e.*

curves that are *closed at one end only*. The ellipse in (*a*) is a *double-lipped* curve.

In (*c*) the plane of section KL enters a generator VE at K and is continued down through the cone in a direction parallel to the axis VH. The outline of the section in this case is a curve of an entirely different character from that obtained in either (*a*) or (*b*) and is known as a **Hyperbola**. The properties of this curve are so complex and are so rarely met with in practical work that no attempt is made to deal with them within the limits of this work. The problems which immediately follow show those features with which it is essential that the craftsman should be familiar.

Elliptical Section of a Cone.

PROBLEM.—The plan *acb* and the elevation *a'c'b'* of a cone are shown, Fig. 255. *Find the true shape of a section on the line* RS, *and draw a plan of the outline of the section.*

Draw a series of straight lines such as AB, CD, etc., parallel to the base of the cone *a'b'*. Consider these lines as representing the *edge view of horizontal planes* which cut through the cone at various levels parallel to its base. The number of these levels chosen and the position at which they cut through the cone is immaterial, though it should be clear that no such horizontal sections need be taken above *n'* or below *h'*, *i.e.* the limits of the plane of section. The plans of these horizontal sections are circles as shown at 1, 2, 3, 4, etc., in plan. Consider the section plane AB. Measure the distance *o*B, and with *c* (in plan) as centre and radius = *o*B, describe the circular arc 1*dd* in plan. From *d'*, the point at which the section line AB crosses the section line RS, draw a projector at right angles to GL to intersect the circle 1 in plan at *d* and *d*. These are the plans of two points on the outline of the section, one at the back of the cone and one at the front. In other words, *d'* in elevation represents two points on the elliptical section, these two points being at the same level above the ground and immediately behind each other.

Consider next the horizontal section at CD in the elevation. Measure the half width *o*D, and with *c*, in plan, as centre, and radius = *o*D, describe the circular arc 2*ee* in plan. This is part plan of the horizontal section at CD. From *e'*, which is the elevation of the two points in which the section RS intersects the section CD, draw a projector to intersect the arc 2, in plan, at *e* and *e*. Proceed in the same way with each of the other sections at 3, 4, 5, and 6 in the elevation, obtaining in each case two points such as *mm* on the plan. Projectors from *h'* and *n'* determine the position of the points *h* and *n*, the extremities of the major axis in plan. A free curve *hdemn*, etc., is the *plan of the outline* of the elliptical section. It is clear, however, that this is not the *true shape* of the section, since it is the plan of a surface

RS *inclined to the* HP. Two methods of determining the *true shape* of the section are shown.

First method (*shown to the left of the elevation*).—Draw a line HN parallel to the section line RS. Set out projectors from *n'* and *h'* at right angles to RS and meeting HN at H and N. Then HN is the true length of the longest axis of the section on RS. Bisect HN and draw the bisector M'M at right angles to HN. This is the *correct position* for the minor axis, since it is *midway between* H and N, the

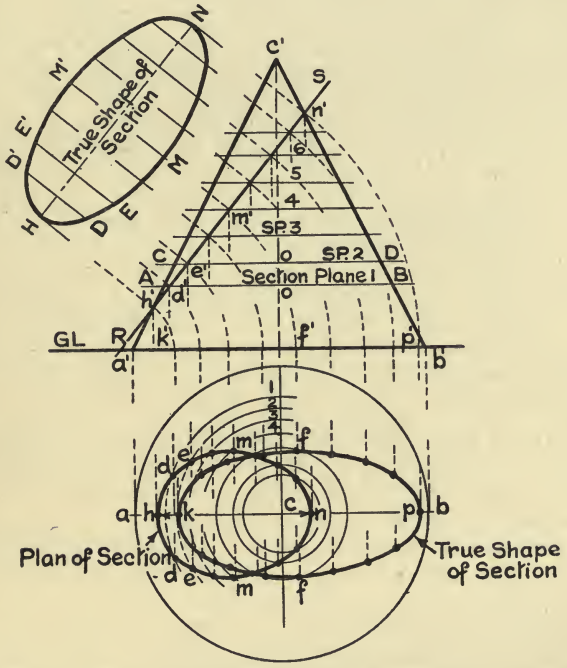


FIG. 256.—Method of finding the true shape of an elliptical section of cone.

extremities of the major axis, but the *length* of the minor axis must be transferred from *mm*, in plan, *i.e.* M'M = *mm*. Note that the distance *mm* in the plan is the *real distance* apart of two points which are represented by *m'* in the elevation. These two points, which are situated respectively at the front and back of the cone, are *both at the same level.* Hence, *mm* is the plan of a horizontal line, or one that is *parallel* to the ground plane, and in this respect differs from the line *hn* in plan, which is the plan of a line *h'n'* in elevation, *inclined* to the ground plane. Thus it follows that *hn* in plan is *not* the correct length for the major axis HN of the ellipse, while *mm* in plan *is* the correct length for the minor axis

MM′ on the true shape. The same reasoning holds good with regard to any other horizontal line such as that shown at *ee* in plan and *e′* in elevation. From *e′* draw a projector at right angles to the section line RS and crossing HN, making the length EE′ on the true shape equal to *ee* measured from the plan. Similarly, the points DD′ are obtained by means of a projector from *d′* in the elevation, making DD′ = *dd* in plan. By this means twelve other points, such as DE, D′E′, are obtained in addition to the points H, M′, N, M, and the ellipse may be obtained by drawing a free curve through these points.

Second method (*shown on plan*).—Rabat (or swing) the plane of section into parallelism with the ground plane. In this position it will show its true shape, *kfpf*. Imagine the hinge for rabatment to pass through R on the ground line. Thus, with R as centre, and R′*d′*, R′*e′*, R′*m′*, R′*n′*, etc., describe the arcs *h′k′*, *m′f′*, *n′p′*, etc., to intersect the ground line at *k′*, *f′*, *p′*, etc., and from the latter points draw projectors at right angles to GL as at *k′k*, *f′f*, *p′p*, etc. From the various points *h*, *d*, *m*, *n*, etc., on the plan of the ellipse, draw projectors parallel to GL to intersect the respective projectors, *e.g. hk* intersects *k′k* at *k*; *mf* intersects *f′f* at *f*; *np* intersects *p′p* at *p*. The arcs *n′p′*, *m′f′*, etc., are the elevations of the circular paths of the points *n′*, *m′*, etc., as the plane of section is rabatted upon *h′* as a hinge, while the straight lines *hk*, *mf*, *np*, etc., are the plans of the paths of the same points as the rabatment takes place. By combining these two views the ultimate positions of the points *k*, *f*, *p*, etc., are found, and an ellipse through these points is the true shape of the section.

Fig. 257 is another practical illustration of an elliptical section of a cone. A conical buoy is heeled over by the wind in such a way that its axis EF is inclined at 60° to the water line. *Find the true shape of the water line on the cone.*

From what has already been said it should be clear that in this case the surface of the water is an *imaginary section plane* which cuts the cone in such a way as to produce an *elliptical section*, since the plane of section enters a generator at *b*, on one side of the cone, and leaves the cone at a point *a′* on a generator on the opposite side of the cone. Hence, *a′b′* is the true length of the major axis of the elliptical section, and this length is projected on to a line *ab* parallel to the water line. Bisect *ab* at *o* and draw a line *d′d″* through *o* and at right angles to *ab*. The *minor axis* of an ellipse must necessarily be situated *midway* between the ends of the major axis. Hence the line *d′od″* fixes the *position* of the minor axis of the elliptical section. To obtain the *correct length* of the minor axis an alternative method is shown at *d* in the elevation. Produce *d′d″* in plan to meet the water line at *o′*, and through *o′* draw a line *pr* at right angles to the axis (EF) of the cone. A section of the cone at *pr* would be a *circular section*, since it is a section of a right cone taken in a position *parallel to*

the base. The semicircle *p*F*r* represents *one-half* of such a section on the line *pr*. From *o'* set out a line *d* at right angles to *pr* to meet the semicircle. This length, *d*, is *half* the total distance through the cone from front to back on the level of *o'*, *i.e.* at the water line, and the semi-minor axes *d'* and *d''* are each

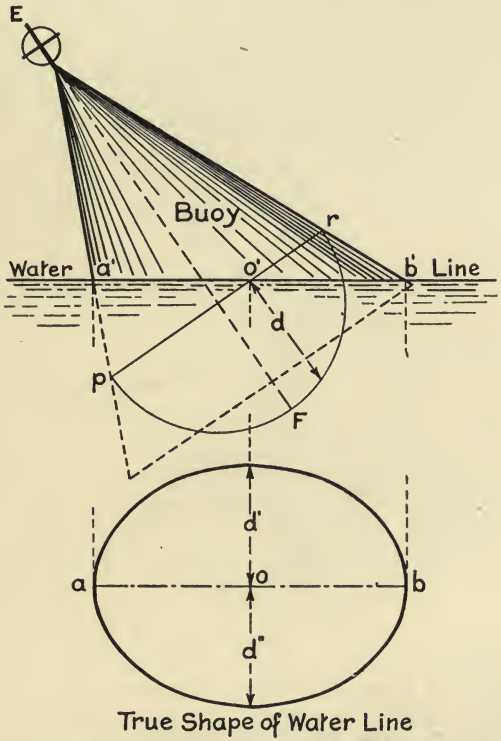


FIG. 257.—Finding true shape of water line on elliptical buoy.

made equal to *d* in elevation, the ellipse being drawn by means of a *trammel*, as explained in Chapter IX, Plane Curves.

Parabolic Section of a Cone.—Fig. 258 represents a hollow cone, made from sheet metal. This cone is to be cut through along the line *op*, parallel to *v's'*, and the part of the cone *v's'*, to the right of *op*, removed. *Find the true shape of a piece of sheet metal for soldering in position to fill up the opening at op.*

Draw any number of horizontal lines, *aa*, *bb*, *cc*, etc. Sections

at these levels would be *circular* sections. *One-half* of each section is shown. Thus, with 1, 2, and 3 as centres, and 1*a*, 2*b*, 3*c* as radii, describe the semicircles as shown, to represent the *half-section* at levels *a*, *b*, and *c* respectively. The plane of section *op* crosses the horizontal sections at points such as 4 and 5. From these points set out lines, as at *r*″ and *r*′, to meet the semicircles, as shown. Transfer the length *r*″ on to a line R″, drawn from 4, and at right angles to *o*′ *p*′. In the same way, make the lengths R′ and R equal to *r*′ and *r* respectively. Find the position of the point *o*′, which is the vertex of the curve, by projecting from *o* and at right angles to *o*′*p*′. Draw a *half-plan*

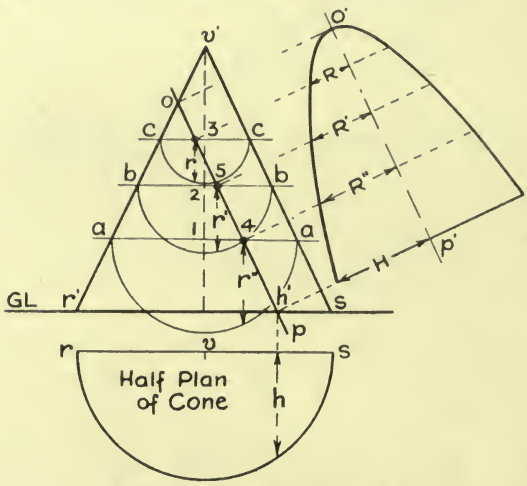


FIG. 258.—Finding true shape of parabolic section of cone.

of the cone, and from *h*′, at which the plane of section *op* intersects the base of the cone, draw a projector at right angles to GL, and so determine the length *h* which is the *half-width* of the parabolic section *at the level of the base of the cone.* Transfer this length *h* to the ordinate at H, and draw a free curve through the various points so found. The other half of the curve, to the right of *o*′*p*, is symmetrical about the axis *o*′*p*′, and may be obtained by setting off the lengths R, R′, R″, and H, on the right-hand side of *o*′*p*′.

Fig. 259 shows an alternative method of solving the foregoing problem by *rabatting* the plane of section about *oo* as a hinge.

Draw lines 1′,1′ ; 2′,2′, etc., to represent horizontal sections of the cone, and obtain the plans of these sections by drawing *concentric circles* in plan, with *v* as centre. Thus, with *v* as centre

and radius = o1′ in the elevation, describe the arc 1,5, which is part of the plan of the horizontal section at level 1′. Having obtained these circles in plan, draw projectors from 5′, 6′, 7′ etc., in elevation, to meet these arcs at 5, 6, 7, etc., in plan. The curve *od*860 is the *plan* of the parabolic section. The *true shape* may be obtained as follows. With o′ as centre and radii o′5′, o′6′, etc., describe arcs such as *d*′D′ to meet the ground line as at D′, and from the points *d*, 7, 5, etc., in plan, draw lines parallel to the ground line. By drawing projectors from the points D′, etc., on

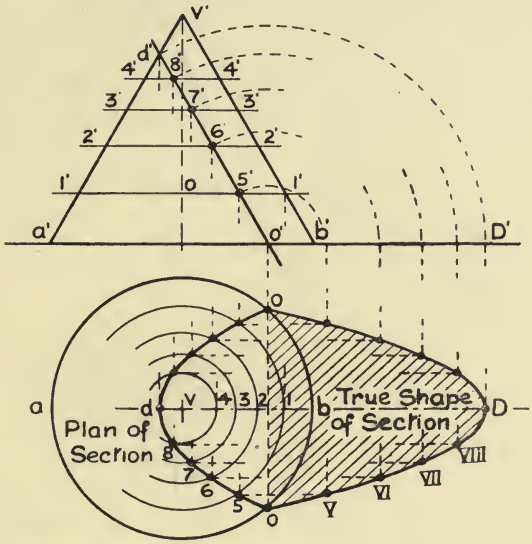


FIG. 259.—Alternative method of finding true shape of parabolic section of cone.

the ground line, to meet the lines *d*D, etc., in plan, the points V, VI, VII, VIII, etc., are found. The curve *o*,V,VI,D,*o* is the *true shape* of the **parabolic** outline of the section.

Hyperbolic Section of a Cone.—Fig. 260 (*a*) and (*b*) shows two methods of finding the true shape of a vertical section through a cone. Any sectional cut, such as that on the line *cd* in (*a*), parallel to the axis of the cone, gives a surface bounded by a hyperbolic curve, as shown on the right-hand side of (*a*).

A solid wooden cone, *a*′*v*′*b*′, Fig. 260, rests with its base upon the bed of a circular saw. The cone is then moved forward so that the saw cuts completely through it in line with *cd*. *Find the true shape of the cut surface.*

First method.—Draw any convenient number of horizontal lines, as at A, B, C, D, E. Draw semicircles to represent the

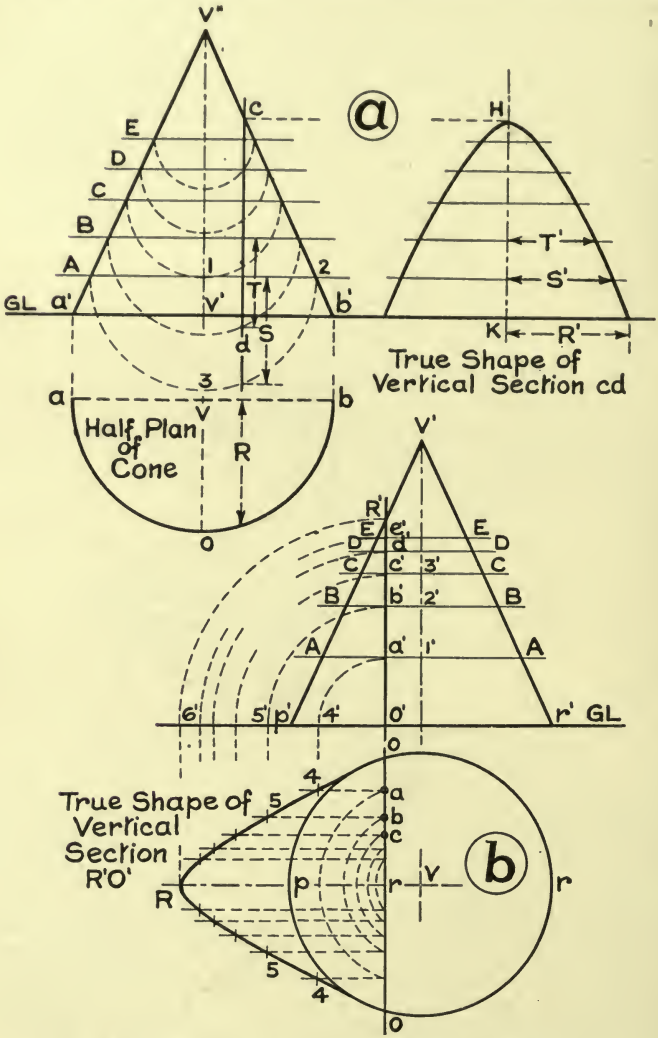


FIG. 260.—Alternative methods (*a*) and (*b*) of finding true shape of hyperbolic section of cone.

half-sections at each of these levels. Thus, with 1 as centre, and radius 1,2, describe the semicircle 2,3,A, which is the true shape

of half the horizontal section of the cone at the level A. Draw the *half-plan* of cone *aob*, as shown. The plan of the *whole* cone is not needed in this case, but may be drawn if necessary to make the working more clear. Consider first the intersection of the plane *cd* and the base of the cone. The full diameter of the base of the cone is *vo* × 2, or *ab*. Thus, if the section *cd* were taken in such a position as to include the axis of the cone (*i.e. vv'* in the elevation) the half-width of sectional cut at the base of the cone would be *vo*; but since the vertical section *cd* is taken at some distance away from the axis of the cone and *nearer b'*, the *half-width* of the sectional cut at the base of the cone must be *less* than *vo*, and would continue to become shorter and shorter as the position chosen for the section plane approached nearer to *b'*. Thus in the semicircle *aob*, which is the *half-plan* of the base of the cone, draw a line R in line with *cd* in the elevation. Then R is the *half-width* of the sectional surface at the base of the cone, and this width, R', set off on each side of a central axis HK, gives two points on the outline of the sectional cut. Consider next the horizontal section at A2. The radius of this section is 1,3, but it should be clear that the width of the hyperbolic section *cd* at this level must be *less* than the *full diameter* of the cone at this level. Thus the half-width of the section at this level is measured at S. Similarly, the half-width of the vertical section at the level B is measured at T, and so on for each of the remaining levels, measuring in each case from the particular level line, *e.g.* A2, to the point at which the vertical section line (*cd*) cuts the semicircle, A,3,2. Set off the respective half-widths of section S', T', etc., as shown, on either side of the axis HK, and draw a free curve through the points so found.

Second method.—This solution, which involves the principle of *rabatment* already described fully in the foregoing problems, should not need any extended explanation. Horizontal sections AA, etc., are taken as before; and arcs of circles are drawn in plan to represent these horizontal sections. The line *oo*, which is the plan of the vertical section plane R'*o*, cuts the arcs representing the respective horizontal sections at *a*, *b*, *c*, etc. The line *oo* is used as a hinge, and the whole plane of section is rabatted or swung over until it rests upon the HP. The front view of any points *a'*, *b'* as they swing upon the plane, is represented by arcs *a'*4', *b'*5'; and the paths in the plan of these points as the plane rotates about *oo* are straight lines such as *a*4, *b*5, etc., at right angles to the hinge *oo*. Thus a projector drawn from 4' and at right angles to GL, to meet *a*4 at 4, gives the position of two points 4, 4, on the outline of the plane of section, when it has reached the HP. Similarly, a projector from 5' on the ground

line determines the position of two further points 5, 5, on the outline of the true shape of the section.

Intermediate points are obtained in the same way, and the vertex of the curve R is obtained by means of an arc R′6′ in the elevation, and a projector from 6′ to meet the line *r*R at R in plan. Then the curved line (a hyperbola) *o*4,5R5,4*o*, is the true shape of the section at R′*o*′.

CHAPTER XIV

INTERSECTION OF INCLINED AND HORIZONTAL MOULDINGS

Raking Mouldings.—The small inset sketch (*c*), Fig. 261, serves as a key to the diagrams which immediately follow, and should be studied in conjunction with them. The opening at (*c*) may be taken as representing an entrance doorway from a vestibule or lobby. The overdoor is finished with a pediment *r* surmounted by two **raking** or *inclined* mouldings, such as *rh*, which have to be mitred to intersect correctly with a *horizontal* moulding at *h*. Both the horizontal moulding and the raking mouldings must be of the *same character*, *i.e.* must have the same details, *e.g.* OG curves, flats, beads, etc., must correspond in character in each case; but if moulding of exactly the *same* section is used for both raking and horizontal mouldings it is practically *impossible to obtain a correct intersection* at the mitred corner without a considerable amount of paring and easing, and this would not be permissible in first-class work. Hence it becomes necessary to determine accurately what the section of the raking moulding should be, in order to intersect correctly with a horizontal moulding of any given section.

A, Fig. 261, is the cross section of a horizontal moulding which is to be fixed in a position indicated by *h* in the inset sketch (*c*). *Find the correct section for a raking moulding* B *inclined at* 30° *to the horizontal and intersecting with the horizontal moulding* A. *Find also the correct shape for returning the ends of the raking mouldings* C *in a manner similar to that indicated at r in the inset sketch* (*c*).

Raking Mould.—Set out the outline of the cross section A of the horizontal moulding as shown, arranging the back surface of the moulding in a vertical position, and draw a line *ab* parallel to the top edge of the moulding. Select any number of points, 3, 4, 5, 6, 7, etc., on curved part of the OG, and from these points and the points 1, 2, and 8, 9, draw lines at the correct raking angle, *i e.* 30° to the horizontal, as at 4IV, 6VI, etc. From

the same points 1, 2, 3, etc., on the cross section A, draw vertical projectors to meet *ab* at the points 2, 4, 6, etc. At any convenient point draw a line PR at right angles to the raking lines and draw a line *cd* parallel to the *rake*, making $cd = ab$, *i.e.* the thickness of the moulding. Transfer the distances *a*9, *a*6, *a*4, etc., to the line *cd*, making $d9' = a9$; $d6' = a6$; $d8' = a8$, etc., and from the points 2′, 4′, 6′, 8′, etc., on *cd* draw projectors parallel to PR to meet the raking lines 2II, 4IV, etc., as shown at IV and

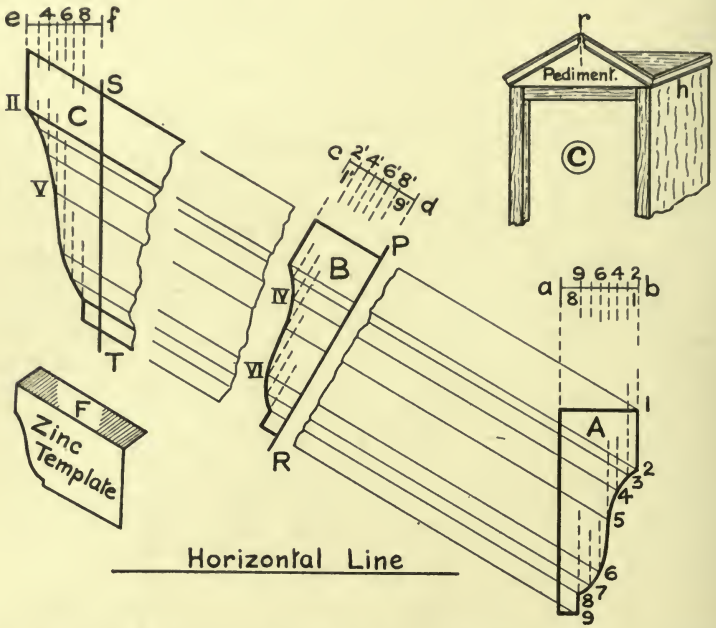


FIG. 261.—Correct sections for raking (B) and return (C) mouldings to intersect with horizontal moulding A. (*e*) Sketch of pediment showing raking mouldings. (*f*) Zinc template for marking *return* mould at top of pediment.

VI. Draw a fair curve through these points and determine the position of the *flats* by drawing projectors from 2′ and 9′, parallel to PR, and between the raking lines passing through 1 and 2, 8 and 9, etc., on the section A. The finished outline B is the correct section for the raking moulding to intersect correctly with a moulding of section A fixed in a horizontal position.

It is quite possible that in a large public building several hundred pediments such as that illustrated would have to be finished, and in such a case two separate bundles of moulding

would require to be machined. For such purposes the milling cutter for shaping the one lot of moulding would require to have an outline A, while another milling cutter of outline B would have to be used for machining the other lot of moulding. The extra expense of providing a second milling cutter would be more than repaid by the labour saved in mitreing the mouldings and fixing in position.

Return Mould.—The correct shape for the return mould is determined as follows. Draw a vertical line ST and a horizontal measuring line $ef = ab$, arranging the point f in line with ST. Transfer the distances $a,8$; $a,6$; etc., from ab to ef ; and from these points on ef draw vertical projectors to meet the raking lines coming from the various points 2, 3, 4, 5, etc., on the cross section A. Thus V on the return mould is the intersection of a vertical projector from point 5 on ef and a raking line from 5 on the cross section A. Draw a fair curve through the points on the OG part of the return mould and fill in the straight parts representing the *flats* in line with e and 8 on the measuring line ef. It sometimes happens that a large number of pairs of raking mouldings, all sloping at the same angle, have to be worked to the correct return mould. In such cases it would be well worth while at the start to determine the correct shape for the return mould : mark the required outline on a piece of sheet zinc, cut it out, and file it up to the finished shape as shown, turning over a small piece of the zinc F at right angles to the remainder of the sheet metal in such a way as to form a *fence* or stop for holding against the top edge of the moulding, while the pencil or scriber is used for marking the required outline on the moulding by means of the sheet metal *template*. By this means guesswork is avoided, there is no danger of spoiling the return end of the moulding after having fitted the mitred end correctly, and the usual haphazard method of taking parings off here and there and trusting largely to judgment is superseded.

Raking Mouldings for Curved Pediment.—Fig. 262 shows the method of treatment when the pediment is *curved* and the raking mouldings, instead of being cut from a straight length, have to be worked to the required *sweep* or curvature of the pediment. H is the cross section of a horizontal moulding which is to be fixed in a position corresponding to that of h in the inset sketch (c), Fig. 261, while the curved lines 4IV, etc., indicate the curvature of the raking mouldings.

Select any number of points 2, 3, 4, etc., on H, as before, and from these points draw vertical projectors to determine the position of the points 2′, 3′, 6′, etc., on de, and from the points

1, 2, 3, 4, etc., on H draw concentric arcs such as 4IV, etc., these lines being arcs of circles struck from a centre X, which falls outside the limits of this page. In order to obtain the section for the raking moulding K, draw any line RP, to radiate from the centre X from which the arcs 4IV, etc., were struck. Set out a line R1 at right angles to RP and transfer the various distances *d6'*, *d3'*, etc., from *de* to R1. From the points 1, 3, 6, etc., on R1 draw projectors parallel to RP, and meeting the respective arcs 4IV, etc., at points IV on K. The top flat portion of the moulding is obtained by means of a projector from 1 on R1, and should be drawn parallel to RP and between the curved lines coming from 1 and 2 on the cross section H.

The line CL indicates the vertical centre line of the pediment, and the position of the *returned* ends of the raking mouldings is

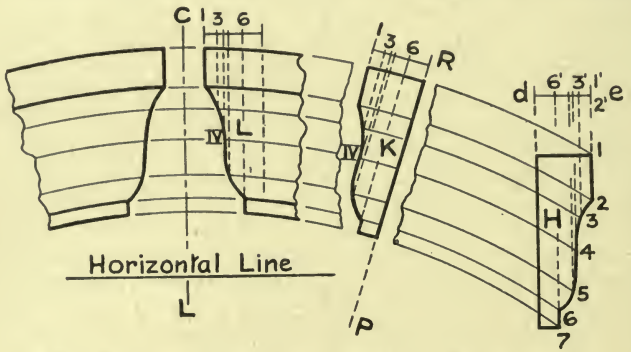


FIG. 262.—Raking and return moulds for curved pediment.

obtained by marking off equal distances, C1, on either side of C. Draw a line 1,6 at right angles to CL, *i.e.* horizontal, and transfer the points 1, 2, 3, etc., from *de* to the line 1,6 in L. From these points draw projectors parallel to CL to meet the respective curved raking lines. Thus, IV on L is the intersection of a vertical projector from 4 on the line C6 and a curved raking line from 4 on the cross section H. Where such work is to be executed in hard wood and French polished, it is absolutely essential that the curved raking moulding shall be of such a section as to intersect correctly with the horizontal moulding H, as no *stopping* and very little *easing* is permissible at the mitre. A simple geometrical construction on a setting-out board or the surface of a table will determine the correct section K for the raking mould and the proper outline for returning this length of moulding at L. These two examples, Figs. 261 and 262, cover practically all the usual cases that arise in practical work, and as

the methods used in these examples are applicable to all such cases, there is no need to introduce further examples; but the student is strongly urged to master the principles involved by working first the examples as shown, and afterwards by substituting a *different cross section* for the horizontal moulding in A and H respectively. Employing the same methods as are employed in the given cases, endeavour to obtain the section of the raking moulding and correct returning mould outline in the new cases set. The practical man will be anxious to employ a practical test of the accuracy of his methods. This can be done quite easily by using two short ends of any moulding of the same section, and using the front and end of a rectangular box against which to hold the two pieces of moulding, as shown in inset sketch (*c*), Fig. 261. Having satisfied himself that two mouldings of the *same section* will not intersect properly at the mitre, let him determine geometrically the *correct section* for the raking mould, and having altered one of the pieces of moulding or made a new piece to this section, try it again in place.

CHAPTER XV

DEVELOPMENTS

Cylinder.—The term **development,** as used in this connection, implies the *unfolding and opening out of the surface* of a geometrical solid in such a way as to show how a piece of sheet material must be set out for covering the surface of the object. Fig. 263 serves as an illustration of the **development of a cylinder,** of which the plan, *ab*, and elevation, *a'b'*, are given. If a cylindrical object, such as a column, of diameter D and length

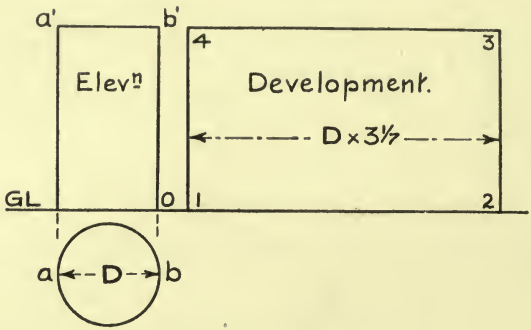


FIG. 263.—Development of cylinder.

ob' is to have its surface covered with sheet lead, it is necessary to determine the correct size and shape of all parts to be covered. It is clear that two *discs* of sheet lead will be required, of diameter D, for covering the **ends** of the cylindrical column. For covering the *curved* surface of the cylinder a **rectangular** piece of lead, 1,4,3,2, is required, the *width* of the rectangle 1,4 being equal to the *length* of the cylinder *ob'*. The length of the rectangle 1,2 is determined by reference to the circle *ab* in plan. If a tape is used for measuring the distance *around the curved surface* of the cylinder at any given level above GL, the

length of tape is equal to the *circumference* of the circle *ab* in plan, *i.e.* πD, or roughly $3\frac{1}{7}$ times the diameter D of the circle *ab*. The length 1,2 of the development is the neat measurement around the curved surface of the cylinder, assuming that the two edges of the sheet metal at 1,4 and 2,3 are to *butt* against each other. An extra allowance must be made for *lapping* over if such is necessary for any case which arises in practice.

Frustum of Cylinder.—Fig. 264 shows the plan, elevation, and development of a cylinder which is cut by an inclined plane at *a'd'*. The development is obtained as follows: Set out a rectangle to represent the development of the curved surface of a whole cylinder of diameter *ad* and height *dd'* by the method explained in the previous problem. Divide the plan circle into

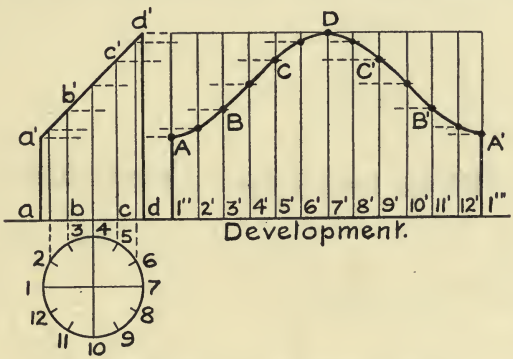


FIG. 264.—Development of frustum of cylinder.

any convenient number of equal parts at 1, 2, 3, . . . 12, and from these points draw projectors at right angles to GL to determine the position of the respective lines *bb'*, *cc'*, *dd'*, etc., in the elevation. Each of these lines (*e.g. cc'*) is the elevation of two *stripes* or imaginary lines on the cylinder, one situated on the curved surface at the *front* of the cylinder and one at the *back*. Hence, these stripes divide the curved surface of the cylinder into twelve *equal* and *parallel* strips; so that if the total length, AA', of the development is divided into twelve equal spaces at 2', 3', 4', etc., and parallel lines as at 7'D are drawn, the piece of material for the curved surface is assumed to have been *folded out flat*, with the various stripes, such as *dd'*, *cc'*, etc., on it. Assuming that the joint in the sheet metal is to be made along the shortest side of the cylinder, *i.e.* at *aa'*, the width of the development at each end, A1', A'1, must be equal in length to *aa'* on the elevation. Similarly at the centre of the length of

development the width of the sheet material, D7′, must be equal to *dd′* on the elevation, and thus, by drawing projectors from the points *c′d′*, etc., in elevation to meet the respective parallel lines as at B, C, etc., on the development, it is possible to draw the curve ABCDC′B′A′, which with the straight edges A1′ and the long edge 1′1′, is the development of the curved surface.

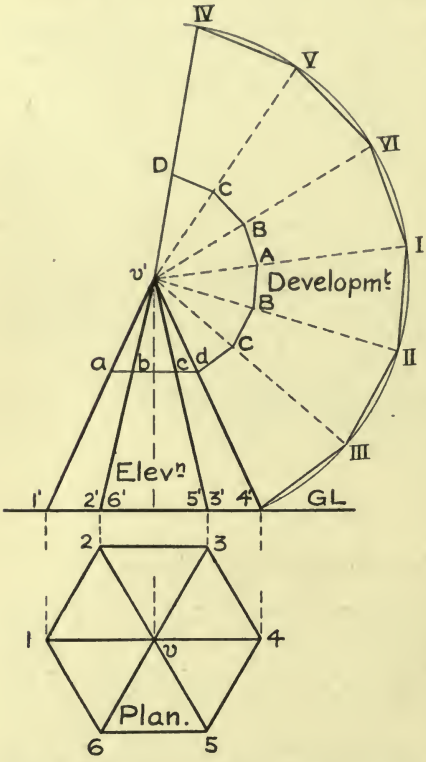


FIG. 265.—Development of hollow hexagonal pyramid.

If the *ends* of the cylinder are also to be covered, a *circular* piece of diameter 1,7, in plan, is required for the *lower* end of the cylinder, and an *elliptical* piece for the *sloping top end*, the longer axis of the ellipse being *a′b′* in the elevation, and the shorter axis 4,10 in plan, *i.e.* the diameter of the plan circle.

Hexagonal Pyramid.—A hollow box, Fig. 265, is to be made up from sheet zinc in the form of a hexagonal pyramid, as shown in plan and elevation. It is afterwards found necessary to cut off the top portion of the pyramid above *ad*, including the vertex *v*. Draw a development of the remaining portion (*frustum*), assuming that the whole pyramid is made from one piece of sheet material.

With *v′* as centre and radius *v′*4′, describe the arc 4′IV. Measure the length of a base edge, *e.g.* 3,4, in plan, and step this length *six* times along the arc 4′IV at III II, I, etc. Join *v′*III, *v′*II, *v′*I, etc., and join 4′III, III, II., etc. With centre *v′* and radius *v′d′*, describe the arc *d* . . B . . D, and join *d*C, CB, BA, etc. This gives the correct shape for cutting the sheet metal for making the *frustum* of the pyramid, *i.e.* the part below *ad*. To make the *whole* pyramid the complete piece *v′*IV,IV,V . . . III4 is required. The *broken lines* at CIII, B2, etc., indicate that the

metal is to be *scored* in order to facilitate bending along these lines, while the sheet is *cut completely through* along the *solid lines* shown on the development.

Before leaving this problem, note that, since the sloping edge $v4$ of the pyramid in plan is parallel to the ground line, the real length of this sloping edge appears at $v'4'$ in the elevation. This is important in view of the problem which immediately follows this.

Octagonal Pyramid.—A turret, Fig. 266, having an octagonal base, 1,2,3,4, etc., and height ov', is cut by an inclined plane RS. Draw the development of the lower part of the turret, *i.e.* find the true shape of a piece of sheet lead or other sheet material for covering the turret from the level of GL up to RS. Draw the octagon which in plan represents the base edges of the pyramid, and draw the diagonals 1,5, 2,6, etc., which are the plans of the slant edges of the turret. From V in plan draw a projector vo at right angles to GL, and set off the height of the turret ov' above GL. Determine the elevations on GL of the eight points on the base of the turret, as at 1′, 2′, etc., and complete the elevation by joining 1′, v', 2′, v', etc. Draw a line, RS, to indicate the height at which the top of the turret is cut off. Before proceeding to draw the development note that neither $v4$ nor $v5$ in plan are *parallel* to GL. Hence $v'4$, the *elevation of these slant edges* of the pyramid, **does not show their real length in elevation.** To obtain the *real length*, proceed as follows: With v (in plan) as centre, and radius $v4$, describe an arc $4b$ to meet a line vb parallel to GL. From b draw a projector at right angles to GL to fix the position of b', and join $v'b'$. Then $v'b'$ is the *true length* of one of the slant edges $v4$, after it has been swung into a position such that it is parallel to the VP, and the true length $v'b'$ is the radius for striking the arc b' . . . II . . . v' for the development. On this arc set out the *equal* distances b'IV, IV, III, etc., equal to the length of one of the *base edges* 1,8. As there are eight sloping faces to the turret, the development must include eight strips such as $v'b'$IV. The solid lines at $v'b'$ and v'V indicate that the metal is cut through along these lines, while the dotted lines v'VI, v'VII, etc., show the lines along which the metal must be *scored* so that it may bend correctly to form the slant edges of the turret. If the whole turret is to be covered, the whole development v'V, I, b' will be required. The true shape of the material required for covering the lower part of the turret is obtained as follows: From all points such as R, a, etc., at which the inclined section line RS intersects the slant edges

of the turret draw level lines RR′, *aa′*, etc., to meet the true length line *v′b′* at R′, *a′*, etc. Assuming that only one joint is to be made in the metal, and that the joint is to occur along the slant edge *v*5, the dotted lines on the development should be numbered to correspond; thus the two extreme edges of the development must meet to form the joint along the slant edge *v*5, while the next pair of lines on the development should be

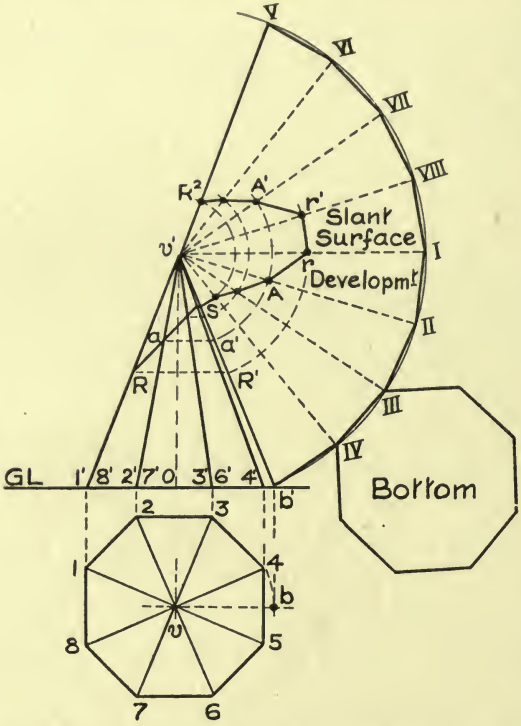


FIG. 266.—Development of sheet metal for covering an octagonal turret.

numbered *v′*IV and *v*VI, respectively, to indicate that they are to form the slant edges *v*4 and *v*6 of the turret. If the numbering of the points on the lower edge of the development is carried out correctly, the position of the following points may readily be fixed; the radius *v′*R′, centre *v′*, fixes the position of *r* and *r′* on *v′*I and *v′*VIII, respectively. . Similarly, radius *v′a′* gives the position of A and A′ on the next two lines *v′*II and *v* VII, while radius *v′*S fixes the position of the point R², which has to

coincide with S when the development is folded up into place. Join R^2 . . A . . $r'r$ . . A . . S by straight lines to represent the top edge of the development of the sheet material for fitting around the turret along the line RS. If a hollow pyramid such as this is to be made in sheet metal, aa octagonal piece, as shown, must be cut to make the bottom.

Cone.—Fig. 267, (*a*), (*b*) and (*c*) illustrate the method of finding the development or true shape of a piece of sheet material for covering the *curved* surface of a conical solid such as a **spire** or the end of an ornamental wooden **finial.** Referring to (*a*), the plan *avb*, and elevation *a'v'b'*, of a conical solid are

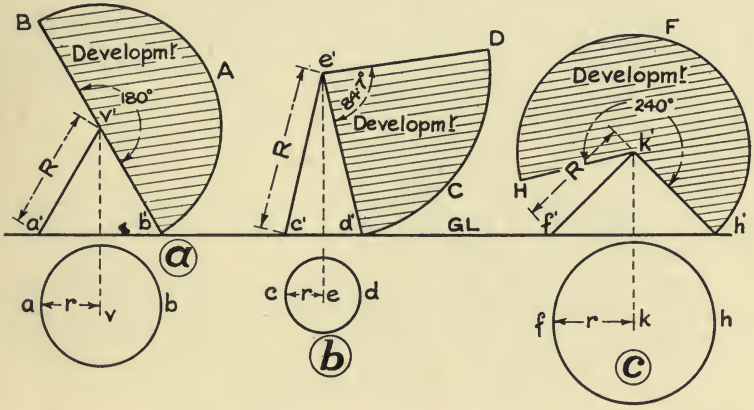


FIG. 267.—Development of cones.

shown, **r** in plan being the *radius of the base* of the cone and **R** in elevation being the *slant height* of the cone. With centre *v'* and radius *v'b'* = slant height of cone describe an arc *b'*AB. This curve *b'*AB is the shape to which the sheet material must be cut in order that it may fit around the circular base of the cone. The development is completed by drawing two straight lines *v'b'* and *v'*B to include an angle which varies according to the relation which exists between the radius of the base (*r*) and the slant height (R). A *ratio* or comparison of these two terms is made thus, R : *r* (read " as R is to *r* "). By making a *double ratio*, using 360° as the *third* term, it is possible to find the *fourth* term of the double ratio *x*, which is the **magnitude**, in degrees, **of the angle required** between the two straight edges of the development. In the given case *r* = ·75 and R = 1·5 units, and the ratio would be stated as follows :—

$$R : r :: 360° : x$$
i.e. 1·5 : ·75 :: 360 : angle required
$$\frac{360 \times \cdot 75}{1 \cdot 5} = 180°$$

Hence the development is completed by drawing a straight line v'B in line with $v'b$, thus making theoretically an angle of 180°. The piece of material is then cut to the straight lines Bv', $v'b'$; and the curved line BAb'.

The second example (*b*) shows how the development varies in shape when the cone has a *small base* and a *greater height* than that in (*a*). In this case the curved edge of the development d'CD is described with e' as centre and radius $e'd'$, and the statement necessary for obtaining the magnitude of the angle at e' is as follows: R = $2\frac{1}{8}$; $r = \frac{1}{2}$; *i.e.* R = 17 units; $r = 4$.

$$R : r :: 360° : x$$
i.e. 17 : 4 :: 360 : angle required
$$\frac{360 \times 4}{17} = 84 \cdot 7°$$

Hence the development of such a conical surface is bounded by two straight lines e'D, $e'd'$, which include an angle of 84·7° and an arc of a circle d'CD.

The third example (*c*) shows the development of a cone, the *height* of which is *small in proportion to the diameter of the base.* In this case R = $1\frac{1}{2}$ and r = 1, *i.e.* R = 12 units and r = 8 units. Cancelling R : r as 3 : 2 and the ratio for determining the required angle would then appear as—

$$R : r :: 360 : x$$
i.e., 3 : 2 :: 360 : required angle
$$= \frac{360 \times 2}{3} = 240°$$

Hence the development is completed by drawing the circular arc h',F,H, with centre k' and radius $k'h'$, and setting out a line k'H at an angle of 240° with $k'h'$, as shown. In each of the three cases (*a*), (*b*) and (*c*) the development is that required for covering the whole of the slant surface.

Frustum of Cone.—Fig. 268, (*a*), (*b*) and (*c*) show the method of development of a conical surface when *part of the cone is removed.* In (*a*) a funnel-shaped vessel, plan *ab*, elevation $a'b'de$. is required. Find the true shape of a piece of tin for making the funnel.

With centre c' and radii $c'b'$, $c'e$, draw two arcs b'B, eD to meet a line c'B making an angle $b'c'$B with $c'b'$, the magnitude of

the angle being found by the method already explained in the foregoing problems. Then the piece of metal required for making the funnel is that shown, bounded by the two straight lines *b'e*, BD, and the two arcs of circles D*e*, and B*b'*.

In (*b*) a conical vessel *mon*, *m'o'n'*, is cut by an inclined plane along *hr*. *Draw the development of the part of the cone from the base up to the inclined plane hr.* First obtain the development of the whole cone in the manner already described, at *o'*NM*n'*. Divide the plan circle of the cone *mn* into *any convenient number* of *equal* parts, say 12, at 1, 2, 3, 4, etc., and from these points draw the radial lines 01, 02, etc., in plan to represent *imaginary lines* running from the *vertex* of the cone

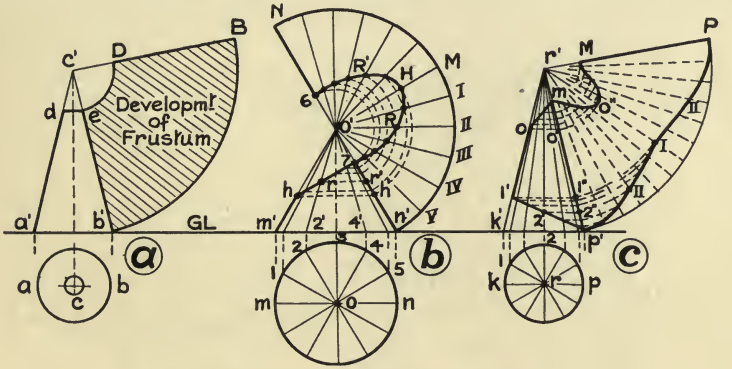


FIG. 268.—Development of frusta of cones.

to its *base*. Determine the elevation of these imaginary lines as at 2', 4', etc., and from the points *h*, *r*, etc., at which these imaginary lines cross the inclined plane of section *hr*, draw *horizontal projectors rr'*, *hh'*, to meet the line *on'*, which is the **true length** of each of the imaginary lines drawn from the vertex to the base of the cone. Divide the arc *n'*MN into the *same number* of equal parts as the base of the cone has been already divided into and draw the lines *o'*V, *o'*IV, etc., the numbers V, etc., corresponding in position with those on the base circle of the cone. Assuming that the metal is to be jointed along the line *on*, determine the position of the respective points RHR', etc., on the various lines *o*I, *o*M, etc., on the development, and draw the curved line RHR', etc. This is the line to which the top edge of the metal must be cut in order that it may fit up properly at *hr*, while the straight lines *n*6, *n*7, give the straight edges for making the joint, and the arc NM*n'* is the line for cutting the metal to make the base of the cone.

In (*c*) a hollow conical metal vessel is cut *obliquely* at both ends as shown at 1′*p*′, *om*, to make a spout for a vessel such as an oilcan. Draw a development of the piece of sheet metal required for making the spout.

The whole development *r*′P*p* is drawn as before, and this development divided into twelve equal parts, as also is the base of the cone. Horizontal projectors are drawn from 1′2′ and 0′, etc., to meet the true length line *r*′*p*′. The radius *r*′*m* gives

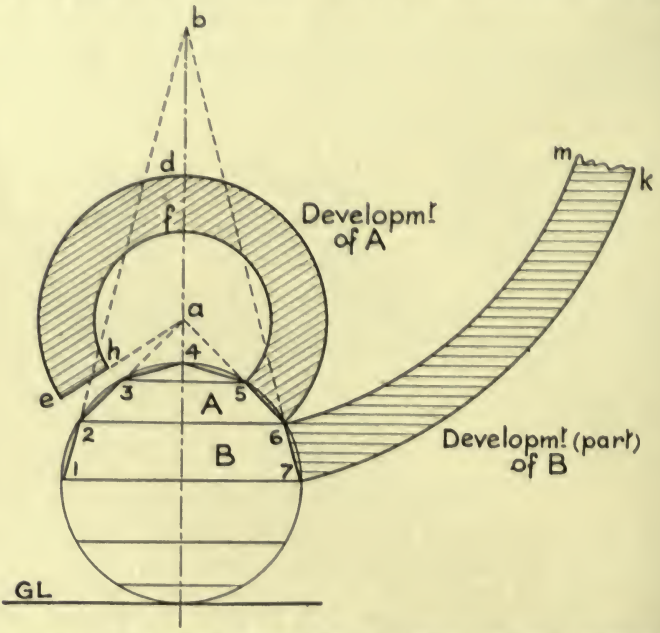


FIG. 269.—True shape (development) of sheet copper for covering a spherical cupola in zones.

the position of M. Similarly radius *r*′*o*′ determines *o*″ ; radius *r*′1″ determines I ; *r*′2″ determines II, II, etc., and the shape of the metal required is that bounded by the straight lines MP, *mp*′, and the curvilinear lines *mo*″M, and *p*′II,I,P.

Spherical Cupola developed in Zones.—Fig. 269 is an elevation of a spherical wooden cupola which is to be covered with sheet copper laid on in such a way as to form **zones** A, B, etc., on the cupola. Find the true shape of these *zones* or strips of sheet copper.

Divide the quarter circle 1,4 into the number of zones

required, say 3, and draw horizontal lines 1,7, 2,6, 3,5 as the elevation of the joints between the respective zones. To obtain the development of the zone B, join 1,2, and produce to meet the centre line 4*a* at *b*, and join *b*,6,7. Consider *b*,1,7 as the elevation of a cone fitting down after the fashion of a *candle extinguisher* over the part of the sphere between the lines 2,6 and 1,7. This is not possible from a *practical* point of view, because of the fullness of the curved part of the sphere between 1 and 2, and between 6 and 7, but is used merely as a *theoretical* illustration.

With centre *b* and radii *b*6 and *b*7 describe the arcs 6*m*, 7*k*. Then the strip 6*mk*7 is the development of that part of the cone *b*1,7, between the levels 1,7 and 2,6. The whole development is not shown, but could be found by employing the ratio R : *r* as before.

The development of each of the other zones is carried out in a similar manner. Thus *a* is the vertex of the imaginary conical extinguisher *a*2,6 which fits over the sphere between the levels 2,6 and 3,5, and the development of the zone 3,5,6,2 is found by using radii *a*5, *a*6 to describe the arcs 5*fh*, 6*de*, respectively. These arcs, with the two straight lines 3,2 and 5,6 are the outline of the development of the zone 3,5,6,2. It is obvious, however, that as these developments are cut out of a *flat* sheet of copper, the *centre* of each strip, *i.e.* midway between 6*m* and 7*k* and between 5*fh* and 6*de*, must be *bossed out* in order to fit the *curvature of the sphere* at its fullest point between 1 and 2, 2 and 3, etc.

Spherical Cupola developed in Lunes.—Fig. 270 shows an alternative method of covering the cupola already referred to, Fig. 269. In this case it is required to cover the surface of the cupola with sheet metal, laid on in the form of **lunes** as shown; each lune being made up of *two equal halves* jointed together along the *equator* or mid line of the cupola.

Draw two circles *af*, *a'f'* to represent, respectively, the plan and elevation of the spherical cupola. Divide the plan circle into a convenient number of equal parts, say 6, according to the number of *lunes* required, and draw the diameters *af*, *ed*, etc., and in a similar manner divide the quadrant *a'c'* in elevation into, say, 5 equal parts at *o*, 1′, 2′, 3′, and draw level lines through these points as at *on*, 1′,8′, etc. Since *a'c'* is the elevation and *ac* is the plan of a quarter of a *great circle* of the sphere, find the plans of 0,1′,2′, etc., at 1,2,3, etc., in plan. With *c* as centre and radii *c*1, *c*2, *c*3, etc., in plan, describe the arcs 3,6, 2,5, 1,4, etc., to meet *ce* at 6, 5, 4. These latter points are the plans of

points on part of another *great circle* of the sphere, but their heights are the same as those of 0′, 1′, 2′, 3′ respectively on the elevation. Hence the elevation of point 4 must occur vertically above 4 (in plan) and on the level of 1′ in the elevation, because 1 and 4 (in plan) are *equidistant from the centre of the sphere.* Similarly the elevation of 5 and 6 must occur vertically above the plans of these points, and on the levels 2′ and 3′ respectively in elevation, because 5 and 2, 6 and 3 are equidistant from the centre of the sphere in plan. The curve through the points $k4'5'6'c'$ and other points n, etc., equidistant from $c'o'$ is an ellipse $c'e'o'h'$, which is the elevation of the lune nearest the front of the sphere. To obtain the development of half this lune, assuming that the latter is made up of two equal portions jointed along $e'h'$, proceed as follows. Make EH = $e'h$ taken from the elevation. Bisect EH and draw a line RC, making RC = $\frac{1}{4}$ of the circumference of the circle $a'c'f'o'$, *i.e.* $\frac{af \times 3\frac{1}{7}}{4}$. Divide RC into the same number of equal parts as $a'c'$ has been divided, *i.e.* 5, and draw ordinates KN, etc., at the respective points of division, and transfer the half widths mk, 8′4′, etc., from the elevation along KM, IV, VIII, etc., respectively, so that the whole ordinate KN = kn, and so on. Draw the curves EKC and HNC. These two curves and the straight line EH form the outline of the *template* ECH, to which each of the twelve half lunes can be cut for covering the cupola. Here, as in the case of the zones described in the foregoing problem, it would be necessary to do a certain amount of *beating* or *dressing* at R and along the line RC in such a way as to drive this part of the metal out and so form a *curved surface* instead of a *flat sheet*, as each lune would be when first cut out.

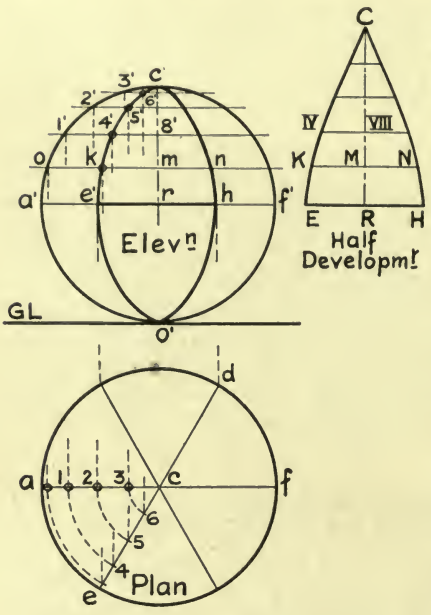


FIG. 270.—True shape (development) of lunes for covering a spherical cupola.

The plan and elevation of a dome are shown (*a*), Fig. 271, *horizontal sections of which are regular octagons. Complete the elevation. Draw a development of one of the eight curved faces of the dome.* (*Board of Education Exam.* 1904.)

The solution is worked separately at (*b*) for the sake of clearness. It will be observed that in the elevation (*a*), the examiner has been careful to omit those lines which involve most

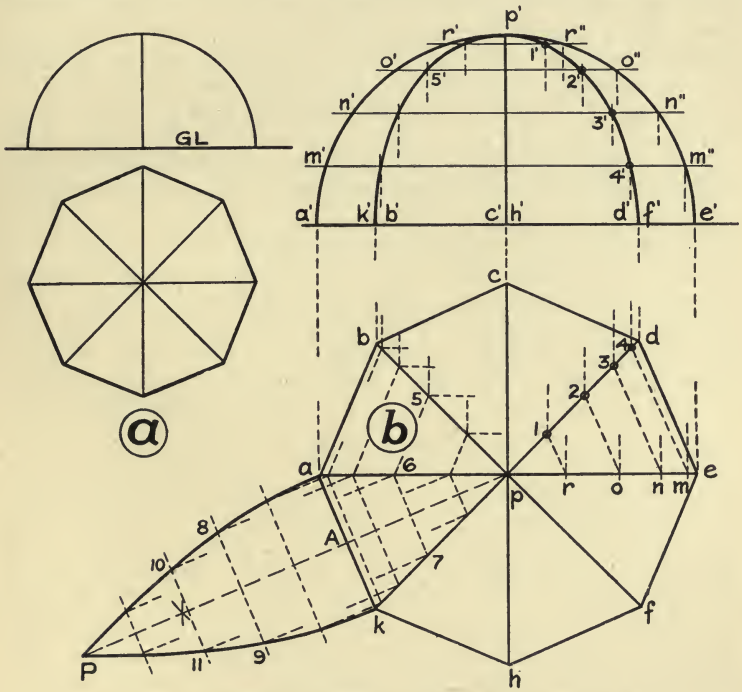


FIG. 271.—True shape of covering material for octagonal dome.

difficulty in determining their proper position. He is careful to state, however, that *horizontal sections* of the dome are *regular octagons*, and this statement forms a basis for the solution.

Draw a regular octagon *abcd*, etc., and its diagonals *ae*, *bf*, etc. This is the complete plan of the dome. Determine a point *c'* on GL, and in line with *ch* in plan, and describe the semicircle *a'p'e'* with radius = *ap* in plan. Draw the vertical line *c'p'*. Divide the semicircle in elevation into any convenient number of parts and draw level lines as at *m'*, *n'*, *o'*, *r'*. Consider the level line *m'm"*. This is the elevation of an imaginary horizontal

plane of section which cuts through the dome at that level, and as all horizontal sections of the dome are regular octagons, the plan of the section on $m'm''$ will be an octagonal parallel to the octagon *abcde*, etc. Hence a projector from m'' in elevation determines the position of one of the eight points on this horizontal section at m, and a line $m4$ represents one of the sides of the octagonal section. From 4 on *pd* in plan, draw a vertical projector to determine the position of 4′ in elevation on the level of $m'm.''$ Proceed in a similar manner to draw lines $n3$, $o2$, $r1$ in plan, by drawing projectors from n'', o'', r'' in elevation, and by drawing vertical projectors upwards from 1, 2, and 3, respectively, find the position of 1′ on level $r'r''$, 2′ on level $o'o''$, and 3′ on level $n'n''$, and draw a curve $p'1'2'$ . . . d', to represent the elevation of the rib *pf*, which, as is evident in plan, is not parallel to the GL, and will therefore appear as a quarter ellipse in elevation. Determine points such as 5 on *bp* by drawing projectors from 2 parallel to GL, and from the points 5, etc., so found, draw projectors to determine the position of 5′, etc., on the various levels in elevation, and draw the curve $k'5'p'$, which is the elevation of the ribs *kp* and *bp* shown in plan. This completes the elevation. The development of any one of the eight curved surfaces is obtained in the manner shown at *a*P*k*. From *p*, the centre of the octagon, draw *p*P at right angles to *ak*, and make AP = $\frac{1}{4}$ of the circumference of a circle of diameter *ae*, *i.e.* $\frac{ae \times 3\frac{1}{7}}{4}$. Divide AP into the same number of equal parts as $a'p'$ in elevation, *i.e.* 5, and draw ordinates as at 8, 9, etc. From the points 6, 7, etc., on *ap* and *kp*, respectively, draw projectors at right angles to *ak*, to meet the proper ordinates 8, 9, etc., as shown ; thus 6, 7 gives the position for the points 10 and 11 on the development. Through the points so found, draw the curves P,10,8,*a* ; P,11,9,*k*. These two curved lines, with the straight line *ak*, are the outline of the *template* or *pattern* for setting out each of the eight pieces of sheet lead or other material required for covering the dome. If for practical reasons these pieces are considered to be too large to lay on in one piece, a joint could be made at 8,9 on development, and each piece of lead laid on in two parts, *a*,8,9,*k*, and 8,P,9, allowing a little extra material to the right of 8,9, so that the upper piece 8P9 might lap slightly over the lower piece *a*8,9*k* when in position.

CHAPTER XVI

INTERPENETRATIONS OF SOLIDS

THE following examples are introduced in order to give an idea of cases which involve the **interpenetration** or *intersection* of two solids. In many such cases it is impracticable to determine the *development* of the surface of either solid until the correct line of *intersection* or interpenetration has been determined.

Intersection of Two Pipes of Square Section.—In Fig. 272 P and R are two *hollow* metal pipes of *square section* meeting

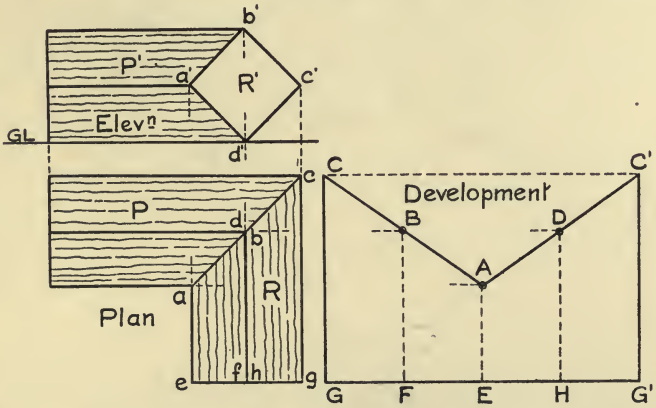


FIG. 272.—Interpenetration of two hollow pipes P and R of square section.

each other at right angles along the line *ac*. This is a comparatively simple case in which the line of intersection of the two pipes is represented in plan by the one straight line *ac*, and in elevation by the lines *a'b'*, *a'd'*.

The true shape of the piece of metal for making the pipe R is found as follows. Referring to the elevation, the line *a'b'*—or, in fact, either of the sides of the square *a'b'c'd'*,—gives the true

length of the line *ef* or *hg* in plan. The length GG′ of the development must therefore be made equal to four times the length of *a′b′*, and the width GC = *gc* on the plan. On the development make the distances GF, FE, EH, HG each one-fourth of GG′, *i.e.* = *a′b′*, and draw GC, FB, EA, HD, and G′C′ at right angles to GG′. From the point *db* in plan draw a projector to determine the position of B and D on the

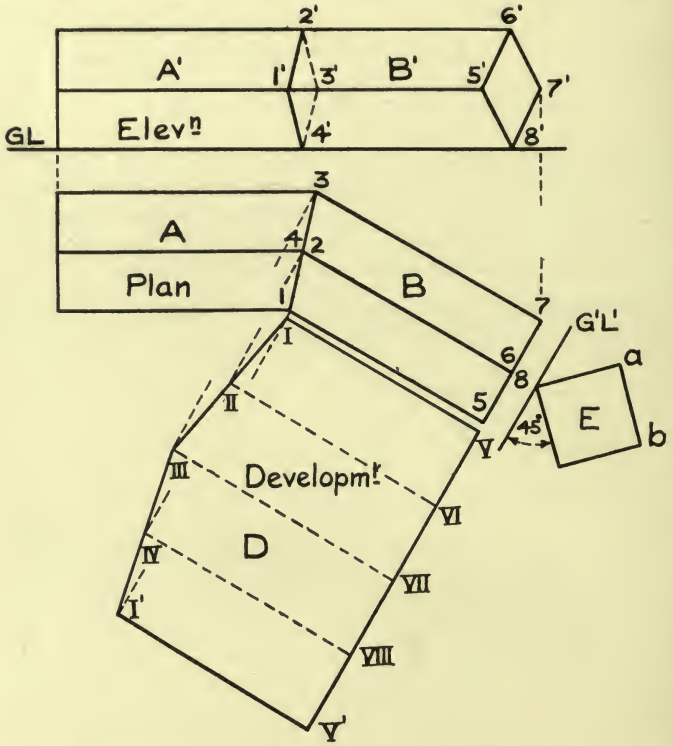


FIG. 273.—Interpenetration of two pipes A and B of square section meeting at an obtuse angle.

development. Similarly, the point A on the development is obtained by means of a projector from *a* on the plan. The solid lines GC, CA, AC′, C′G′, and GG′ represent the outline to which the sheet metal must be cut, while the broken lines FB, EA, and HD show where the metal must be scored or half cut in order to make it bend in such a way as to make a pipe of square section.

Two Square Pipes meeting at Obtuse Angle.—Fig. 273 shows two pipes A and B of *square section* meeting at an *obtuse angle*, along the line 1,3 in plan. Draw the plan as shown, A and B, and make an end elevation E of the pipe B on a ground line G′L′, taken at right angles to the pipe B, *i.e.* square with 3,7. From *b* on E draw a projector to determine the position of the line 2,6, which in B represents the plan of the top and bottom edges of the pipe. Both branches of the pipe are horizontal, and the elevation is obtained as follows. Draw lines 4′,8′, 1′,7′, 2′,6′ parallel to the GL and at heights equal to that of *a* and *b* respectively above G′L′. From the point 1 in plan draw a projector at right angles to GL to determine the position of 1′ on the line representing the edge 1′,7′ in elevation. Similarly the point 2′ on the elevation of the top edge is obtained from 2 in plan, and 3′ and 4′ from 3 and 4 respectively in plan. The solid lines 1′,2′ and 1′,4′ represent the part of the intersection on the nearest side to the observer, while the dotted lines 2′,3′, 3′,4′, are the elevations of the unseen part of the intersection. The elevation of the end of the pipe 5′,6′,7′,8′, is obtained in a similar manner by means of projectors from 5, 6, 7 and 8 in plan.

The total length of the sheet metal VV′ required for the development D must be made equal to four times the length of the side *ab* of the square E. The lines V,I ; VI,II ; . . . V′I′ are drawn parallel to 1,5 on the plan, making the distances V,VI ; VI,VII ; etc., each equal to *ab*, *i.e.* one-fourth of VV′. By means of a projector from 1 in plan determine the position of I and I′ on the development, and in the same way the points II and IV, and the point III are found by drawing projectors respectively from 2 and 3 on the plan. The metal should be cut from the sheet to the lines V,I, I,III, III,I′, etc., and *half-cut lines* made at II,VI, III,VII, etc., to *facilitate bending* so as to make a pipe of square section.

Intersection of Two Cylindrical Pipes.—Fig. 274, A and B, is the plan of two *cylindrical* pipes meeting at right angles. The plan of the intersection in such a case is a straight line *ck*. The elevation of the pipe A is shown at A′, while the elevation of the pipe B is represented by the circle *c′f′k′n′*, and the elevation of all points, such as *e*, *f*, *g*, etc., on the intersection of the two pipes appears on the circle *c′f′k′* at B′.

The development of either pipe is obtained as follows:—Divide the circle at B′ into *any convenient number of equal parts*, as shown at *d′*, *e′*, *f′*, etc. These points are the elevations of imaginary *stripes* or lines running along the pipe B parallel to its axis. The plans of these stripes is obtained by drawing

projectors from d', e', f', etc., in elevation to meet the intersection line *ck* in plan at *e*, *f*, *g*, etc. From these points draw the lines $e2, f3, g4$, etc., parallel to *cv*. Then the spaces $ef32, fg43$, etc., are the plans of two of the twelve equal and parallel widths into which the pipe has been divided.

The *length* of the development RR′ is made equal to πvr, *i.e.* $3\frac{1}{7}$ times the diameter of the pipe B, and the greatest *width* of the metal required at RK and R′K′ is made equal to *rk* on the plan. Divide RR′ into the same number of equal parts as the circle B′ has been divided into—in this case 12—and draw lines such as F′F, V′C, N′N, etc., parallel to RK. Assuming that the metal is to be jointed along *kr*, the

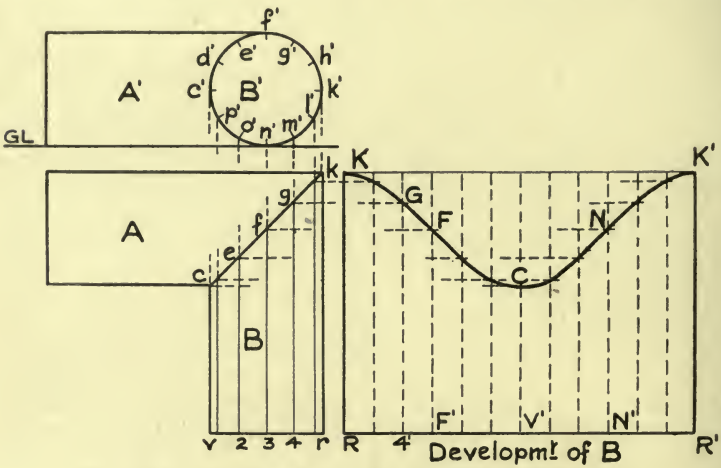


FIG. 274.—Interpenetration of two cylindrical pipes A and B.

length *kr* is projected on to the lines RK and R′K′. The length of the *first stripe* in from *kr* is projected on to the *first parallel line* to the right and left of KR and K′R′ respectively. The length of the next stripe $g4$ is projected on to the second parallel line at each end of the development as at G, and so on, while the length of the shortest part of the development at V′C is obtained from *cv* on the plan. A smooth curve through KGFCNK′ gives the shape to which the sheet metal must be cut for making the joint, and the straight lines KR, K′R′, and RR′ are the lines to which to cut the metal for the sides and end of pipe.

Intersection of Three Cylindrical Pipes.—Fig. 175 represents three pipes A, B, and C of circular section meeting at obtuse angles. The intersections, $d'f'$ and *df*, are straight lines

as shown. A complete elevation of the pipes is not necessary in order to obtain the development.

To obtain the development of the pipe B draw a *semicircle* (*a*) to represent a *half-elevation* of the pipe, and divide this semicircle, as shown, into any convenient number of *equal* parts, say six. The whole pipe may then be assumed to have been marked off into twelve equal strips along the entire length of the pipe. From the points 1, 2, 3, etc., draw projectors parallel to the direction of the pipe B, *i.e.* parallel to *f'f*, and so obtain the plan of the *imaginary lines*, such as *ee'*, which mark the pipe off into twelve equal parts. The length of the development FF′ is made equal to $3\frac{1}{7}$ times the diameter of the pipe B, *i.e.* $\pi 6{,}0$. This length, FF′ is divided into *twelve equal parts*, and lines such as EE, D′D, E′E′ drawn *parallel* to FF. Assuming that the

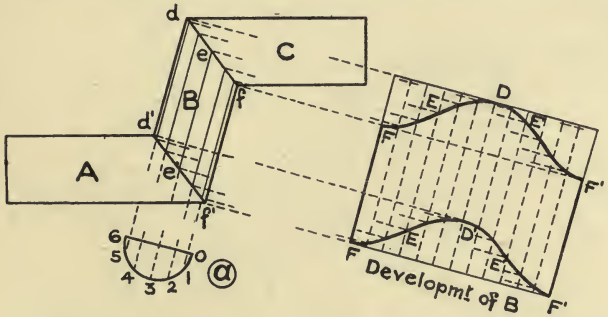


FIG. 275.—Interpenetration of three cylinders A, B and C.

joint in the pipe occurs along the line *ff'* the correct distance across the development at each end FF, F′F′, is obtained from *ff'* in plan, that at EE, which is the third *stripe* in from the ends of the development, is obtained from *ee*, the third parallel line counting from *ff'*; the width DD′ of the development at the centre of its length is obtained from *dd'*, which on the plan represents a distance equal to half the circumference of the pipe measured around the curved surface from *ff'*. As both ends of the pipe B have to be cut to intersect with A and C respectively, two curvilinear lines are required for the ends of the development of the sheet metal as shown at FDF′, and FD′F′, the metal being cut along the straight lines FF, F′F′ also to make a joint along *ff'*.

Intersection of Two Cylindrical Pipes of Different Diameters.—The inset sketch (*a*), Fig. 276, is the incomplete elevation of two cylindrical pipes which meet at right angles.

Complete the elevation so as to show the correct line of intersection of the two pipes, and determine the true shape of the piece of metal required for making the smaller pipe.

Draw (*a*) to a larger scale, as shown at (*b*), and draw the lines *cc*, 5,4 to represent the axis of each pipe. Draw a semicircle (as shown to the left of *b*), making the radius *c'd'* = *ce* on (*b*). This *semicircle* is a *half-end elevation* of the larger pipe. Determine the position of E in line with *c'e'*, and set out E*o'* = H, *i.e.* one-half the diameter of the smaller pipe, and draw *o'o*. Then *e'*E*o'o* is a *half-end elevation* of the *smaller* pipe. Divide the quadrant E into three equal parts on the assumption that the surface of the smaller pipe has been marked off into 12 equal strips, and from 1', 2', 3' draw lines 1'1, 2'2, 3*e*, parallel to E*e'*. Measuring inwards from 9,8 and 7,6, make the distances apart of the stripes

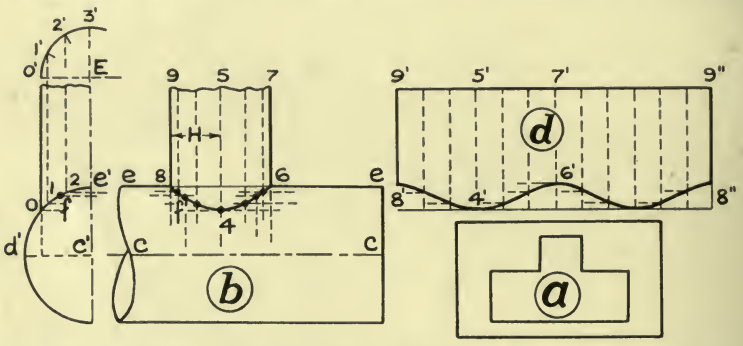


FIG. 276.—Intersection of two cylinders of different diameters meeting at right angles.

5,4, etc., equal to that between *o,o'* and 1'1', *o,o'* and 2,2', etc., and from the point *o* obtain the position of 4 on the middle stripe 5, 4; similarly, by drawing projectors from 1, 2, etc., obtain the position of the remaining points through which the curve of interpenetration 8, 4, 6 must pass.

The development (*d*) is obtained by making the total length 9',9" = $3\frac{1}{7}$ times the diameter 9,7 of the smaller pipe, and dividing this length into 12 equal parts by means of parallel lines such as 5',4', 7',6', etc. Project the length 7,6 from the elevation on to the end lines 9,8 and the middle line 7',6' of the development. The other points on the curvilinear line 8'4'6'8" on (*d*) are obtained in a similar manner from the respective points on the elevation of the interpenetration 8, 4, 6 on (*b*).

Intersection of Cone and Cylinder.—C'C, Fig. 277, are the elevation and half-plan respectively of a *conical spire*, through

which a *semicircular opening* is to be made as shown at $d5$ in elevation. *Determine the plan and elevation of the interpenetration* of the semi-cylindrical opening and the surface of the conical spire, and *find the true shape of a piece of sheet lead for covering the spire.*

The elevation of the interpenetration is the semicircle $m'd5'$, while the plan is represented by the broken lines *mn* and *ob*, and the curved line *n*67*b*, which is obtained as follows. Divide the

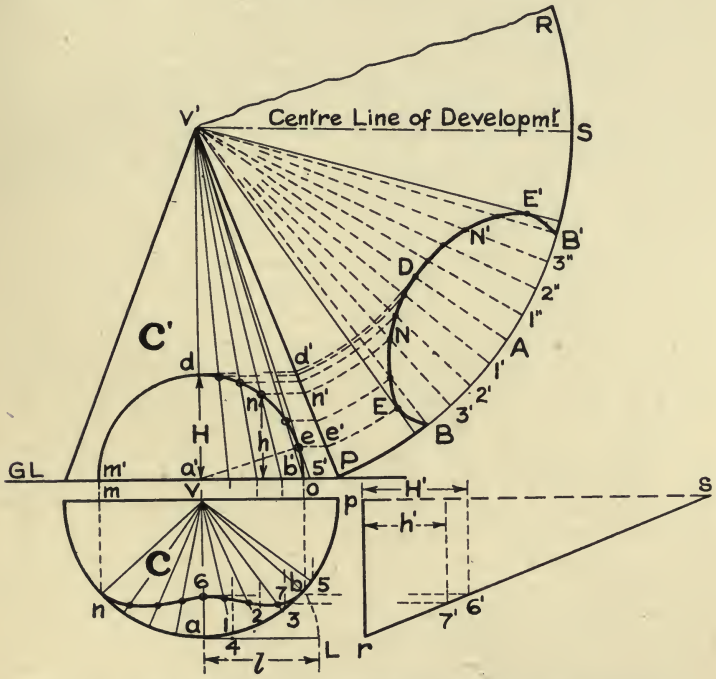


FIG. 277.—Conical spire (c') interpenetrated by semicircular opening H.

arc *ab* in plan into, say, four equal spaces at 1, 2, 3, and draw $v1$, $v2$, $v3$, to represent the plans of imaginary lines drawn from the vertex of the cone to its base. Draw the elevation of these lines as at $v'a'$, $v'n$, $v'b'$, etc. Make a side elevation of the half cone as at *prs*, making $ps = a'v'$ on the elevation. Measure the height H above GL, at which the line $v'a'$ crosses the semicircle $m'db'$, and transfer to the side elevation at H'. Obtain the point 6' by drawing a line parallel to *pr*, and from 6' draw 6'6, parallel to GL. Then 6 is the plan of the point *d*, *i.e.* the highest point of

the opening in the cone. Determine the position of other points on the curve *n67b* in the same way. Thus, measure the height *h* in elevation. This is the height above GL of the point *n* on the line *v*3. Transfer this height to the side elevation at *h'*, and fix the position of 7' by drawing a line parallel to *pr*. From 7' draw 7'7 parallel to GL. Then 7 is the plan of the point *n* on the elevation of the semicircular opening.

The development of the surface of the cone is determined as follows:—Proceed in the usual manner to obtain a development V'PR of the complete cone—see previous problems,—the only added difficulty in this case being that of finding the correct shape of the piece of metal to be cut away to allow the half-cylinder to pass through. The *complete* development is not shown, but the *incomplete half* of the development on the side R of the centre line V'S is similar in every respect to that shown at V'SP. The sheet metal bounded by the straight lines V'S and V'P is one, and the arc PS is the development of *half* the cone. Hence V'A, which is *midway* between V'B and V'P, represents the imaginary line V'*a'* which runs down the front of the cone, and is the *centre line* of the semicircular opening. Measure the length of the arc *ab* in plan, as shown at *l*, thus. With centre *a* and radius *a*1, describe the arc 1,4; with centre 4 and radius 1*b*, describe the arc *b*L. Then a straight line of length *l* is equal in length to the arc *ab*. Transfer this length *l* to the curve PAS (development) on either side of A at B and B'. The arc PAS is so nearly a straight line that the length *l* may be transferred direct to the curve PAS, making AB and A'B' = C. Then the part of the curve BAB' on the development represents the true length of the curve *nab* on the plan C. Divide AB and AB' each into four equal parts at 1', 2', 3'; 1", 2", 3"; and draw the lines V'1', V'2', V'3'; and V'1", V'2", etc. From the points *d*, *n*, *e*, etc., on the elevation C draw level lines *dd'*, *nn'*, etc., and with centre V' and radii V'*d'*, V'*n'*, etc., describe the arcs *d'*D, *n'*NN', *e'*EE', on the respective *stripes*, V'3', V'A, V'3", etc., on the development, and draw the curve BENDN'E'B'. This is the line to which the sheet metal must be cut away so as to fit around the circular opening in C'. It should be noted that the line V'5' is *tangential to the semicircle m'd*5' at *e*. This is an important line, as it gives the limiting points of the curve on the development at E and E.'

Intersection of Cone and Square.—Fig. 278 is the plan and elevation of a cupola S supported on a conical structure C. Find the true shape of a piece of metal for covering the cone in such a way as to fit up tightly under the cupola.

The method of drawing the development is similar to that

already described, and does not need any further explanation. The development of the whole cone $o'v'6'$ is shown at v', VI, 1″,6′. The curved line PAB, etc., for cutting the top end of the metal is obtained as follows :—

Divide the plan circle of the cone into twelve equal parts at 1, 2, 3, . . . 6, etc., and draw lines *vo*, *v*1, *v*2, etc., to represent "stripes" running from the vertex of the cone to its base. Determine the elevation on GL of the points 1, 2, etc., at 1′, 2′, 3′, 4′, etc., and draw the elevation of the imaginary stripes as shown, $v'1'$, $v'2'$, etc. From the points *a*, *b*, etc., at which these stripes intersect the elevation of the cupola, *i.e.* the circle S, draw level lines to meet the line $v'6'$ at points *c*, etc., and with centre v' and radius $v'c$, etc., describe arcs intersecting the respective lines $v'5''$, $v'4''$, etc., as shown at A, B, A′, etc. It should be obvious that the *shortest length*, $b3'$ on the elevation C, will, when levelled across to $v'6'$ at *c*, give the length for marking the *narrowest* parts of the development at B and B′, while the level line from *a* in the elevation will give the length of the part of the development at A and A′. The piece of the metal cut away at PABA′ would fit the semicircular penetration at the front of the cone, and a similar piece would require to be cut out of the metal on the opposite side of the centre line $v'0''$, *i.e.* towards VI, to fit the semicircular opening at the back of the cone. For this purpose the working shown on the part of the development $6'0''$ should be repeated, having first set off a distance equal to $6'0''$ on the arc $0''$VI.

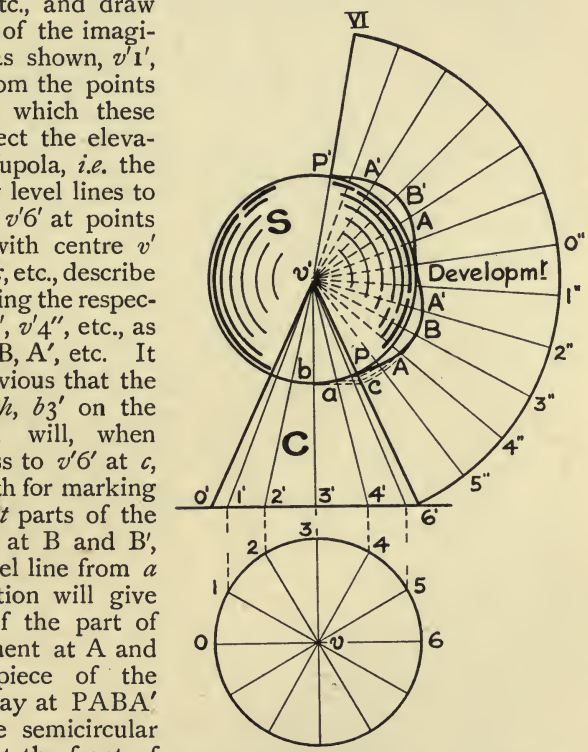


FIG. 278.—Intersection of conical spire and spherical cupola.

CHAPTER XVII

ROOF SURFACES AND BEVELS

A careful study of the problems which immediately precede this should enable the student to understand some of the simple principles which are involved in the determination of the **true shape** and **area** of **roof surfaces**, and the method of obtaining the **correct bevels** for the various roof timbers. It is necessary to know the true shape of the roof surface and determine its *area* before it is possible to estimate for the slating and roof boarding. In the same way it is necessary to know the correct *lengths* of the roof timbers, and the correct *bevels* for cutting them so that the carpenters may commence to prepare the various parts of the roof some time before the bricklayers are *wall plate high*. Unless this is done considerable delay must necessarily occur. If these geometrical principles are thoroughly understood it should be possible for the first hand of carpenters to scale off all lengths of timbers such as hips, valleys, common rafters, jack rafters, ridge, etc., from the plan of the building in the foreman's office, and with the aid of a fairly large setting-out board, improvised for the purpose, to obtain the necessary bevels for giving to the carpenters for cutting the various members of the roof trusses, etc., so that all is ready to fix in position as soon as the wall plates have been laid.

Length of Hip Rafters.—PP′, Fig. 279, are respectively the plan and elevation of a roof over a rectangular building *abcd*. The roof has a "hipped" surface at *aed*, and *bfc*. Determine the true length of one of the **hip rafters** *de*, represented by a single line in plan.

Two methods are shown. The first method will be better understood by reference to the pictorial sketch, Fig. 280. Referring to this sketch, *ab* is the hip rafter, and *ac* its plan. It is obvious that the **hip rafter**, *inclined* as it is to the *horizontal*, cannot show its *true length* in **plan**. The plan will be *shorter*

than the actual length of the hip. Again, *a'e'* in the elevation of the roof, Fig. 279, is not the *real length* of the hip because the top end *e* of the hip is *nearer the imaginary vertical plane* than the end *d*, *i.e.* the hip rafter is a practical illustration of a **doubly-inclined line**, the *true length of which does not appear in either plan or elevation.* In the pictorial sketch, Fig. 280, the height *bc* is the *difference in height* between the lower end *a* of the hip and the upper end *b*. Imagine a right-angled triangle made up of *ac* (the plan of the hip) as a *base*, *bc* (*i.e.* the height *h* in Fig. 279) as a *perpendicular*, and the hip itself *ab* as a *hypotenuse*, the right angle of the triangle being at *c*. Using *ac* as a hinge, rotate this triangle *abc* about *ac* until the plane of the triangle is horizontal, *i.e.* until it lies on the plane of the wall plate level. Then in this position *ab*, the hip would appear its true length because lying on a horizontal plane.

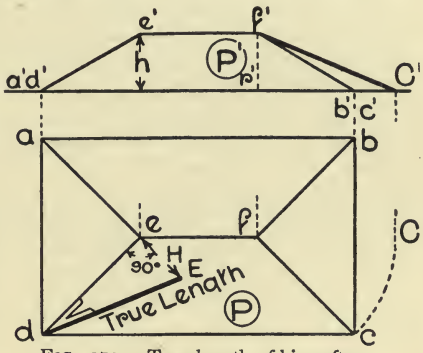


FIG. 279.—True length of hip rafter.

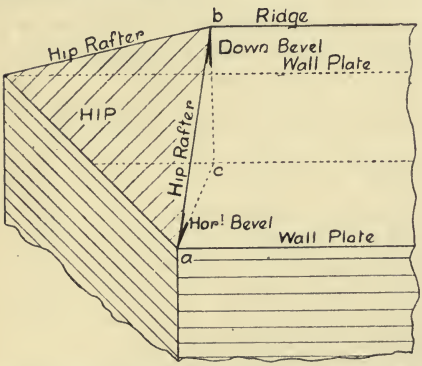


FIG. 280.—Pictorial sketch of roof showing various roof timbers.

In the plan and elevation PP', Fig. 279, this operation is assumed to have been carried out. A line *e*E in plan is set out at right angles to the plan of the hip, *de*, and a length H, equal to the height *h* in elevation, marked along *e*E. Join *d*E. Then the *triangle de*E is the *true shape* of the triangle *abc* in the pictorial sketch, Fig. 280, and *d*E is the *true length* of the **hip rafter**. From the same triangle, *de*E, the **correct bevels** for cutting the top and bottom ends of the hip rafter are obtained. Thus the angle at *d*, *i.e. ed*E, is the *horizontal bevel* for cutting the *lower* end of the hip rafter, while the angle at E, *i.e. e*E*d*, is the *down bevel* or vertical cut for the *top* end of the same timber.

A second method of finding the necessary particulars with regard to the hip rafter is shown at CC′, in Fig. 279. In this case it is assumed that the hip rafter, *cf*, *c′f′*, is rotated about a vertical axis *f′r′*, until the end *c* has reached a point C in line with *f*, *i.e.* when *f* and C are at the same distance from the imaginary vertical plane. During the process of rotation, the upper end of the hip *f* remains in its original position. The lower end *c* travels around in a path which appears as an arc of a circle *c*C in plan. In elevation, however, this path would appear as a horizontal straight line *c′*C′, because *c* has rested upon the imaginary horizontal plane all the while. Then *f′*C′, which shows the hip rafter after it has been rabatted into parallelism with the imaginary vertical plane, is the true length required for the hip, and the angles *r′f′*C′ and *r′*C′*f′* are respectively the correct bevels for cutting the top and bottom ends of the hip rafter.

True Shape of Roof Slope and Length of Jack Rafter.—S, S′, Fig. 281, are the plan and elevation of one hipped surface of a roof. The line 4,5 is the plan of one of the **jack rafters.** Assuming that the roof surface is to be boarded, find (*a*) the *correct bevel* for cutting the *ends of the boards* to fit against the hip rafter ; (*b*) the *true shape and area* of the *hipped surface* ; and (*c*) the *true length* and bevels for cutting any *jack rafter* such as 4,5.

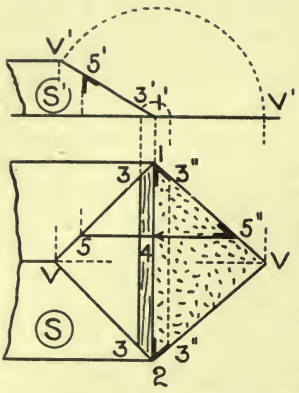


FIG. 281.—True shape of one slope of roof and real angle of jack rafter.

Assuming that the lower edge of the roof surface is used as a hinge of rotation, the plan of the path of V would be a line *v*V, drawn at right angles to the hinge 1,2. The elevation of the path of *v* during rotation, is an arc of a circle *v′*V′, struck with 1′ as centre. From V′, on the ground line, draw a projector V′V at right angles to the ground line, to meet the line *v*V at V. Join 1,V, 2,V. Then the triangle 1V2 is the *true shape* of the hipped surface of the roof. Its *area* is determined by taking $\frac{\text{Base} \times \text{Height}}{2}$, where the line 1,2 is the base and the height is the perpendicular distance of V measured from 1,2. Taking the shaded portion 1,2,3,3, as the plan of one of the roof boards, the true shape of such a board after the roof had been rotated into a

horizontal position would appear as shown at 1,3″,3″,2, and the darker shaded angles at 1 and 2 are the correct bevels for cutting the ends of the boards. Similarly 4,5″ would be the position of the jack rafter, 4,5, after rotation. Hence 4,5″ is the *true length* of the **jack rafter,** and the shaded angle at 5″ is the *cross bevel* for marking on the upper edge of the top end of the jack rafter. Since the jack rafter is parallel to the vertical plane, the *down* bevel or vertical cut for the top end may be obtained directly from the elevation, as shown by the shaded angle at 5′ in the elevation S′.

Dihedral Angle between Two Plane Surfaces—Backing Angle for Hip Rafters.—Fig. 282 shows the plan and elevation

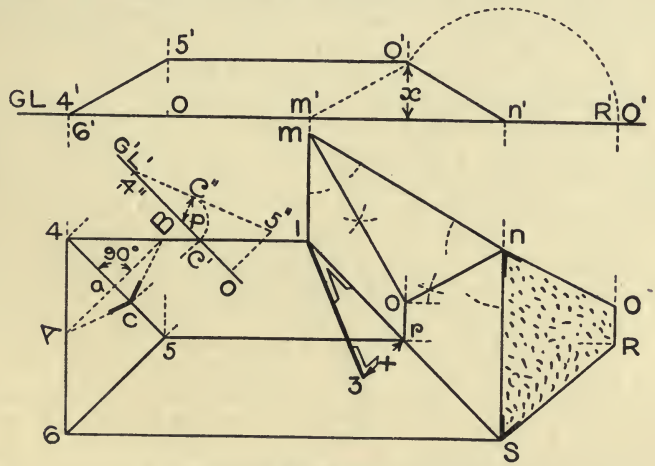


FIG. 282.—Backing angle for hip rafters.

of two hipped roofs meeting at right angles, and forming a *valley* at 1, *r* ; in plan. Determine (*a*) the *true length of the valley rafter* 1, *r* ; (*b*) the *true shape of the hipped surface n, o, r, s ;* and (*c*) the *dihedral angle* between the two roof surfaces 4,5,6, and 4,1,*r*,5.

(*a*) The **true length** of the valley rafter is obtained by the method already fully explained in the previous problem. A line, *r*,3 in plan, is drawn at right angles to the plan of the valley, 1*r* ; a distance X, equal to the height *x* in elevation, is set off along *r*3, and the correct length of the valley 1,3 obtained by completing the triangle 1,3,*r*. (*b*) The **true shape** of the *hipped surface, n, o, r, s,* is obtained by rotating it about *ns* as a hinge, as already explained, and is shown at *n*OR*s*, the shaded

angles at *n* and *s* being the correct angles for cutting the ends of the roof boards. (*c*) The term **dihedral angle** is used in contra-distinction to a *plane angle.* The latter refers to the angle between two *lines*, while the former has reference to the angle between two plane *surfaces.* Quite apart from the question of the correct bevels for cutting the top and bottom ends of the hip rafter 4,5, it is necessary to plane the top edge of the hip rafter in such a way that one half of the thickness of the top edge is in line with the main roof surface 1,4,5,*r*, and the other half in line with the hipped surface 4,5,6. This operation of *chamfering* the top edge of the hip rafter is termed **backing.** In order to determine the correct angle for *backing* the top edge it is necessary to obtain the *true angle* between the two *roof surfaces* in question, as follows. Draw a line AB at right angles to the plan of the intersection of the two surfaces, *i.e.* the hip. This line represents the *horizontal trace*, or *line of intersection of an imaginary plane* which cuts through the hip *at right angles to its slope.* Draw a new ground line G′L′ parallel to 4,5, the plan of the hip, and make a new elevation of the hip on this ground line. Thus, draw projectors from 4 and 5 in plan at right angles to G′L′ ; make the height 0,5″ on this new elevation equal to 0,5′ on the original elevation. Then 4″5″ is a new elevation of the hip, and incidentally gives the true length of the hip. In line with AB mark the position of a point *p* and draw a line *p* at right angles to 4″5″ to represent the profile view of the plane surface which is supposed to cut through the hip. This cutting plane, after passing down through the roof surfaces, will form a **triangle** underneath the roof surfaces, the *base* of the triangle being the line AB, and the *vertex* of the triangle being a point *c* on the hip. Using AB as a hinge, rotate this triangle down until it is laid on the imaginary ground plane. The path of the *vertex* of the triangle is represented in *elevation* by a *dotted curve* c*C*′, and the *plan* of the path of such a point would appear as a line *a*C, *i.e.* **at right angles to the hinge** AB. Then the dotted triangle ACB shows how this triangle would appear when rotation was complete, and the shaded angle at C is the **real angle** between the two plane surfaces, *i.e.* the **dihedral angle** between the hipped part of the roof and the main part. The angle to which the bevel would be set for testing the backed edge of the hip would really be the angle *a*CA, or *one-half* the whole angle ACB. This is a very important principle which should be illustrated by placing two set squares in a sloping position so that their respective edges coincide. The line of intersection of the edges of the two set squares represent the hip rafter, and a sheet of cardboard may be

used to illustrate the plane surface which is supposed to cut through the two roof surfaces (the set squares) at right angles to the slope of the hip. Reference to the pictorial sketch of the ventilating shaft (*a*) Fig. 287 and again to Fig. 288 will also help to make the matter clearer.

Purlin Bevels.—The **purlin** is a roof timber which is fixed parallel to the *wall plates* and *ridge*, and, resting upon the *principal rafters* of the roof trusses, forms an intermediate support for the common rafters between the ridge and the wall plates. The fitting of purlins to an ordinary roof in which there is no hipped surface presents no difficulty. When a hip rafter is necessary, one end of the purlin must be supported by the hip rafter, and the

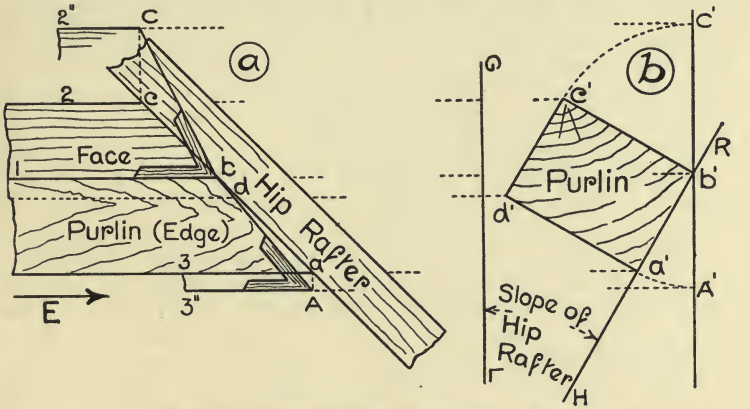


FIG. 283.—Determining correct bevels for purlines.

end of the purlin has to be *splayed* to fit against the side of the hip rafter. *Two bevels* are necessary for marking the end of the purlin ready for sawing off, a *face bevel* and an *edge bevel.* The method of obtaining these is as follows :—

(*a*) Fig. 283 is the *plan* of part of a hip rafter with purlin fitted against it ; (*b*) is an *end elevation* of the purlin on a ground line GL, looking in the direction indicated by the arrow E. The slope of the hip rafter is represented by the line RH. *Determine the two bevels necessary for fitting the purlin against the hip rafter.*

The method employed in this case is another example of *rabatment* of a surface about an arbitrary *axis* or *hinge.* It is necessary to observe that, since *both the face and edge* of the purlin are *inclined to the horizontal*, their true widths *do not show in the plan* (*a*). Thus it follows that the *plan* of the angle 3*ab* is not the

true angle which the cut on the upper edge of the purlin makes with the hip rafter. Similarly, the angle 1*bc* in plan (*a*) is not the *true angle* for marking on the *face* of the purlin. In order that the true shape of the angle may appear it is necessary to assume that the face—or edge—is rotated into a horizontal position. Consider the face of the purlin (1, 2, *c*, *b*, in plan and *c'b'* in elevation). Assume the edge 1*b* as a hinge or *axis of rotation*. Then the arc *c'*C' in (*b*), centre *b'*, represents the elevation of the path of this face as it rotates about 1*b*. The plan of the path of any point such as *c* is represented by a line *c*C *at right angles to the imaginary hinge* 1*b*. Draw a projector from C' in (*b*) to meet *c*C in (*a*) at C. Then C is the new position of this point when the face of the purlin is in a horizontal position. C2'' being the new position of edge *c*2. Thus 1*b*C2'' represents the *developed* face of the purlin and the shaded angle at *b* is the **correct angle** for setting the bevel for marking the **face** of the purlin. Proceed on similar lines to determine the bevel for the edge of purlin. In the elevation (*b*) with *b'* as centre, *b'a'* as radius, describe the arc *a'*A', meeting a horizontal line C'A' at A' (note that C'A' is parallel to the new ground line GL, and may therefore be considered as a *horizontal* line).

In the plan (*a*) draw a line *a*A at right angles to the edge 1*b*, *i.e.* the hinge, and from A' in elevation draw a projector at right angles to GL to determine the position of A in plan. Then A represents the new position of the point *a* when the edge of the purlin has been rotated about 1*b* into a horizontal position; 1,*b*,A,3'' represents the developed edge 1,*b*,*a*,3; and the shaded angle at A is the **correct bevel** for marking on the top or **bottom edge** of the purlin.

Joint between Hip Rafter and Ridge.—Fig. 284 is the plan of part of a ridge showing plans of top ends of hip rafters *birdsmouthed* to fit against the end of the ridge. *Determine the correct bevels for marking the top end of the hip.*

The *plan* of the birdsmouth is shown at 2,3,4, but this is not its *real shape*, because it is the *plan of an angle set out on an inclined surface*, *i.e.* the top edge of the hip Draw a ground line GL parallel to the hip, and obtain the elevation of any point such as o,o' on the top edge of the hip. Draw a line o',3',4' making an angle with GL equal to the actual slope of the hip rafter. From 2, 3, and 4 in plan draw projectors at right angles to GL to meet the line of slope at 2', 3' and 4'. With o' as centre and radii o'3', o'4', o'2', describe arcs to meet GL as at III, and from these points draw projectors, such as III3'', II2'', etc.,

to meet the lines 2,2″, 3,3″, 4,4″, in plan. Join 2″,3″, 3″,4″. Then 2″,3″,4″ is the true shape of the angle 2,3,4; and the correct angle for setting the bevel is that between the lines 2,2″ and 2″,3″, or between 4,4″ and 4″,3″.

Bevel for Top End of Jack Rafter.—A method of determining the "cross" bevel for the top edge of the jack rafter is also shown (Fig. 284). The line 6,5 shows the intersection of the jack rafter and hip rafter. To obtain the *true shape* of this angle, select any imaginary line p,p^2 on the jack rafter as a hinge. Draw

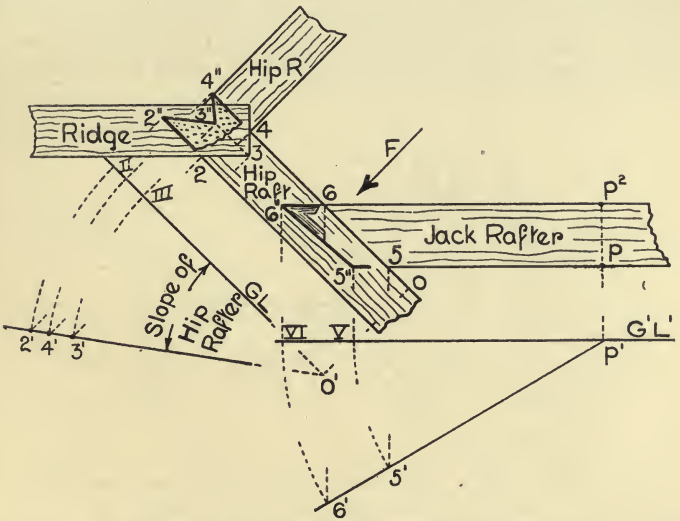


FIG. 284.—Correct angles for cutting jack rafters and birdsmouthing hip rafter.

G′L′ parallel to the jack rafter. Draw a projector from p to determine the position of p' on G′L′. From p' set out $p'6'$ making an angle with G′L′ equal to the slope of the jack rafter. From 5 and 6 in plan draw projectors square with GL to meet the line of slope at 5′ and 6′. Then the line $p'5'6'$ is the elevation of the top face of the rafter, and the points 5 and 6 are included in this face. With p' as centre, radii $p'5'$, $p'6'$, draw the arcs 5′V, 6′VI; and from V and VI draw projectors to meet P5, produced, at 5″, and $p^2$6, produced, at 6″. Join 6″5″. Then the *correct angle* for **top edge of rafter** is shown shaded at 6,6″,5″.

Roof Slopes.—The rectangle PRST, Fig. 285, is the plan of a building. In order to obtain a *larger area of roof surface* on the side sloping towards PT, it is decided to make the slope of

the roof surface at RS = 45° and the slope of each of the remaining three roof surfaces = 30°. *Draw a plan, showing the position of the hip rafters and ridge.*

If all the roof surfaces were *equally inclined* to the horizontal, the position of the hips and ridge could be found by *bisecting the angles* at the corners of the building. The *bisectors* of these angles would represent the **hip rafters**, and the line joining the intersection of these bisectors would be the plan of the ridge. Where the slopes are unequal, as in this case, a special construction is necessary in order to fix the position of the hips, etc., in plan. Using the line PR as a ground line, set out a line R3′ at 45° to represent a *profile view* of the roof surface R*ab*S as seen from the direction indicated by the arrow X. Set up any

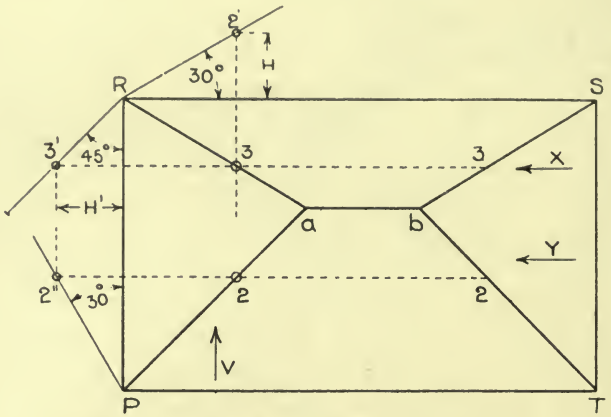


FIG. 285.—Plan of roof RSTP to given slopes.

height H′ (say ½ in.) and draw a line 2″3′ parallel to PR and meeting R3′ at 3′. From 3′ draw a line 3,3 parallel to RS. Then 3,3 is the plan of an *imaginary level line* on the roof surface R*ab*S, at a height of H′ above the wall plates. Similarly, using RS as a ground line set out a line R2′ at 30°, to represent the slope of the roof surface R*a*P as seen from V. Make the height H = H′, and by drawing a line parallel to RS, determine the position 2′. From 2′ draw 3,2 parallel to RP. Then 3,2 is the *plan of a level line* on the roof surface R*ap* at the same height above the wall plates as the level line 3,3. Then 3, the intersection of these two level lines at *equal* heights above the wall plates, must be a point *on the intersection of the two roof surfaces*, and a line from R through 3 determines the *direction*, but not the *length*, of the plan of the hip rafter R*a*. Using a

similar method of construction, set out a line P,2″ at 30°. This is a *profile view* of the roof surface P*ab*T as seen from Y. Make the distance apart of 2″ and PR = H′, and from 2″ draw 2,2 parallel to PT. Then 2,2 is the plan of a third level line at the same height above the wall plates as those already drawn, *i.e.* 3,3 and 3,2 ; and the point 2, which is common to the two levels 3,2 and 2,2, is necessarily *on the intersection of the two roof surfaces* P*ab*T and R*a*P. Hence, draw P*a* through 2 to intersect R3 at *a*. Then R*a* and P*a* are the plans of the two hips. From *a* draw the plan of the ridge parallel to RS, and obtain the position of *b* by making the triangle S*b*T exactly like the triangle R*a*P.

The figure ABCD (Fig. 286) is the plan of a building erected

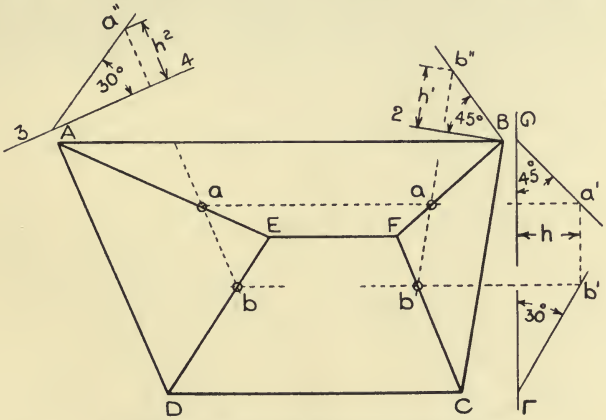


FIG. 286.—Plan of roof ABCD to given slopes.

upon an irregular-shaped piece of ground. The roof is to have two hipped surfaces, the slope of the respective roof surfaces being as follows: AEFB = 45° ; BFC = 45° ; DEFC and AED each 30°. *Draw the plan of the building and determine the position of the hips and ridge.*

Draw a ground line GL at right angles to AB or DC, and set out angles of 45° and 30°, as shown, to represent the profile view of two opposite roof surfaces AEFB and DEFC. Set off any height *h* and draw *a′b′* parallel to GL. From *a′* and *b′* respectively draw *aa* and *bb*, parallel to AB. Then *aa* and *bb* are the *plans* of two *level lines* on opposite surfaces of the roof, and at *equal heights* above the wall plates. Draw a ground line 2B and set out B*b*″ at an angle of 45° to represent the slope of the surface BFC. Set up a height *h′* equal to that already set out at *h*, and by drawing a line parallel to 2B obtain the position

of the roof surface BED, and obtain the po
at a height h^2 equal to that at h. From a'
AD. Then since ab is the plan of a level
AED at the *same height* above the wall pla
aa and bb on the adjacent roof surfaces,
which are each *common to two adjacent surfa*
be *on the intersection* of these adjacent roo
drawn through a, and DE through b, ar
remaining two hip rafters, and a line EF is th

CHAPTER XVIII

OBLIQUE CUTS IN CARPENTRY AND JOINERY

Bevels for Ventilating Shaft.—(*b*) Fig. 287 is the front elevation of a tapered ventilating shaft, the shape of which is shown more clearly in the pictorial sketch (*a*). The shaft is an example of a *hollow frustum* of a pyramid, with square ends, the

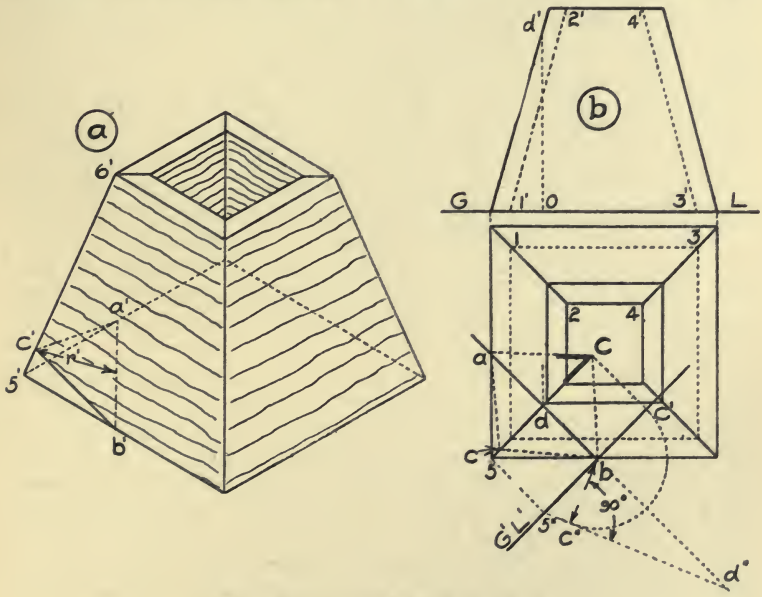


FIG. 287.—Tapered ventilating shaft.

over-all measurements being as follows: Large end, 2 ft. 8 ins.; small end, 1 ft. 4 ins; distance between ends, 2 ft. 4 ins. The shaft is made from four pieces of wood $1\frac{1}{2}$ ins. thick.

Draw an elevation (*a*) *and from this elevation project a plan*

as shown. Determine the real anlge between any two of the sides which are bevelled to fit together at the four corners, and find the correct bevel for shooting or planing the edges of the two pieces of wood to make the joints at the corners.

The dotted lines 1′,2′, 3′,4′ in the elevation (*b*) represent the thickness of the wood. From the points in which these dotted lines meet the ground line GL, draw projectors to determine the position of the dotted lines 1,3, etc., in plan, and by drawing projectors from the points 2′, 4′ in elevation, obtain the solid lines 2,4, etc., in plan. The completed plan is made up of four squares, three of which are solid lines, and the fourth a dotted line, to represent the unseen inside bottom edge of the shaft. The diagonals 1,2, 3,4, etc., in plan represent the sloping corners, along which the two boards are mitred to fit each other. In setting out the wood for making such a piece of work two things must be known, (1) the *true shape of the outside* (*or inside*) *face* of each of the four pieces; *i.e.* a **face template** for marking on the face of the board from which each of the sides is cut. The method of determining this has already been described (see Fig. 238). (2) The *angle* to which the bevel must be set for *testing the edges* of the wood as the board is undercut to the required slope for mitreing against the other board forming the joint at one corner. This bevel, as already shown in dealing with the *backing angle* for hip rafters, depends upon the **dihedral angle** between the two surfaces which meet at the jointed edge.

This is obtained as follows: Draw a line *ab* at right angles to the plan of the intersection at one of the corners, *cd*. Consider this as the line in which a plane surface meets the ground after cutting through the corner in such a way as to be square with the sloping jointed edge.

Referring to the sketch (*a*) the line *a′b′* corresponds with this line *ab* in plan; *r′* is a line so placed as to be at *right angles to the jointed edge*, 5′6′, and the triangle *a′c′b′* is the part of the cutting plane *underneath* the two sides of the shaft, after the plane has passed through the corner at *c′* and cut downwards in a sloping direction towards *a′b′*. In order to show this in the plan, a new ground line G′L′ is drawn, and a new elevation of the sloping edge 5″*d*″ is obtained by projecting from 5 and *d* in plan, making the height *bd*″ on the new elevation equal to the height *od′* on the original elevation. Draw *bc*″ at right angles to 5″*d*″ to represent the profile view of the plane surface which is supposed to cut through the corner of the shaft at *c*. Then the triangle *acb* is the *plan* of the triangle *a′c′b′* in the sketch (*a*). Rotate this triangle about the line *ab* as a hinge. The elevation of the path of *c* as the triangle is rotated is an arc of a circle

$c''C'$, and the plan of its path is dC drawn at right angles to the hinge ab. Then the triangle aCb shows this triangle lying upon the ground after having been rotated about ab. The angle aCb is the **dihedral angle** between the surfaces (the boards) forming one of the jointed edges. The shaded angle at C, *i.e.* the angle aCd and one half the dihedral angle aCb, is the **correct angle** for setting the bevel for *mitreing the boards* to fit at one of the jointed corners.

Fig. 289 shows the plan 1,2,3,4, and elevation $1'5'6'2'$, of four thin pieces of wood jointed together along the edges 1,5, 2,6, 3,7, and 4,8. Four metal stiffeners, one of which is shown

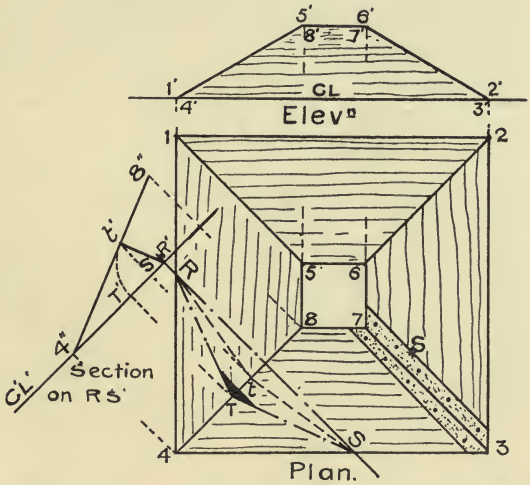


FIG. 288.—Correct angle between two inclined surfaces.

in plan at S, on the edge 3,7, are to be riveted on the edges of the wood to hold the two pieces together at the corners. Find the correct angle for making a template to be used for testing the angle between the faces of the stiffener so that it may bed properly upon each of the two pieces of wood.

The angle between the two under faces of the stiffener must be made to correspond with the *dihedral angle* between the two boards which are jointed together at that particular corner. This is obtained in the manner already described. Draw RS at right angles to 4,8. Make a new elevation 4″,8″, of the sloping edge 4,8 on a ground line G′L′ parallel to 4,8, in plan, making the height of 8″ above the new ground line G′L′ equal to the height of 8′ above the original ground line GL. Draw R′t′ at right

angles to 4″8″. With R′ as centre, and R′*t*′ as radius, describe the arc *t*′T′, and from T draw a projector T′T to determine the position of T on 4,8, after the triangle R*t*S has been rotated into the horizontal plane about RS as an axis. Then the angle RTS is the **dihedral angle** between two boards which meet at one of the corners, and the shaded portion at T is the correct shape of a *template* for testing the under side of the metal stiffeners.

(*p*) Fig. 289 is the plan, and (*p*′) is the elevation of a sloping piece of wood which is bevelled in such a way as to fit down

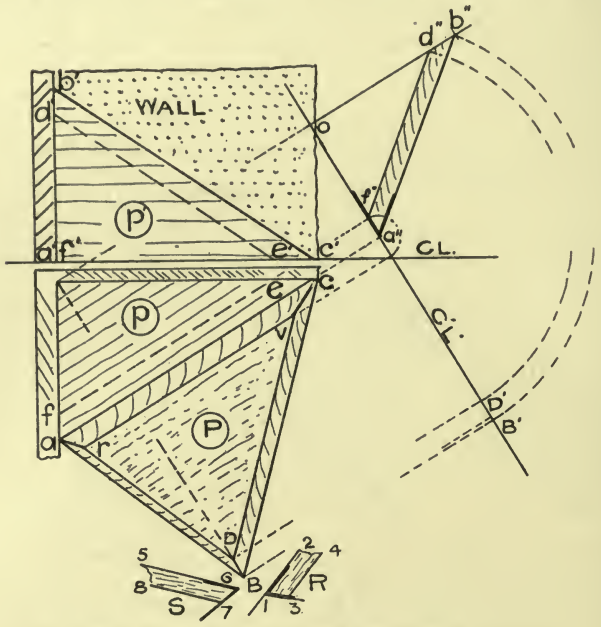


FIG. 289.—Bevels for fitting board (*pp*′) down to floor, etc.

tightly on the floor at *ac*, and against two vertical surfaces along *ab* and *bc*. Determine (1) *a face template for marking* the *top face of the wood* to the required shape. (2) The *correct bevels* for planing the bottom edge and the two inclined edges of the wood so that it may fit against the floor and the upright surfaces.

Draw a new ground line G′L′ at right angles to *ac*, the plan of the outer bottom edge, and determine the position of *a″* on G′L′ and in line with *ac* in plan. From *b* in plan draw a projector *bo* at right angles to G′L′ and set up a height *ob″* equal to *a′b′* on the original elevation (*p*′). Join *a″b″*. This is a profile

view of the top surface of the sloping board. From the dotted line *ef*, which is the plan of the inside bottom edge of the board, draw a projector (not shown) to determine the position of *f''* on G'L', and draw *f''d''* parallel to *a''b''*. Then the distance apart of *f''d''* and *a''b''* is the *actual thickness* of the board; and the shaded angle at *a''* is the *correct bevel* for planing the **bottom edge** of the board to fit down upon the *floor*. With *a''* as centre and radii *a''b''*, *a''d''*, describe the circular arcs *b''*B', *d''*D'. From *b* in plan (*p*) draw a projector *b*B at right angles to the imaginary axis of rotation *ac*, and from B' and D' draw projectors at right angles to G'L' to meet *b*B (in plan) at B. Then the triangle *a*B*c* is the *correct shape* of **face template** required, *i.e.* *a*B*c* is the plan of the board after it has been rotated about *ab* and is lying with its face upon the ground. In order to complete this new plan draw projectors from *e* and *f* in plan, at right angles to the hinge *ac*, to represent the plans of the paths of the *inner edges of the mitres* as the board is rotated over into the ground plane. The inner triangle, parallel to *ac*B, represents the under edges of the board after it has been planed, so as to fit against the horizontal and vertical surfaces. One of these mitres DB is used for the second part of the construction. Thus, draw a line 1,2 at right angles to the edge *a*B of the developed board, and draw 3,4 parallel to 1,2, making the distance apart equal to the true thickness of the board as shown in the profile view *a''b''d''f''*. Mark a point 1 on 1,3, in line with *a*B, and a point 3 on 3,4 in line with D*r*. Join 1,3: then the *shaded angle* 2,1,3 is the *correct setting of the bevel* for testing the *left-hand edge ab* of the board. Similarly, draw 5,6 at right angles to B*c*, and draw 7,8 parallel to 5,6, and at a distance from it equal to the thickness of the wood. Mark a point 6 in line with B*c*, and a point 7 in line with D*v*, and join 6, 7. Then the *shaded angle* 5,6,7 is the *correct bevel* for the *edge of the board* which fits against the **wall** at the back.

The plan and elevation of a strut S,S', Fig. 290, are given. The foot of the strut is splayed to fit down upon the floor and the top end is cut to fit into the angle between two walls WW, which meet at right angles, and the strut is cut horizontally so as to take the weight of the floor joists, J, above. *Determine the correct bevels for cutting the top and bottom ends of the strut.*

Assume that the face 1',2',3',4,'5' of the strut is swung around upon 3',2' (*i.e.* a vertical line) as an axis until it is parallel to the vertical plane. Thus, draw a line PR in plan parallel to GL and passing through the point 2,3. With 2,3 as centre, radii 2,5; 2,1; 2,4, describe arcs 5*b*, 1*a*, and 4,6, and from *ab* and 6 draw projectors at right angles to GL to determine

the position of a', b', and $6'$ in the elevation. Join $2'a'$, $6'b'$. Then the dotted figure $2',3',6',b'a'$ is the *true shape of the* face *of the strut* after it is swung into parallelism with the back wall. In this position the true shape of the bevels for marking on the *face of the strut* appear as shown by the shaded angles D and E, that at H being the bevel for cutting the *foot of the strut*. It should be noted that, as the top end of the strut is cut horizontally to fit against the under side of the joists, the angle 3,7,8

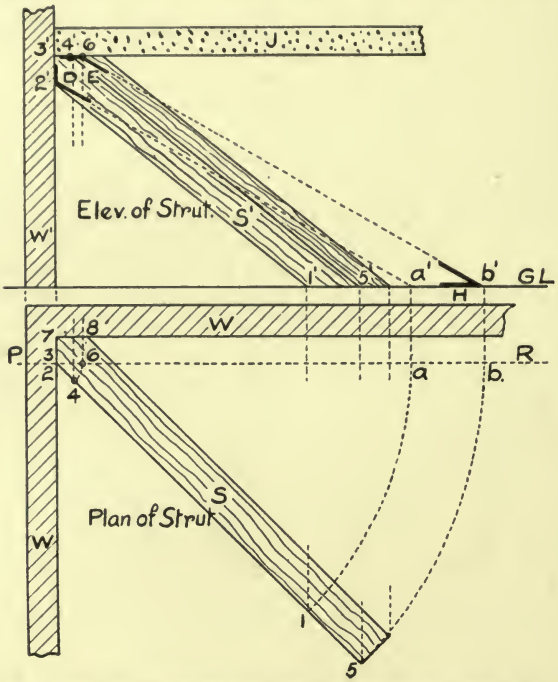


FIG. 290.—Correct angles for top and bottom ends of strut S S'.

in plan (*i.e.* a right angle) is the correct angle for setting out on the top edge of the strut, after cutting to the bevel E.

P, P', Fig. 291, are the plan and elevation of a post of square section, and S, S' represent a strut, the section of which is shown by dotted lines at *tt*. The strut is to be cut to *birdsmouth* around the post. *Determine the correct bevels for marking the top end of the strut for fitting around the post.*

The method of working in this case is based upon the theory of rabatment explained in the foregoing problems. The upper

face 5′3′1′6′ is *not parallel* to either of the imaginary planes of projection, and, consequently, *does not show its true width in elevation.* If this face is rotated about 5′,3′ as an axis of rotation until it is in a vertical position, *i.e.* parallel to the VP, its width would be as shown at T, *i.e.* equal to *tt* taken from the sectional drawing. Hence draw *b*9 parallel to 5′3′, and making T = *tt*. Draw 1′9 at right angles to the axis of rotation 5′3′ to meet *b*9 at 9. Join 2′9. Then the angle *b*9,3′ is the *correct bevel* for marking the **top face** of the strut. The same method is adopted for obtaining the bevel for the under surface 5′,3′,1″,7

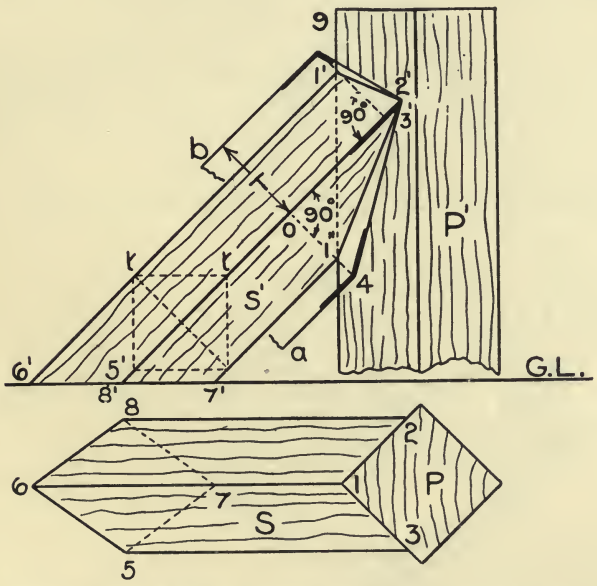


FIG. 291.—Correct angles for cutting strut SS′ to fit against a post PP′.

of the strut. Draw *a*4 parallel to 7′1″, make 04 = *tt*, and join 4,3′. Then *a*4,3′,5′ is the true shape of this face, and the correct bevel for marking on this face may be obtained either at 3′ (*shaded angle*) if working from the *top* edge of the strut, or at 4 if working from the *bottom* edge of the strut.

Fig. 292 shows the plan and part elevation of a turret roof. The plan of the roof is a regular hexagon. The problem is introduced as a further exercise in the application of the principles already demonstrated, and does not need any lengthy explanation. *pt* is the correct *length* of one of the six **hip rafters**, assuming that the top end is splayed to fit against the upright

face of a central post. The angle at T shows the *correct bevel* for fitting the *top end of the hip rafter p*3 up to the post, and that at P for fitting the *bottom* end of the hip down to the plate. The angle at H is the correct *backing angle* for the upper edges of each of the hip rafters, and is obtained by means of an auxiliary construction shown at RS in plan, and *ab* in the new elevation on G′L′. The triangle 1,7,2 is the *true shape* of one

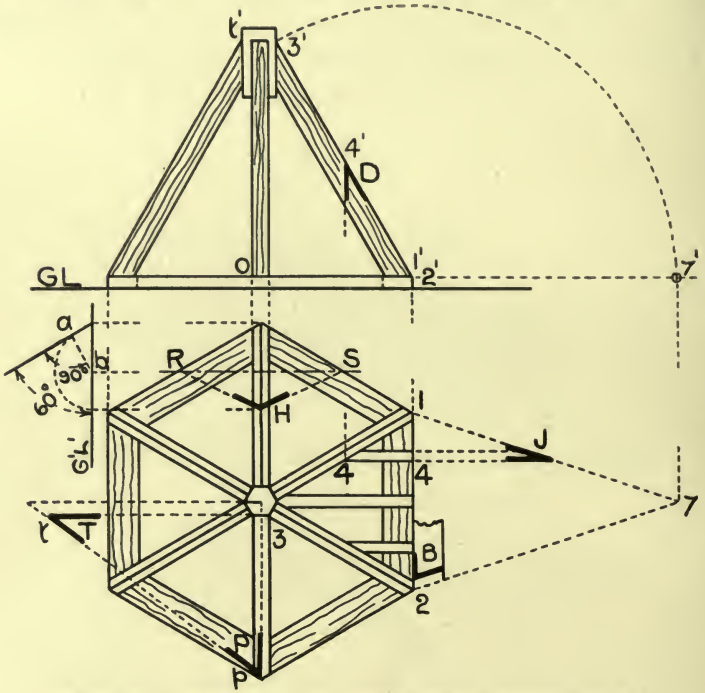


FIG. 292.—Correct angles for various timbers in octagonal turret.

of the developed roof surfaces ; 4J is the length of one of the jack rafters, shown at 4,4 in plan. The angle at J is the *correct bevel* for marking the **top edge** of the *jack rafters*, *i.e.* the **cross bevel**, and that at D is the *correct bevel* for marking the **face** of the jack rafter 4,4 and the angle at B is that to which the bevel should be set for marking the *ends of the roof boards* which are cut to fit between the rafters 1,3 and 2,3.

CHAPTER XIX

PROJECTION OF SHADOWS

THE term **shadow** as used in this chapter may denote either of two conceptions. It may mean the *space deprived of light* by interposing some opaque body between a luminous source and a plane surface; or, again, it may mean the *outline of the projection* of an object upon a surface made by imaginary projectors, *i.e.* the rays of light from the luminous source. Thus when the sun is shining on the east front of a house, the opaque body (the house) deprives the western side of the house of the sun's rays. The western (or shadow) side of the house would thus be an example of the first definition given above. On the other hand, the rays of light which pass through the various points on the outline of the roof and the chimney stacks pass on in a straight course, and eventually reach the ground or some other surface on the western side of the house, making an outline of the *projection* of the top of the house on the receiving surface—the back garden. The *cast shadow* of the house would thus be an illustration of the second definition given above.

Position of the Luminous Source—Parallel and Divergent Rays.—The rays of light which, being interrupted, produce a shadow, may proceed from some source of illumination *near* the object, such as an electric filament lamp, or may proceed from some source at an *infinite distance away* from the object, *e.g.* the sun. In the first case the rays would **diverge,** but in the second case the divergence would be so slight that they may be taken as **parallel** rays. In most of the examples which follow, the latter type is taken, and the rays which, being interrupted by the opaque body, are assumed to *travel in parallel straight lines.*

Line of Separation of Shadow Area.—When an object casts a shadow, one portion of the object itself must be in darkness; and the line which separates the lighted portion of the object from that which is not illuminated is termed the **line**

of separation. This line plays an important part in the solution of problems on shadows, since it is the *projection* of the **line of separation** upon the receiving surface which gives the *outline* of the **cast shadow** on that surface.

Shadows cast wholly upon One Plane.—Fig. 293 (*a*), (*b*), and (*c*) are examples of shadows which are *projected* in such a way as to fall entirely upon the HP, the **cross-hatched** part of the diagram in each case representing the **cast shadow.**

EXAMPLE.—A thin rectangular sheet of cardboard *abcd* in (*a*) is held in such a way that the front elevation of the card is an edge view *a'd'b'c'*. Rays of light from a distant source of illumination cause the cardboard to cast a shadow on the HP. The rays of light are assumed to be parallel; hence it is only necessary to show the plan of any single ray R and the elevation of any ray R' in order to indicate that all the rays proceed along

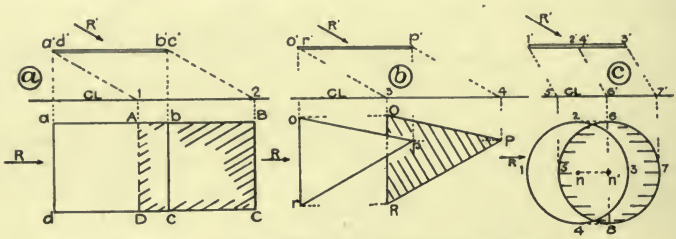


FIG. 293.—Shadows produced by parallel rays.

lines *parallel* to the VP, but *inclined* to the HP. Determine the cast shadow of the card upon the HP.

Through *b'* draw the *elevation* of the ray *b*2 which passes through *b* and is parallel to R'. This ray meets the ground line at 2. From *b* in plan draw the *plan* of the ray *b*B which passes through *b* parallel to R. From 2 draw 2B in order to determine the point B at which the ray actually meets the HP. Then B is the shadow on the HP of the corner, *b*, of the card. The ray *c'*2 may also be taken as the elevation of the ray which passes through *c*. Draw *c*C parallel to R, to represent the plan of the ray shown at *c'*2 in the elevation. From 2 draw 2C at right angles to the GL. Then C is the **cast shadow** of the corner *c* of the card. In a similar manner draw *a'*1 to represent the *elevation*, and *a*D to represent the plan of the ray which passes through the corner *a* of the card, and from 1 draw 1D. The point A is obtained by using the same ray *d'* in elevation and projecting the line 1A from 1 at which the ray meets the ground line In this way the plan of each of the four corners of the

card is obtained, and it should be obvious that, since the edge *bc* of the card is a straight edge, its shadow if cast upon a *plane surface* will also be a *straight line*, *i.e.* the line BC. In the same way AB is the shadow cast by the edge *ab*, and DC and AD are the cast shadows respectively of the edges *dc* and *ad* of the card.

In (*b*) the shaded area OPR is the shadow cast upon the HP by a triangular-shaped card shown in plan at *opr*, and in elevation by the horizontal line *o'p'r'*. The method of determining the cast shadow OPR is similar to that already described in (*a*), and should be easily followed.

In (*c*) the cast shadow of a circular disc is shown. Any points such as 1, 2, 3, 4 on the plan are selected, and the elevations of these points are found at 1′, 2′, 3′, 4′ on the horizontal line 1′3′, which represents the edge view of the circular disc.

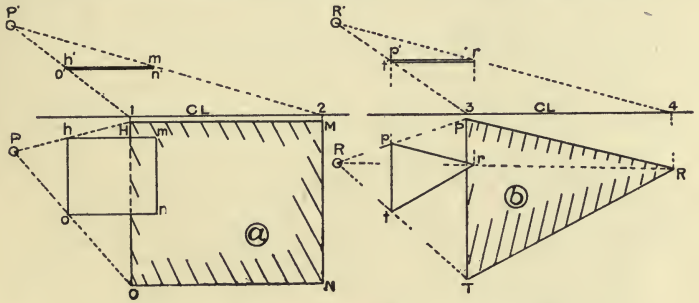


FIG. 294.—Shadows produced by divergent rays.

Rays, such as 3′,7, 4′,6′, 1′,5′, are then drawn parallel to R′, and rays 2,6, etc., in plan are drawn parallel to R. Projectors from 7′, etc., on the ground line determine the position of the shadow of the respective points at 5, 6, 7, 8, etc. A curve through these points is a circle.

From an observation of these three cases, (*a*), (*b*), and (*c*), it should be obvious that when a plane surface is held in a position such that it is *parallel to the HP*, and the *illumination proceeds from a distant source*, *i.e.* the rays of light are *parallel*, the **cast shadow of the object is exactly the same size and shape as the original.** Thus, the cast shadow of the circular disc in (*c*) could have been obtained by determining the shadow, *n′*, of the centre *n* of the disc, and describing a circle 5, 6, 7, 8 with *n′* as centre and radius = *n*1, *i.e.* the radius of the original card.

(*a*) and (*b*), Fig. 294, are introduced in order to show the method of determining the cast shadow where the rays proceed from a **point of light** P, P′ in (*a*); and R, R′ in (*b*). A thin

rectangular sheet of cardboard, *h*, *m*, *n*, *o*, in (*a*) is placed in a position parallel to the HP, so that the edge of the card appears in elevation as a straight line *h'm'*, parallel to the GL line. From P', the elevation of the source of light, draw rays P'*m'* and P'*o'*, and produce to meet the GL at 1 and 2. From P, the plan of the source of light, draw rays P*h*, P*m*, P*n*, and P*o*, and produce in such a way that projectors from 1 and 2 determine the position of the points H, M, N, O, which are the cast shadows of the four corners of the card. The straight lines, HM, MN, etc., which join these points, are the shadows cast upon the HP by the edges of the cardboard, and the shaded area HONM is the shadow of the card.

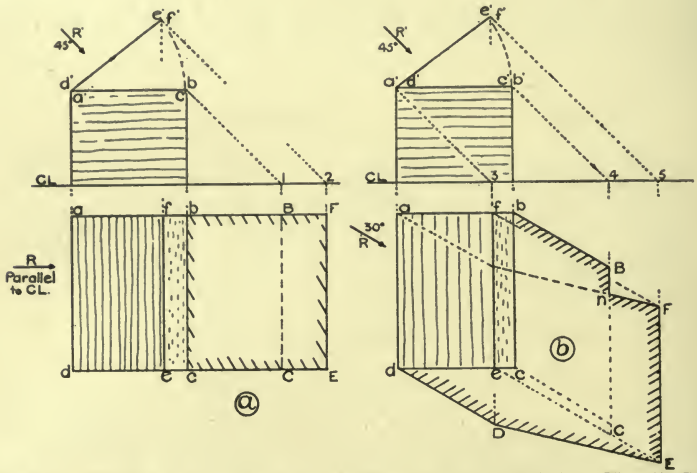


FIG. 295.—Shadow of box cast upon the HP. (*a*) Rays parallel to VP. (*b*) Rays inclined outwards at 30° to VP.

In (*b*) R, R' is the source of light, and *prt* is a triangular piece of card, the shadow of which, PRT, is obtained in a manner similar to that described in (*a*).

Shadows cast by Rectangular Solids.—(*a*) Fig. 295 shows the plan and elevation of a **rectangular box** *abcd*, the lid of which is partly open. The rays of light R, R' are inclined at 45° to the HP, but are parallel to the VP. *Determine the cast shadow of the box.*

The point which is lettered *c'b'* in elevation, is the elevation of the long front edge *bc* of the box shown in plan. Similarly, the point *e'f'* is the elevation of the long front edge of the lid,

shown at *ef* in plan. From these points draw the elevation of the rays *f*′2′ and *b*′1′, respectively, parallel to the ray R′, and from the points *f*, *b*, *c*, and *e* in plan draw rays parallel to R, *i.e.* parallel to the ground line. A projector from 2 determines the position of F and E, the shadows cast by the corners *f* and *e* of the lid, while a projector from 1 determines the position of the points B and C, the shadows cast by the two top front corners, *b* and *c*, of the box itself. Note that the lines B*b* and C*c* are the shadows cast by the *upright* edges of the box.

(*b*) Fig. 295 represents the cast shadow of the same box when the plan of the rays of light are *inclined to both planes of projection* as shown (45° in elevation and 30° in plan), instead of being parallel to the VP in plan as in the last case (*a*). The cast shadow FE of the top front edge of the lid is obtained by means of a ray *f*5 in the elevation at 45° to the GL and a ray *f*F in plan at 30° to the GL. The line DE is the cast shadow of the top edge of the lid *nearest* to the observer, and is obtained by means of the rays *a*′3 in elevation, and *d*D in plan, respectively. The line *d*D is the cast shadow of the upright edge of the box itself, while the line *b*B is the cast shadow of the upright edge of the box on the side *farthest* from the observer.

(*a*) Fig. 296 shows the cast shadow of a **chimney stack**, the rays of light being inclined at 45° to the GL in elevation and 30° to the GL in plan. Before commencing to draw the cast shadow it is advisable to consider which faces of the chimney stack are illuminated and which are in darkness. Considering first the *top* portion of the chimney as shown at *abcd* in plan, it is clear that as the rays are proceeding along lines at 30° to, and *inclined away from*, the GL, the *two upright faces ab* and *bc* will *catch the light* fully, while the two upright faces *cd* and *ad* are *in darkness*. Hence it is the cast shadows of the two top edges *cd* and *ad* (*i.e.* CD and AD on the *shaded* part of the diagram) which form the outer limits of the cast shadow. Having determined the shadow cast by the edges *cd* and *ad*, the shadow of the vertical edge at *c* should be determined. This is obtained by means of a ray which meets the ground line at 7 and a projector from 7 meets a ray from *c* in plan at *m*. Hence C*m* is the cast shadow of the vertical edge at *c*. Similarly the line *mn* is the shadow cast by the back inside edge of the upper part of the chimney, and 2*n* is the cast shadow of the upright edge of the back of the lower part of the chimney. By means of rays drawn from *a*′ and *e*′ in the elevation and from the point *ae* in plan, the line AE, which is the cast shadow of the vertical edge *ae*, is determined. In the same way E,IV is the cast shadow cast by the lower edge of the top part of the chimney, while IV4

is the cast shadow of the vertical edge of the lower part of the chimney.

(*b*) Fig. 296 shows the shadow cast by a **stone obelisk** which is made up of *two square-based pyramids.* The construction follows closely along the lines already indicated; thus the cast shadow of the vertex of the pyramid *v'* is obtained by means of rays from *v'* in elevation and *v* in plan and is shown at V. The points E and F on the shadow are obtained by means of rays from the point 5',6' in the elevation, and from the points 5 and 6 in plan. Having determined VF and VE, the shadows of the two edges of the *upper* portion of the obelisk, join E2 and F4 to represent the shadow cast by two of the sloping edges of the *lower* part of the obelisk. It should be

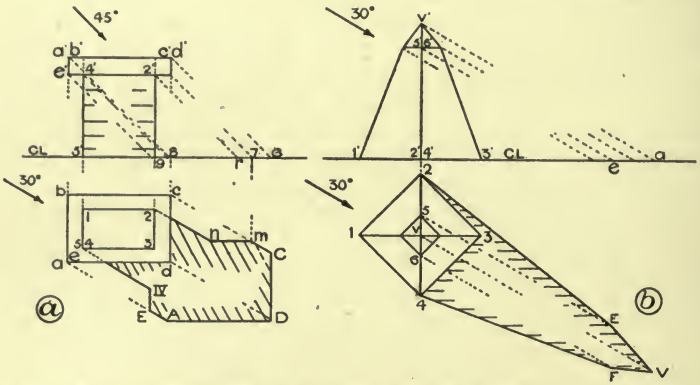


FIG. 296.—(*a*) Cast shadow of chimney. (*b*) Cast shadow of stone obelisk.

noted in this connection that where points such as 2 and 4 are *resting upon the plane* upon which the cast shadow is being projected, such points can **have no cast shadows,** or theoretically *the shadows cast by the points* 2 and 4 *coincide with the points themselves.* Thus, having drawn the lines E2 and F4, these lines contain the shadows of all points on the respective edges 5,2 and 6,4 of the frustum which forms the lower part of the obelisk.

Shadows cast partly upon the VP and partly upon the HP.—(*a*) and (*b*), Fig. 297, illustrate a type of problem in which the position of the solid with reference to the respective planes or the direction of the rays of light is such that the shadow does not fall entirely upon one plane of projection.

In (*b*), which is the simpler of the two cases, the plan of the rays is inclined *towards* the VP, with the result that part of

the shadow falls upon the VP. For the sake of clearness, the unilluminated faces 1,5,2 and 3,5,2 of the pyramid are shaded. If there were no VP, the shadow of the vertex 5 would occur at the point 6, which is found by drawing a ray 5′7 in elevation, and 5,6 in plan, the projector from 7 on the GL being drawn *above* the GL. The reason for this will be better understood if the VP is imagined to be a *transparent* plane through which the ray from the vertex 5′ passes and eventually reaches the ground at 6. Draw the lines 6,1 6,3 to represent the cast shadow of the pyramid as it would appear if projected entirely upon the HP, and at the point 8, at which the ray from 5 in plan crosses the GL, draw a vertical projector to intersect the ray 5′,7 in elevation at 9. This is the point at which the

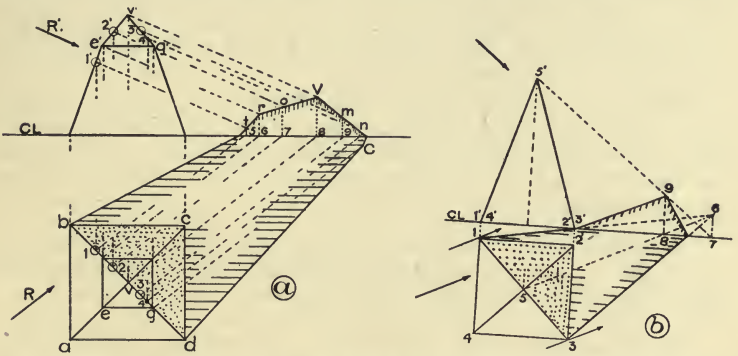


FIG. 297.—Shadows cast partly on each of the two planes of projection.

ray from the vertex of the pyramid penetrates the VP, and lines drawn from 9 to meet the GL, as indicated by the cross-hatched portion of the diagram, gives the boundary of that part of the cast shadow which would occur upon the VP.

EXAMPLE.—*Copy the plan and elevation* (b) *three times the sizes given and determine the cast shadow.*

The object shown in (*a*) Fig. 297 affords a further illustration of a cast shadow which falls partly upon each of the planes of projection. The dotted area shown in plan between the diagonal *bd* and the base edges *bc* and *cd* is the plan of that part of the object which is not illuminated. Hence it is the lines *bv*, *dv* in plan which form the *line of demarcation* between the **illuminated part** of the object (*i.e.* the faces *bva* and *dva*) and the faces *bvc* and *cvd* which are **in darkness.** These *lines of demarcation* are important in that the cast shadow of the

edges which they represent gives the outline of the cast shadow of the whole object.

The cast shadow of the vertex V on the VP is obtained by the method shown in (*b*), which has been explained already. A ray *v*8 parallel to R and another ray *v'*V parallel to R' are drawn, and the position of V on the ray *v'*V determined by drawing a projector 8V at right angles to the GL. Referring again to the plan, it is found that, by drawing rays from *g* in plan and *g'* in elevation parallel to R and R' respectively, the shadow of the point *g*, *i.e.* the top end of the sloping edge *dg*, occurs on the HP at G. Join *d*G. This is the cast shadow of the edge *dg*. Select any points such as 3 and 4 on the edge *vg* in plan, and determine the elevation of these points at 3' and 4'. By means of rays in plan and elevation it is found that the shadows of these points occur on the VP at *m* and *n*. Draw a straight line from V through *m* and *n* to meet the GL. The short piece of line between G and the GL near *n* is the shadow of that part of the edge *vg* which occurs on the HP. In the same way, by using points 1 and 2 on the opposite sloping edge *bv* of the pyramid and drawing rays such as 2,7, 2',0, the position of other points 0, *r*, and *t* on the cast shadow may be determined.

EXAMPLE.—*Draw the given plan and elevation* (a) *three times the size of the original, and with rays as shown at* R *and* R' *determine the cast shadow of the object on the respective planes.*

Shadows cast on Successive Horizontal Planes.—The small sketch (*b*) Fig. 298 is introduced as a key to the solution of a type of case such as that shown in (*a*), where the shadow of an object falls *partly upon one horizontal plane and partly on another*.

In (*b*) BB' is a square stone slab, and CC' is a square-based turret resting upon the stone slab.

Determine the shadow of the square slab as cast upon the HP and the shadow of the turret as cast partly upon the upper face of the square slab and partly upon the HP.

In such cases it is usual to assume **imaginary horizontal planes** at various heights above the HP, and to determine the shadow cast upon each successive plane. Thus if a horizontal plane is assumed at the height of the top face of the square slab, a ray from *v''* in the elevation would meet this imaginary horizontal plane at *o*. From *v* in plan draw the ray *v*V' parallel to A and determine the position of V' by means of a projector *o*V' at right angles to GL. Join V' to *a* and the opposite bottom corner of the plan of the pyramid (not lettered). Then the

thickened portion of the line V'*a* is the shadow of the sloping edge of the pyramid as cast upon the *top face of the square slab.* Produce *va'* to 3 in elevation. Then *v'*3' is the elevation of one edge of the pyramid, *assuming that all its edges were increased in length* sufficiently to make it rest upon the HP instead of upon the square slab. Produce the plan of *va* to 3, and from 3' on the GL draw a projector at right angles to GL to determine the position of the two points numbered 3 in plan. Join V3. This is the shadow of the sloping edge *v*3 as cast upon the HP. Only the thickened parts V4 and V5 of these lines are required, the remainder of the lines V3 falling within the cast shadow of the square slab *n*,2,2,*m*, which is determined by the method already described. The whole cast shadow is made up of the cross-hatched part *n*,2,4,V,5,2,*m*, this being the shadow on the HP ; and the cross-hatched part between V' and *a* which represents the cast shadow of the pyramid on the top face of the slab.

EXAMPLE.—*Copy the given plan and elevation four times the given size and make the pyramid twice the given height as compared with the width at base. Determine the cast shadow of the objects when the rays are inclined as shown at* A' *in elevation and the plan of the rays* A *is inclined towards the GL at an angle of* 10°.

(*a*) Fig. 298 is a somewhat more difficult case of the same type as that already described in (*b*).

A square-based pyramid rests upon the top face of a square stone slab, the latter resting upon another larger square stone slab.

Determine the shadow of the pyramid as cast partly upon the top faces of the successive stone slabs and partly upon the HP, and determine also the cast shadow of the upper slab upon the top face of the lower one, and the cast shadow of the bottom slab on the HP.

In order to determine the point V, which is the shadow of the vertex of the pyramid as cast upon the HP, produce the edges *v'a'*, and *v'e'*, respectively, to 1 and 2, indicated by small circles on the ground line. Find the plans of these points at 1 and 2 on the lines *va* and *ve* produced. Draw rays *v'*V', *v*V, and draw a projector from V' to determine the position of V. Join V1, V2 to represent the shadow of the pyramid on the GL as it would appear *if there were no square slabs.* Assume imaginary horizontal planes at the level of the top of the respective square slabs as shown at *e'n'* in elevation. From the point *n'* at which the ray *v'*V' meets the plane of the top face of the upper slab draw a projector to determine the position of *n* in plan. Join *ne* and *na*. The only parts of these lines required are the lines *ar* and

eo, which is the cast shadow of the sloping edges of the pyramid as it falls upon the top face of the upper slab. Proceed in the same way to deal with the point at which the ray $v'V'$ in elevation meets the level of the assumed horizontal plane through $b'c'$. This gives the shadow of the *sloping* edges of the pyramid as cast upon the *top face of the lower slab.* It will be found, however, that these lines are not needed when the lines 6,7 and 8,9, which are the shadows of the *vertical corners* of the upper slab as cast upon the *upper face* of the lower slab, are determined, since the one shadow, *i.e.* of the sloping edges of the pyramid, is included *within the shadow area* of the slab itself, and would not

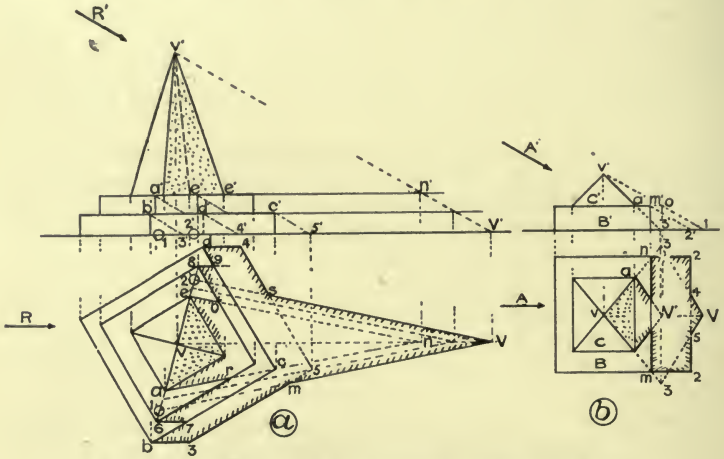


FIG. 298.—Shadows cast upon successive horizontal planes.

therefore appear as a separate shadow. In the same way, part of the cast shadow of the lower slab between the points $s5$ and $m5$ is not required, since it falls within the shadow area sVm of the cast shadow of the pyramid on the ground. The method of determining the shadow of the lower step as cast upon the HP is similar to that already described, *e.g.* rays $c'5'$, $b'3'$, and $d'5'$, are drawn in order to determine the position of the points 4, 5, and 3 in plan, on the rays drawn from *d*, *c*, and *b* respectively. Thus the whole shadow is made up of the following details: *ar* (on the top slab), part of the edge of the top slab at *r*6, 6,7 (on the top face of the lower slab), the part 7*b* of the edge of the lower slab; the shadow of the bottom slab as cast upon the HP at *b*3, 3*m*; the shadow of the pyramid on the HP at *m*V5; and the remaining lines 5,4, 4*d*, *d*9, 9,8, 8*o*, and *oe*.

EXAMPLE.—*Reproduce the plan and elevation of the solids, as shown in Fig.* 298, *double the sizes shown, and determine the cast shadow of the group of solids when the rays of light are as shown in elevation but the plan of the rays is inclined outwards at* 10° *to the ground line.*

Shadows cast upon Curved Surfaces.—The two cases which immediately follow, Figs. 299 and 300, are introduced in order to illustrate the fact that when the *shadow of a straight line*, or the straight edge of a solid is cast upon a *curved surface*, the outline of the **cast shadow is a curved line.**

In Fig. 299 the plan and elevation, VV′, of a **pyramidal ventilating shaft** is shown, standing upon the ground near a **semi-cylindrical brick tunnel** TT′; the rays of line proceeding along lines parallel to A and A′.

Determine the shadow cast by the ventilating shaft upon the outer or convex surface of the tunnel.

The semicircle in T′ is the *elevation of all points on the convex surface* of the tunnel. Hence a ray *e′*E′ meets the semicircle at E′, which is the *elevation* of the point E in plan at which the ray of light would be interrupted by the curved surface of the tunnel, the ray *e*E in plan having been drawn parallel to A. Similarly E′ is also the elevation of the point C in plan which is determined by means of a ray *c*C from the corner *c* of the ventilating shaft. Hence the line CE is the cast shadow of the straight edge of the upper part of the ventilating shaft; and since it occurs in a position *parallel to the edge* of the cylindrical tunnel, the shadow will be a *straight* line.

Consider next the shadow cast by one of the *sloping* edges *ef*, *e′f′*, of the shaft. Select any points in plan such as 4, 5, 6 on this edge and determine the elevation of these points at 4′, 5′, 6′; and from the latter points draw rays as shown at 6′,8, 5′,9, etc. From the points 8, 9, etc., in elevation determine the position of the points IV, V, VI in plan on rays drawn from 4, 5, and 6, parallel to A. Through these points draw the curve E,VI,V,IV, which is the cast shadow of the sloping edge *ef*. Similarly, by selecting points 1, 2, 3 on the top horizontal edge of the ventilating shaft, find the elevations of these points at 1′, 2′, 3′, and draw the rays 1′*n*, 2′*o*, 3′*p*, and from the points *n*, *o*, *p* draw projectors at right angles to the GL to determine the position of the points I, II, III on rays drawn from 1, 2, 3 in plan parallel to A. Then the **curve** A,I,II,III,C is the cast shadow of the horizontal edge *ac* upon the *convex* surface of the tunnel. The **straight line** from *b* drawn towards A is the shadow of the sloping edge *ab* as cast *upon the ground*. Its shadow does not fall *wholly* upon the ground, however, and in

order to determine the character of that part of the shadow of *ab* which falls upon the curved surface of the tunnel any number of rays are drawn as shown at *m* from points intermediate between *a'* and *b'* in elevation and *nearer* the end *a'*. Find the plans of these points on *ab* in plan (not shown on diagram), and

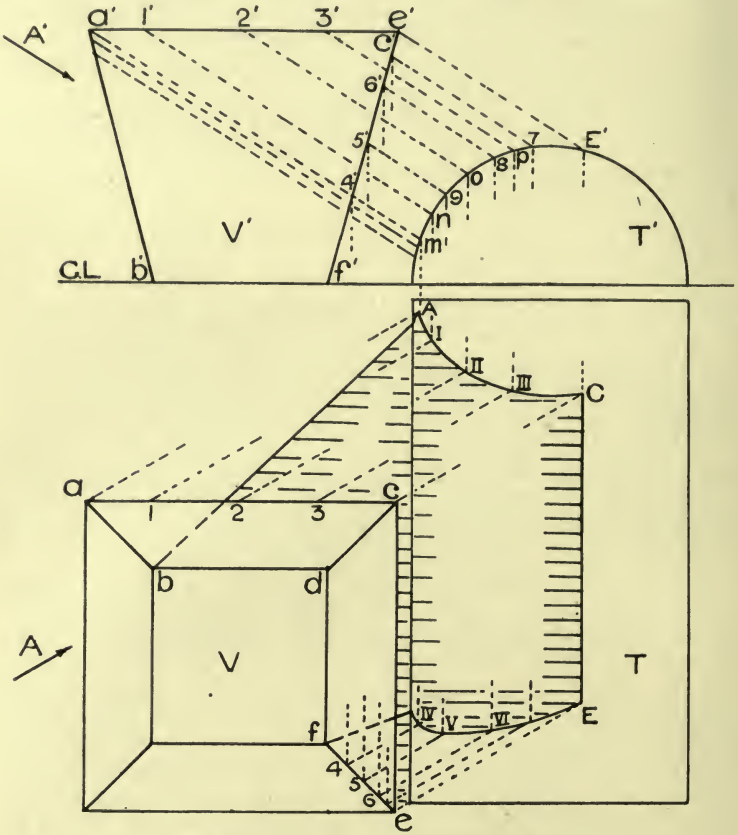


FIG. 299.—Shadow cast by ventilating shaft.

draw rays parallel to A in plan, and thus determine the cast shadow of the respective points. Through these points draw the short piece of curved line near A to intersect the edge of the tunnel, and from this point of intersection draw a straight line to *b* to represent the cast shadow of the straight edge *ab* as cast upon the ground. The shadow of the sloping edge *ef* falls

wholly upon the HP and is shown partly as a broken line between *f* and IV.

Shadow of One Part of an Object falling upon a Curved Surface of the Same Object.—In Fig. 300 a **cylindrical column** surmounted by an **octagonal cap** is shown in plan and elevation. *Determine* (a) *the shadow of the cap as cast upon the curved*

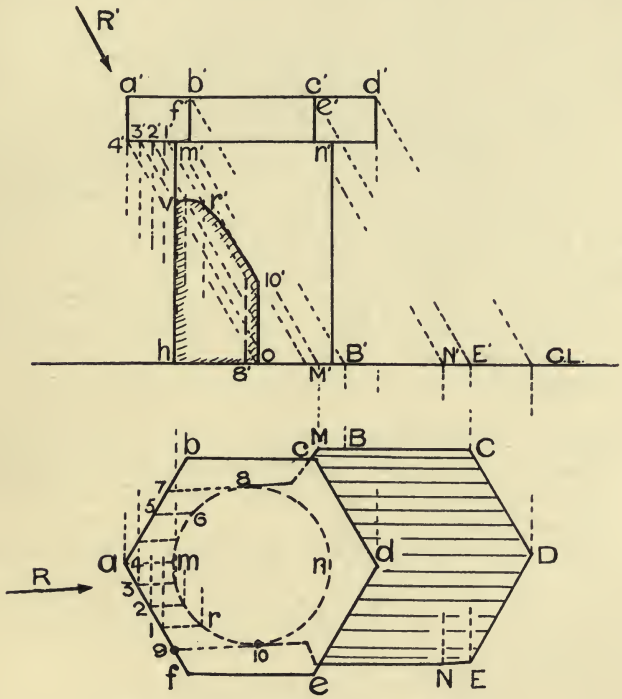


FIG. 300.—Cast shadow of column and cap.

surface of the column, and (b) *the cast shadow of the column and cap upon the HP.*

The solution of (*b*) is obtained by the method already described fully in previous problems, and as the necessary working lines are all shown in the diagram, it should be clear that the lines making up the figure 8,M,B,C,D,E,N,10, are the cast shadows of the respective edges of the cap, etc., the rays at 7,8, 9,10, which are drawn parallel to R and tangent to the circle *m*,8,*n*,10, giving the shadow of the cylindrical column as cast upon the HP.

The solution of (*a*) needs closer consideration. Select any points such as 1, 2, 3, and 5, 7, on the lines *af* and *ab* in plan. Determine the elevation of these points at 1′, 2′, 3′. From points 1, 2, 3 draw rays parallel to R to meet the circle at *r*, etc., and from 1′,2′,3′ draw rays indefinite in length and parallel to R′. From the points *r*, *m*, 6, etc., at which the plan rays meet the circle *m*,8,*n*,10, draw projectors at right angles to the GL to determine the elevation of points such as *r′* on the rays from 1′, 2′, 3′, etc., in elevation. Then the curved line 10′*r′*V is the shadow of the *straight* edge on the under side of the cap, *af*, as projected upon the *curved* surface of the column. The straight line *o*,10′ shown in elevation is the line which divides the shadow area from the illuminated part of the column. In the diagram, however, the shaded portion, owing to an error, really represents the *illuminated* portion of the column ; while the part *vm′n′*B′*o*10′, which should be *shaded*, is the part of the column which is **not** illuminated. The broken vertical line at 8′, with a curved line, represents the elevation of the shadow of the under side of the cap as thrown upon the curved surface of the column *farthest away from the observer*, and is determined by means of rays such as 7,8, and 5,6 in plan and corresponding rays in elevation.

EXAMPLE 1.—*Reproduce the plan and elevation of the solids shown in Fig.* 299 *twice the given size, and determine the cast shadow when the rays of light are as shown at A′ in elevation and the rays in plan are inclined* away from *the VP at* 10°.

EXAMPLE 2.—*Reproduce the plan and elevation of the column and cap shown in Fig.* 300 *twice the given size, and determine the cast shadow of the whole solid on the ground and the shadow of the cap cast on the column when the rays are as shown in elevation but inclined away from GL in plan at an angle of* 10°.

CHAPTER XX

PICTORIAL PROJECTION

Advantages of a Pictorial View of an Object.—It has already been shown (Chap. XII) that in orthographic projection at least *two separate views* of an object must be drawn in order to give a clear idea as to the *shape*, *size*, and *position* of the object. A certain degree of technical training is necessary in *reading* such a drawing to enable an observer to combine, mentally, the plan and elevation of an object in such a manner that a clear and definite mental picture of the character of the object is obtained. Those who have not received the necessary training find it difficult to *read* from an orthographic projection. Hence it is sometimes necessary for a foreman to be able to make sketches of details of construction in such a way as to be intelligible even to the uninitiated. To the latter class of persons a **picture**, or *pictorial drawing*, appeals more than any other type of drawing, since the **one drawing shows the length, width, and height** (or thickness) of the object. The student of building construction, or carpentry and joinery, will frequently find it necessary to make pictorial sketches in order to elucidate the various details of construction, such as the *joints in woodwork construction;* the various *methods of bonding in brickwork;* and the methods of fitting and preparing separate blocks of stone in *masonry work*. The architect and draughtsman, too, is sometimes called upon to prepare a pictorial view of a building which it is proposed to erect, such that the prospective client may be able, with his very limited knowledge of technical drawing, to form a more definite idea as to how the building will appear after erection. Thus a careful study of the detailed description of the various types of pictorial projection which immediately follow should prove of general interest in the classroom, the workshop, and the drawing office. Three types of Pictorial Projection will be demonstrated: (1) **Perspective**, (2) **Isometric**, and (3) **Oblique Projection.**

Perspective Projection.—The term *perspective* means literally to *see through;* hence the basis of this system of projection may be explained as follows:—

In making a perspective view of an object, O, Fig. 301, which is shown resting upon a **Horizontal Plane or Ground Plane,** the observer is assumed to be situated in such a position that a **Transparent Plane** is interposed between him and the object as shown. Rays of light from the respective points (such as *a*) on the object penetrate the *transparent plane*, and, passing on in straight lines, converge to a point at the observer's eye. If the various points (such as *a'*) at which these rays penetrate the plane are joined together by imaginary lines drawn *on the surface of the transparent plane*, a view such as P is obtained, and it is the purpose of the observer to transfer to his paper a *reduced counter-*

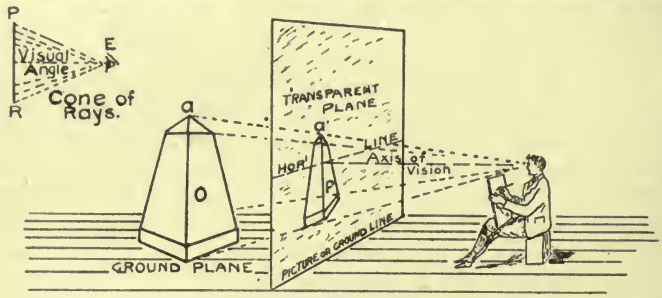


FIG. 301.—Diagram illustrating theory of perspective projection.

part of the projection P, and thus make a drawing which shall represent the object, O, *as it appears to him* when viewed from the point of view he has selected.

In this type of projection, therefore, the basal principle is that the object is viewed through an imaginary screen or **transparent plane surface.** If the observer moves to a position further to the left or right, still keeping at the same distance from the object, he will see more of left side or right side respectively of the object, and the imaginary projection on the transparent plane will differ accordingly. Before proceeding to explain in detail the method of making a Perspective Projection it is necessary to explain certain principles which constitute the basis of this type of projection.

(1) The **rays of light** which make it possible for the observer in Fig. 301 to see the object O are *straight lines.*

(2) These *rays* form a *cone of rays*, the vertex of which is at the observer's eye, and the angle at the eye of the observer (see inset sketch, Fig. 301) is called the **visual angle.**

(3) The *apparent size of the object* O depends upon the *visual angle* it subtends, *i.e.* if the observer moves to a position *farther away* from the object, the visual angle at the observer's eye is *smaller*, and consequently the image of the object as projected upon the transparent screen is smaller.

(4) From (3), above, it follows that objects of *the same size* but at *different distances from the observer* appear to him to *vary in size*.

(5) The line of intersection of the imaginary *Transparent Plane* and the *Ground Plane* is called the **Picture Line** or **Ground Line.**

(6) A straight line from the observer's eye, parallel to the *ground plane* at a height of, say, 5 ft. above the ground is called the **Axis of Vision.**

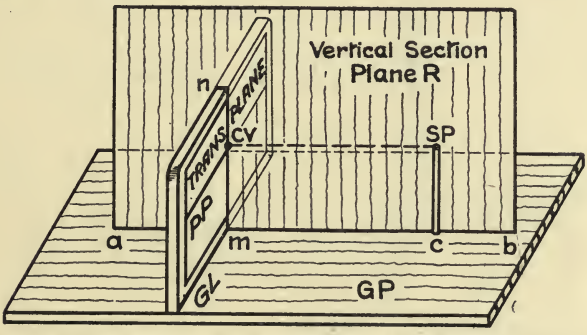


FIG. 302.—Model illustrating direct method of making a perspective projection.

(7) A straight line drawn upon the *Transparent Plane*, parallel to the *Ground Plane* and *at the same height above the Ground Plane* as the Axis of Vision is known as the **Horizontal Line.**

Direct Method of Perspective Projection.—It is not within the scope of the present work to attempt an elaborate and comprehensive study of the various methods of Perspective Projection, and for that reason the ordinary method taught in art classes and known as the **School of Art Method** is not considered in these pages, but a more **Direct Method** substituted, which, while it has certain very definite limitations, will yet serve the purpose of the practical man who has a fair knowledge of the principles of Solid Geometry. This **direct** method will be better understood by reference to the diagrammatical sketch, Fig. 302 and the sectional view, Fig. 302. The sketch, Fig. 302, represents a model illustrating some of the most important principles. GP is the *Ground*

Plane, PP is the *Transparent Plane* or *Picture Plane*; SP is a point called the *Station Point*, at the height of the observer's eye above the ground plane. A horizontal line from SP meets the P plane in a point, CV (*centre of vision*), while a line through CV parallel to the ground plane is shown in order to represent the *Horizontal Line.*

Suppose a Vertical Plane of section R to cut through the whole model and to include the axis of vision, *i.e.* the line from SP to CV. This section plane would intersect the Ground Plane along the line *ab*, and the Picture Plane along the line *mn*. The

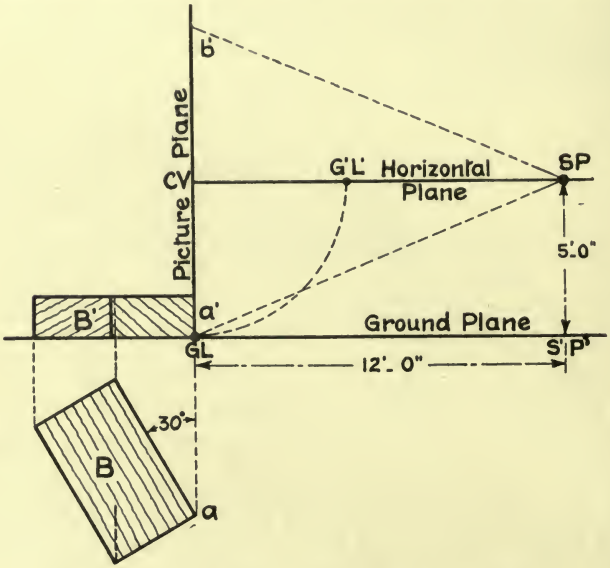


FIG. 303.—Sectional view of model illustrated in Fig. 302.

line of intersection *ab* would include the point *c*, which is the plan of the *station point* SP, while the vertical line of section *mn* would include the point CV at which the *Axis of Vision* meets the *Horizontal Line.*

Fig. 303 shows a sectional view made by the Vertical Plane of section in Fig. 302, the Ground Plane and Picture Plane being represented respectively by a horizontal and a vertical line. The observer is in this case stationed at a point 12 ft. away from the Picture Plane, and his eye is assumed to be 5 ft. above the Ground Plane. The plan of the station point SP would appear as shown at S′P′. The Axis of Vision is shown by a line from SP to CV,

while the *cone of rays* is represented by the triangle b',SP,a'. The magnitude of the *visual angle* at SP in this cone of rays depends upon the *size of the object* and the *position selected for the station point;* but it should be noted that there is a *limit to the visual angle of the human eye*, and 40° would be a reasonably large angle which should not be exceeded in arranging the magnitude of the visual angle.

A rectangular block of wood, BB′, Fig. 303, is arranged as shown, in such a position that one of its corners touches the Picture Plane at *aa′*, and one of its long edges in plan makes an angle of 30° with the Picture Plane. From the plan, B, the elevation B′ of

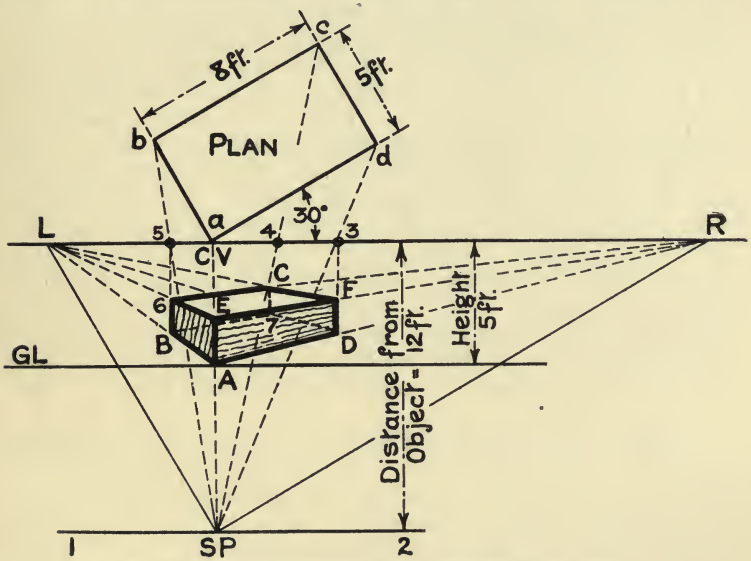


FIG. 304.—Perspective view of rectangular block of stone.

this block can be projected as shown. From the fact that one vertical edge is in contact with the Picture Plane, it follows that this edge will *show its real length* upon that plane, while the other vertical edges of the block, although *equal in length* to the one at *aa′*, will appear *shorter* in length because they are *farther removed* from the Picture Plane. A comparison of these two diagrams, Figs. 302 and 303, will help to make clear the method of arranging a Perspective Projection such as that described in the following paragraphs.

Perspective Drawing of a Rectangular Block.—A rectangular block of masonry, Fig. 304, is 8 ft. long, 5 ft. wide,

and 2 ft. thick. *Make a perspective drawing of the block on the assumption that one of the* 8 *ft.* × 5 *ft. faces of the block rests upon the ground; that one of the* 2*-ft. edges of the block is in contact with the Picture Plane; and that one of the* 8 *ft.* × 2 *ft. faces of the block is inclined to the Picture Plane at* 30°.

Draw a line LR to represent an *edge view*, or plan, of the Picture Plane *as seen from above;* and at a distance of 12 ft. (to scale) away from LR draw a line 1,2, which is to include the station point SP. Draw the plan, *abcd*, of the block to scale as shown, arranging the point *a* on the line LR, and the side *ad* at 30° to LR. At a distance of 5 ft. (to scale) from LR draw the ground line, GL. The position of this line will be made clear by reference to the sectional view, Fig. 303, where the Picture Plane is assumed to be *swung upon CV as a hinge* in such a way that the point marked GL traces a curve (a quadrant) from GL to G′L′, so that the position of GL (G′L′) is now such that it is 5 ft. away from CV and *intermediate between* CV and SP. In making the Perspective Drawing on a flat sheet of paper, the plane of the surface of the paper is assumed to be the surface of the Picture Plane after it has been swung or *rabatted* upon the Horizontal Line as a hinge. Referring again to Fig. 304, from *a* draw a line at right angles to LR to meet the line 1,2 at SP and intersecting GL at A. At A set up AE equal to the actual thickness of the block (2 ft.). *No other measurement of any sort is necessary* in preparing the Perspective Drawing; *all other lengths being determined by means of construction lines.*

Position of Vanishing Points.—In making a Perspective Drawing, those edges of the object which are known to be parallel *appear to approach nearer to one another* or *vanish* as they recede farther from the observer. In order to determine the correct amount of slope which must be given to any line such as AD or AB on the drawing, it is necessary to fix the position of two important points on the line LR as shown at L and R, which are respectively the **Left Vanishing Point** and **Right Vanishing Point.** These are obtained by setting out lines from SP to L *parallel to the edge ab on the plan* of the block, and from SP to R *parallel to the edge ad on the plan.* All lines representing edges of the block to the right of the **salient corner** AE will vanish towards R, while those representing edges to the left of AE will vanish towards L. Thus having set out the thickness of the block AE, draw lines ER and AR to the right vanishing point and AL, EL towards the left vanishing point, as shown. From the four points *a*, *b*, *c*, and *d* on the plan, draw lines converging towards SP. At the point 3 at which the line from *d* to

SP crosses the line LR, draw a projector 3D at right angles to LR to intersect the vanishing lines AR and ER at D and F. Then FD represents a vertical edge of the block which is really *equal in length* to the edge AE, but is *farther away* from the observer, and therefore *appears* shorter. Similarly, by drawing a projector from 5 at right angles to LR to meet the lines AL and EL, the position of the vertical edge B6 is determined. From 6 draw a vanishing line 6R, and from F draw a similar line FL. These lines intersect at C, which is the top corner of the block farthest away from the observer. The unseen edges of the block are shown as broken lines, B7, D7, and C7, and are

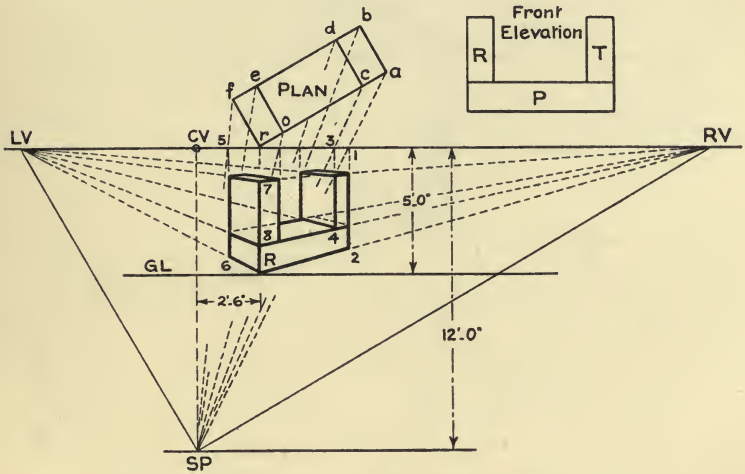


FIG. 305.—Perspective view of three rectangular blocks.

obtained by means of vanishing lines BR, DL, and a vertical line C7.

The inset sketch, Fig. 305, shows the elevation of **three rectangular blocks**, R, P, and T, the blocks R and T standing vertically upon the top face of the block P. *Make a Perspective Projection of the three blocks, assuming that they are arranged as indicated in the given plan; and that the nearest corner of the three blocks is 2 ft. 6 ins. to the right of the axis of vision, and 12 ft. away from the observer.*

Make the Plan and Elevation as shown. The latter is needed for giving the *heights* of the respective points on the Perspective Drawing. Draw the lines LV, RV, and GL, 5 ft. apart (to scale) as before, and fix the position of the line, 12 ft. away from LV, RV, on which the point SP will occur. From *r*, in plan, draw a

projector *r*R, to meet the GL, and at a distance of 2 ft. 6 ins. (to scale) from R*r* draw the line from CV to SP to represent the *axis of vision*. From SP draw lines to RV and LV, *parallel to the edges ra* and *rf* in plan, and thus fix the position of the *vanishing points* RV and LV, towards which all lines on the Perspective view, which represent *horizontal lines on the object*, must vanish. Set out on R*r* the height R7, equal to the total height of the blocks, *measured from the elevation*, and the height R8 equal to the thickness of the lower block P, and *no*

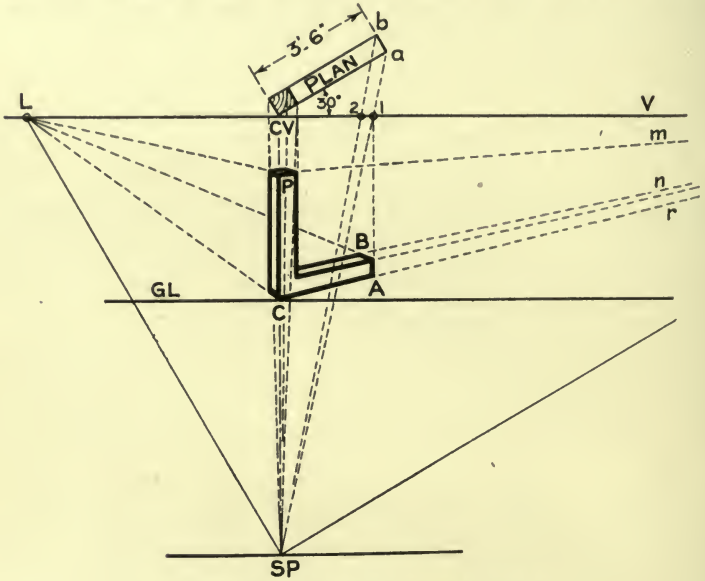


FIG. 306.—Perspective view of block letter L.

further measurements, either of heights or lengths, are necessary in making the perspective view. Draw the vanishing lines 7 to RV and 7 to LV, and 8–LV, 8–RV, and from each of the points *a*, *b*, *d*, *e*, *f*, etc., on the plan, draw lines converging to SP. By means of projectors from points 1, 3, 5, etc., determine the position of the vertical edges of the solids, noting that the projector from 3 should stop at 4 on the vanishing line 8–RV, and so on. The remainder of the working is shown clearly, and it should not be difficult to complete the drawing.

EXAMPLE.—Make a Perspective Projection of a block letter L, Fig. 306, 3 ft. 6 ins. × 3 ft. 6 ins. × 4 ins. thick, standing in a vertical position with one of its faces making an angle of 30° with the Picture

Plane as shown in the plan, the salient corner of the letter being in line with the Axis of Vision, *i.e.* directly in front of the observer.

Curved Lines in Perspective. Omission of one Vanishing Point.—A **stone slab** with semicircular head is shown in plan and elevation at (*m*), Fig. 307. *Make a Perspective View of the*

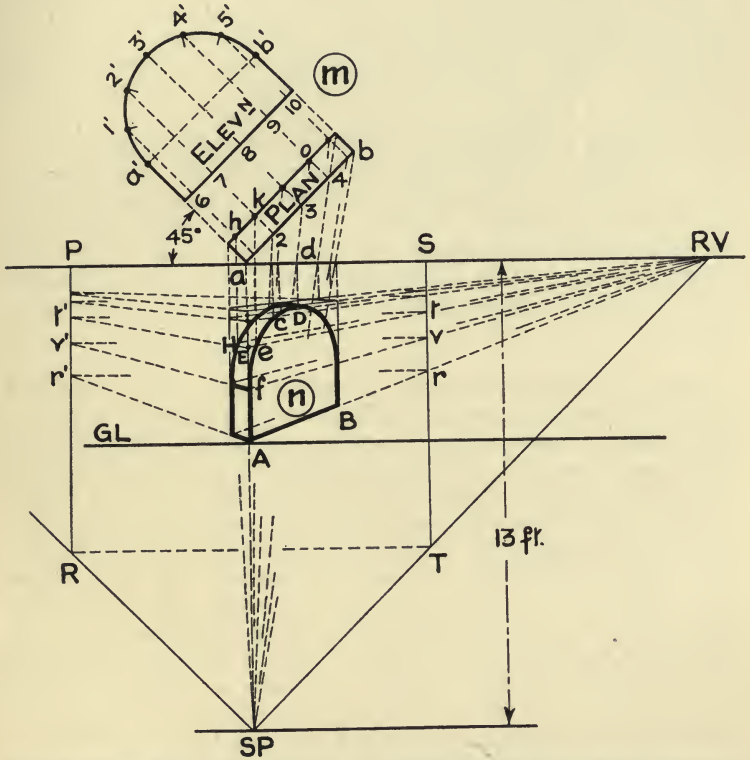


FIG. 307.—(*m*). Plan and elevation. (*n*) Perspective view of semicircular-headed stone slab showing method of working with one vanishing point.

block as shown at (*n*), *the observer being stationed at a point* 13 *ft. from the object and directly in front of it.*

This example is introduced in order to demonstrate the method of dealing with objects in which the outline is made up partly of **curved lines**, and partly with the object of showing how one or other of the **vanishing points may be dispensed with at will.** For this purpose it is assumed that the early part of the working has been completed up to the point where it becomes

necessary to fix the position of the right and left vanishing points. Assuming that the limits of the paper will not allow of the inclusion of the left VP, determine the position of the right VP (RV) in the usual way, by drawing a line from SP parallel to *ab* in plan. From SP draw a line of any length towards the left and parallel to *aa'* in plan. At any point S on the line PS draw a line ST at right angles to PS, and from T draw a line TR parallel to PS and intersecting the left vanishing line SP–R at R. From R draw RP at right angles to the ground line, *i.e.* parallel to ST. If the left vanishing line SP–R is produced to meet the line PS at a point L (not shown), the triangle PLR will be equal and similar to the triangle S,RV,T. Hence, in making the perspective drawing, the vanishing lines from A, F, etc., are drawn in the usual way, to converge towards RV. These lines intersect the line ST at points *r*, *v*, *t*, etc. From these points draw projectors parallel to GL to meet the line PR at *r*, *v*, *t*; then lines drawn towards the left-hand side from A to *r'*, *f* to *v'*, etc., will be as correctly drawn as if drawn to meet at the *Left Vanishing Point.*

Construction necessary for Curved Lines.—Divide the semicircle *a'b'* on the elevation (*m*) into any convenient number of equal parts as shown at 1', 2', 3', 4', 5', and draw projectors 1'6, 2'7, 3'8, etc., parallel to the edge of the slab, *i.e.* at right anglet to *a'b'*. The *straight line ab* is the *plan of the semicircular head* of the slab. Hence the points 2, 3, 4, etc., are the plans of the respective points 2', 3', 4', etc., on the semicircle *a'*3'*b'*. From the points *a* and *b*, and the intermediate points 2, 3, 4, etc., draw projectors, as in the previous cases, *converging towards* SP. The necessary construction for the perspective drawing of the lower or rectangular portion of the slab is shown and is obtained in the manner already indicated. In order to obtain the points *e*, *c*, D, etc., on the curved part of the perspective drawing, it is necessary to measure the lengths of the respective projectors 1'6, 2'7, etc., on the elevation (*m*), and transfer these lengths to the line A*f*, produced as shown at E, the length AE being equal to the length 1'6, and so on. From these points (E, etc.) draw vanishing lines towards RV, and obtain the position of the points *e*, *c*, etc., on these lines by means of projectors from the points, *d*, etc., at which the converging lines from 2, 3, etc., to SP intersect the horizontal line PS. Special attention must be drawn to the fact that in making the perspective projection *no measurements are permissible other than those measured along the vertical line* A*f* (*produced*). A fair curve through the points *f*, *e*, *c*, D, etc., completes the drawing of the *front face* of the slab. Points such

as H on the curve at the *back edge* of the slab are obtained by making use of the lines from points *h*, etc., on the plan of the back edge. Thus, from the point *t*, at which the vanishing line E–RV intersects the line ST, draw a projector *tt'* in order to determine the position of *t'* on PR. From the point E, on A*f* produced, draw the necessary part of the vanishing line E*t'*, and on this line find the position of the point H by making use of the line from *h* to SP. By making use of the points K, *o*, etc., on the plan, other points on the curved line at the *top back edge* of the slab may be obtained in a similar manner.

Perspective of Circular Slab.—The plan and elevation of a **circular slab** are shown at (*p*), Fig. 308, the diameter of the circular face of the slab being equal to *a'g'* and the thickness of the slab being equal to *mg*. The slab is standing with the *plane* of its circular faces *vertical*, and *inclined* to the *Picture Plane* at an angle of 30°. *Make a Perspective Projection of the slab, assuming that the curved surface of the slab at* A *touches the Picture Plane, and that the observer is stationed at a point* 12 *ft. from the object and* 2 *ft. to the left of it.*

In dealing with solids such as this, it will help very materially in the construction if, as a first step, the perspective projection of a *rectangular block* is drawn, as indicated by the fine solid lines in (*q*). This may be obtained by making use of the points *a*, *g*, *m*, and *n* on the plan, and drawing projectors from points 1, 3, etc., at which the converging lines from *g*, *m*, etc., cross the line LV–RV. The height of the slab on the perspective drawing is determined by measuring the *length of the diameter d'h'* on the *elevation* (*p*), and transferring this length to the line 8,9 on the line representing the *salient corner* of the slab in the perspective view (*q*). *No other measurement but that shown at* 8,9 *is permissible in completing the perspective view of the square slab.*

Having completed this initial part of the construction, the *rectangular slab* may be considered as being just large enough to permit of making a *circular slab* AHGD in (*q*), *i.e.* the curved lines representing the front circular face of the slab must be *tangent to the straight edges* of the square slab of the points A, H, G, and D. Other points such as 6 and 7 are obtained as follows. Divide the circle in elevation (*p*) into any convenient number of parts as shown at *b'*, *c'*, *d'*, *e'*, *f'*, and *g'*, and obtain the plans of these points as shown at *b*, *c*, *d*, etc., on *ag*. From each of the points *b*, *c*, *d*, etc., draw lines converging towards SP, and at the points where these lines cross the line LV–RV, such as 2, 4, etc., draw vertical projectors as shown at 4F, etc. On the

elevation (*p*) draw the level lines PP, RR, SS, etc., having first divided the semicircle P*h*'P into six equal parts. Referring again to the perspective drawing, the length A*r*, measured on the vertical line 8,9, is made equal to the *distance apart of the levels* PP and RR on the *elevation*. Similarly the length *r s* on the

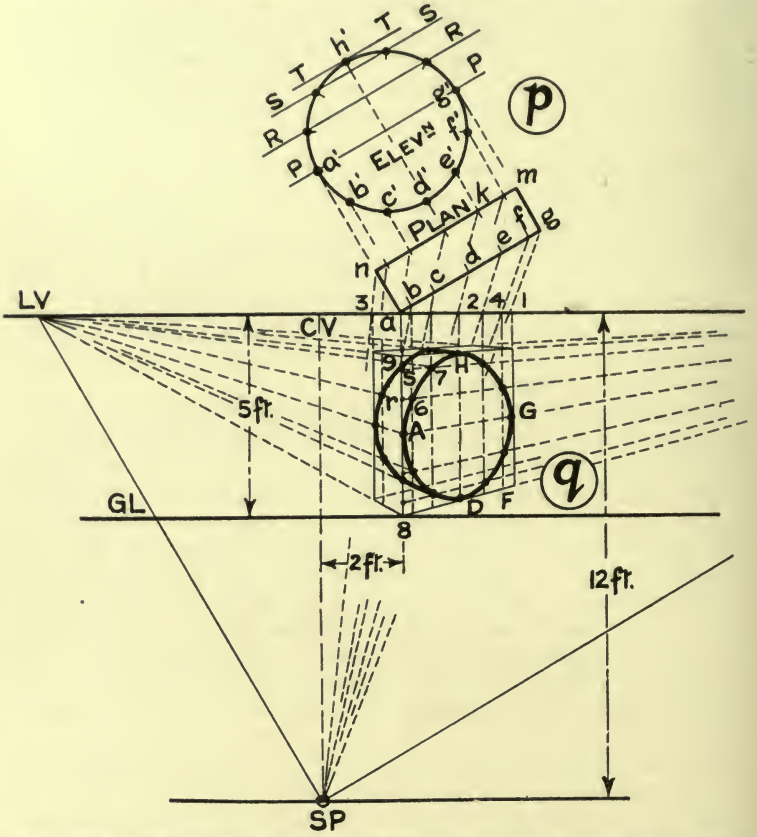


FIG. 308.—(*p*) Plan and elevation. (*q*) Perspective view of circular stone slab.

perspective drawing is equal to the distance apart of the levels RR and SS on the elevation. From the points *r*, *s*, etc., so determined draw vanishing lines towards the right VP and intersecting the respective vertical projectors 4F, etc., at points such as 6,7, etc. Other points on the curve between A and D, and between D and G are obtained in a similar manner, making use of the *distance apart of the respective levels* PP, RR, etc., and

transferring these distances to the line 8,9, measuring from A downwards towards 8. The curved line to represent the outline of the circular face which is *farthest away from the observer* may be determined by making use of the points *k*, etc., on the line *mn* in plan. (NB.—The right VP is omitted in order to save room on the page.)

A **Hollow Fire Brick** is shown in plan at (*a*) Fig. 359, and

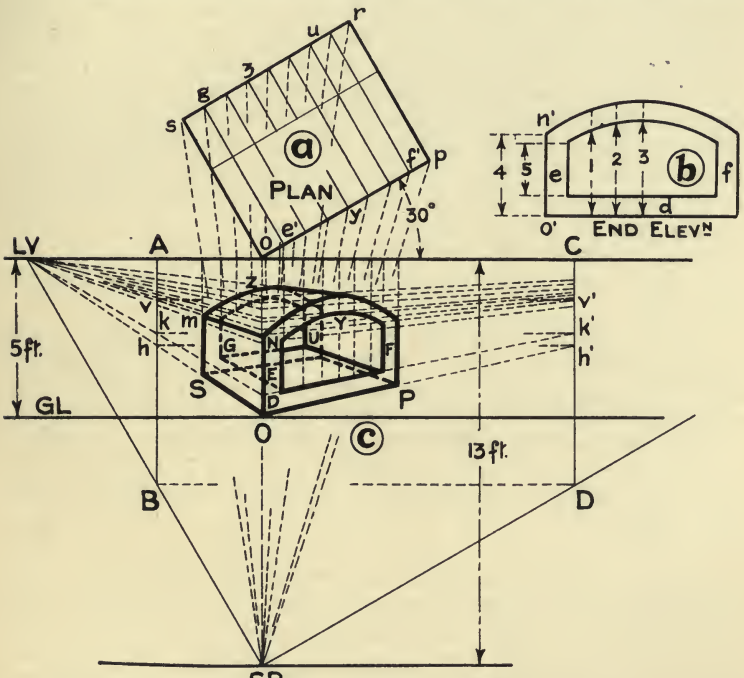


FIG. 309.—(*a*) Plan. (*b*) End elevation. (*c*) Perspective view of hollow fire brick.

an end elevation is given at (*b*). *Make a Perspective View* (*c*) *of the block, assuming that the observer is standing directly in front of it, that the nearest corner of the block touches the Picture Plane at o, and the plane of one end op is inclined at* 30° *to the* HL.

In making the Perspective Projection (*c*) only the *left* VP has been used. The *right* VP is *dispensed with* and in place of it a line CD, *equal* and *parallel to* AB, is employed, as shown, for determining the position of the points *v'h'k'*, through which to draw the vanishing lines O*h'*, D*k'*, N*v'*, etc. As before the

lines SP–D and SP–B are arranged parallel respectively to *op* and *os* on the plan (*a*).

The main part of the construction does not differ materially from that already outlined in previous problems, and it is only necessary to draw attention to one or two details which are peculiar to this case. The height of the front edge ON on the perspective view is made equal to that of *o'n'* on the elevation (*b*); the length OD, measured along ON, is made equal to the thickness of the *floor* of the block at *d* in the elevation. The thickness of the *walls* of the block at E and F *are not measured* but are determined by means of projectors from *e'* and *f'* on the plan (*a*), while the position of the back vertical edges at G and U are determined by means of projectors from *g* and *u* on the plan (*a*). The respective heights, numbered 1, 2, 3, etc., on (*b*), are transferred to the line ON in (*c*). From these points vanishing lines are drawn towards CD, and on these lines the position of points Y, etc., are found by drawing projectors from *y*, etc., on the plan (*a*). Similarly for points on the curve at the back end of the block, such as Z, the points *z*, etc., on the line *sr* in plan are used.

An intelligent application of the principles enunciated in the foregoing paragraphs should make it possible for the student to deal with any practical case which may arise. Naturally, the ambitious student will desire further information, and as a result of the preparatory work given in this connection, he will be more ready to take up the detailed study of Perspective Projection in a more comprehensive manner than the limits of this work permit.

Isometric Projection or Conventional Perspective.—To those who have studied the principles of perspective it will be apparent that while Perspective has its own peculiar uses, *it cannot possibly serve as a workshop drawing* since the projection so made is *not drawn to any scale*, the size of the drawing depending partly upon the distance at which the observer is situated from it. Hence it is quite impracticable to take off any working measurements from such a drawing. Isometric Projection, on the other hand, *is essentially a workshop drawing*, and, as will be shown, can be drawn to scale in such a manner that all the necessary measurements may be scaled off just as would be the case in an Orthographic Projection. **Isometric Projection** may be said to have been *invented* by Professor Farish of Cambridge University, who coined the word **Isometric** as being most applicable to this type of drawing. The word *Isometric* is built up from two Greek roots; *isos* (equal) and *metron* (a

measure) ; thus an **Isometric Projection** is one in which there is an **equality of measurement**. This *equality* will be better understood by reference to (*b*) Fig. 310, where the three lines *ab*, *ac*, and *ad*, meeting at *a*, represent the plans of the three edges meeting at one corner of a cube, as those edges would appear to an observer looking down from above, assuming that the cube is suspended by means of a string, one end of which is fixed to one of its corners *a*, while the other end of the string is held in the observer's hand or suspended from the ceiling, the cube meanwhile being allowed to hang freely from the string. The complete plan of such a cube would be as shown in (*a*) Fig. 310. The lengths of the lines *ab*, *ac*, *ad* in (*b*) cannot be of the *same length* as the edges of the cube, since they are the *plans* of three lines *all equally inclined to the ground plane*, but the plans of any such equal edges are all of the same length, *i.e. ab*, *ac*, and *ad*, which are the plans of three equal edges, are themselves equal.

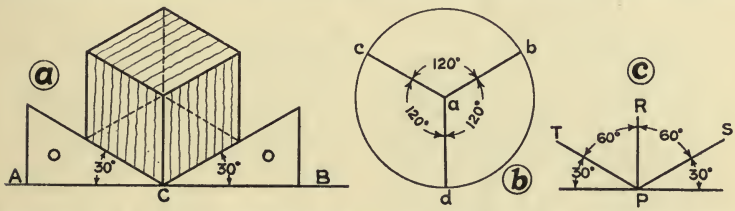


FIG. 310.—(*a*) Use of 30° set-square in isometric projection. (*b*), (*c*) Isometric axes.

The *ratio* or comparison between the **real lengths** of these edges and the **projected lengths** *ab*, *ac*, and *ad* is $\sqrt{3} : \sqrt{2}$ This ratio will be discussed more fully at a later stage. Referring again to (*b*) Fig. 310, the three lines *ab*, *ac*, and *ad* make *equal angles* of 120° as shown. It is upon this fact that the rules for isometrical drawing are based.

Isometric Axes.—The lines *ab*, etc., meeting at *a* are the main lines in the structure of an isometric drawing. Each is an **axis** indicating a *direction in which measurements must be made* when carrying out the pictorial projection of the object. In practice, however, it is found more convenient to arrange these axes as shown in (*c*), where it is assumed that the axis RP replaces the axis *ad* in (*b*). Since the angle between any two axes is 120°, RP, which bisects one such angle, will make equal angles of 60° on either side of it as shown in (*c*), and the complementary angles between PT and PS and a horizontal line are respectively 30°.

Use of 30° Angle of Set-Square.—In order to set out

accurately three lines which shall serve as *isometric axes*, the 30° angle of the set-square is employed, as shown in (*a*), where the line AB represents the edge of a T-square against which the set-square is kept in close contact during the operation of setting out the axes.

Isometric Projection of Rectangular Solids.—(*a*) Fig. 311 shows the plan and elevation of a rectangular block of wood, the length of which appears at *ab*, the width at *ac*, and the thickness at *d'e'*. If an Isometric Projection of such a block is to be made, there is a choice of three positions, as shown at (*b*), (*c*), and (*d*). Having first set out the three axes meeting at A, in (*a*) the length of the block is marked along the axis AB towards the right side of the starting point A ; the width is set out towards the left, at AC, and the thickness is marked along the vertical axis at A. In (*c*) the length is arranged along the vertical axis

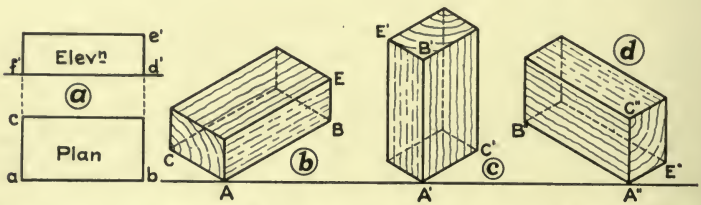


FIG. 311.—(*a*) Plan and elevation of rectangular block. (*b*, *c*, and *d*) Isometric projection of block in three different alternative positions.

A'B' ; the width along A'C', to the right ; and the thickness along the axis to the left of A'. In (*d*) the drawing is so arranged that the width is set out on the vertical axis A''C'' ; the length along A''B'' to the left ; and the thickness along A''E'', to the right. These three drawings (*a*), (*b*), (*c*) are introduced in order to show that in making an Isometric Projection of a solid *the selection of a suitable position is purely arbitrary*, and depends to a large extent upon the nature of the solid. It should be noted, however, that the choice of a suitable position will in many cases help to *do away with* the need for **dotted lines** to represent unseen edges, and thus make the finished drawing more intelligible. Having decided upon the position which is likely to give the best rendering of the object, the three principal measurements are set off along the axes, *measuring from the starting point* A. The drawing is then completed by drawing lines *parallel to one or other of the three axes*. Thus, in (*b*) the length AB and the width AC are set out, and vertical lines such as BE set up at these points to meet a line through E parallel to the

axis AB. It cannot be too clearly emphasized that in making more complicated Isometric Projections *it is not permissible to make any measurements along lines other than the axes, or lines parallel to the axes.*

(*a*) Fig. 312 shows the plan and elevation of a block of wood, one end of which is *chamfered* and the other end *stepped.* An Isometric Projection of the block is shown at (*b*). In making a drawing of an object such as this, it is always advisable to imagine the object to be *enclosed in a rectangular box*, and, first of all, make an Isometric Projection of this rectangular box. In the figure the corners of this *imaginary box* are represented by dotted lines at D, C, etc.; and all subsequent measurements are made *with reference to this rectangular outline;* thus the distance *an*, measured on the plan, is transferred to the construction line

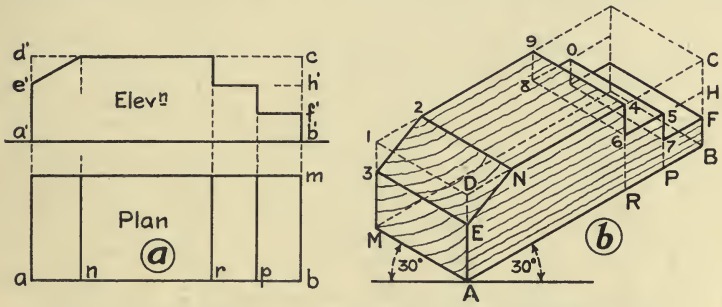


FIG. 312.—Plan, elevation, and isometric projection of chamfered block.

DC, where DN is equal to *an*, and DE = *d'e'*, taken from the elevation. Having fixed the position of the points N and E on the front face DCBA of the imaginary rectangular block, join NE and from N and E draw N2 and E3 parallel to the axis AM; *i.e.* at 30° to the horizontal and towards the left-hand side, and join 2, 3, to represent the back edge of the chamfer. On the vertical line CB make CH, HF, and FB equal respectively to *ch'*, *h'f'*, and *f'b'* on the elevation, and along the line AB, measuring from B, make BP, PR respectively equal to *bp*, *pr* on the plan (*a*). At R and P set up vertical construction lines R4, P5, and from H and F draw lines parallel to AB to intersect R4 at 6 and P5 at 7. Join 4,6; 5,7; 6,5; 7F to represent the outline of the steps on the front face of the block. From 4 draw 4,9 at 30° towards the left, *i.e.* parallel to the axis MA. The remainder of the construction can be completed by drawing lines parallel to one or other of the three isometric arcs. Thus from 9 draw 9,8 vertical, and from 6 draw 6,8 parallel to

4,9 and intersecting 9,8 at 8. From 8 draw 8,0 parallel to the axis AB, and from 5 draw 5,0 parallel to 4,9, etc., etc.

Isometric Projection of Square-based Pyramid.—(*e*) Fig. 313 shows the plan and elevation of a post of square section *abcd*, one end of the post being sawn off obliquely to produce a point as shown in the elevation at *a'v'b'*. The end of the post is an example of a square-based pyramid and will serve as an illustration of the fact that certain geometrical solids *do not appear quite as true to their characteristics when rendered in Isometric as they do when rendered in Oblique Projection.* Thus, in the diagram (*c*) ABCD is the Isometric Projection of the square *abcd* in (*e*) which represents the cross-section of the post. Join DB and AC, and at O, the intersection of the diagonals DB and AC, set up OV = *ov'* on the elevation, this being the difference in length as between the square part of the post and the pointed end. Join VA, VB, VC, and VD to represent the sharp arrises at the pointed end of the post. Note that *only two* of the *four* triangular surfaces at the pointed end appear on the Isometric Projection, viz. AVB and AVD, and these *appear to be very much distorted.* Note also that the line VC, which is hidden behind VA, represents an *arris* whose length is really the same as that represented by VA, while the very small triangles VCB and VCD each represent surfaces which are *equal in area* to AVB and AVD, which is manifestly absurd.

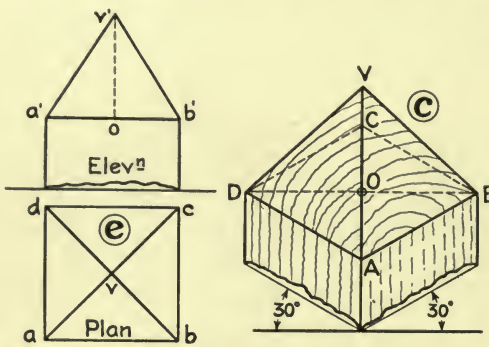


FIG. 313.—Plan, elevation, and isometric projection of post with pointed ends.

EXAMPLE.—*Make an Isometric Projection of a square stone slab 2 ft. 9 ins. × 2 ft. 9 ins. × 8 ins., surmounted by another similar slab 2 ft. 1 in. × 2 ft. 1 in. × 8 ins., and, resting upon the upper slab, a block of stone 2 ft. 6 ins. high × 1 ft. 5 ins. × 1 ft. 5 ins.*

Start with the lowest slab, setting off 2 ft. 9 ins. (to scale) along the axes to right and left and 8 ins. (scale) on the vertical axis. The second slab is 2 ft. 1 in. × 2 ft. 1 in. Thus 2 ft. 9 ins. minus 2 ft. 1 in. = 8 ins.; which leaves a *margin* of 4 ins. in

from the edges *on*, *op*, etc., of the lower slab. Make *o*1, *o*2, *p*4, and *n*3 = 4 ins. (to scale), and from these points draw lines 2*b*, 1*c*, 3*b*, 4c, etc., *parallel to the edge of the lower slab*, and thus determine the correct position for the lower corners *a*, *b*, *c*, *d* of the second slab. Having completed the drawing of this slab make the distances *r*9, 5,7, etc. = 4 ins. (to scale), and employ a similar method to determine the position of the bottom corners *f*, *d*, *e*, etc. of the top block of stone. All that then remains to be done is to make *dh* = 2 ft. 6 ins. (to scale) and complete the top of the block by drawing lines from *h parallel* to *de* and *df* on the lower end.

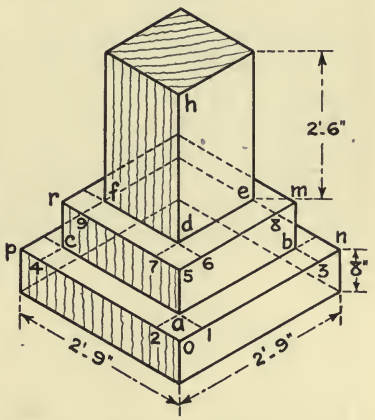


FIG. 314.—Isometric projection of two square slabs and rectangular block.

This example is typical of many cases in which it is necessary to arrange one solid on another in such a way as to show a *margin*, and the working shown may be used in most cases of this type.

Isometric Projection of Principal Rafter.—(*a*) Fig. 315 shows the plan and elevation of the joint between the Tie Beam and Principal Rafter in a wooden roof truss. *Make an Isometric*

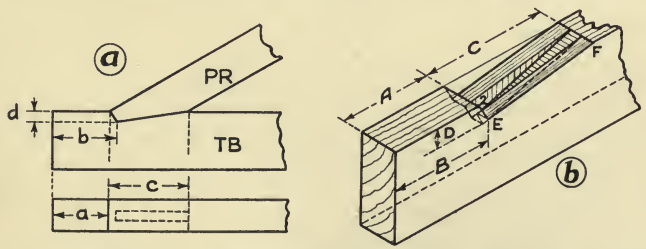


FIG. 315.—(*a*) Plan and elevation of principal rafter and tie beam. (*b*) Isometric projection of tie beam.

Projection of the part of the Tie Beam, showing the Principal Rafter removed.

The main part of the construction here does not differ materially from that already described in previous problems, *e.g.* the distance A, at which to commence the *abutment*, is obtained

from *a* in plan. (N.B.—The Isometric Projection is drawn to a somewhat larger scale than that of the Plan and Elevation.) The length of the *checked* part C is obtained from *c* in plan, while the correct slopes for this part are obtained as follows: measure the distance *b* on the elevation and transfer this distance to the Isometric Projection as shown at B, measuring along a line sloping at 30° towards the right-hand side. Measure the *vertical distance d* on the elevation and transfer this to the Isometric drawing, measuring in a *vertical direction* as shown at D. The intersection of these two lines at E gives a point to which each of the lines EF and E2 may be drawn. The remaining lines on the mortise are drawn *parallel* to EF and E2, and it should not be difficult to complete the Isometric Projection from this point.

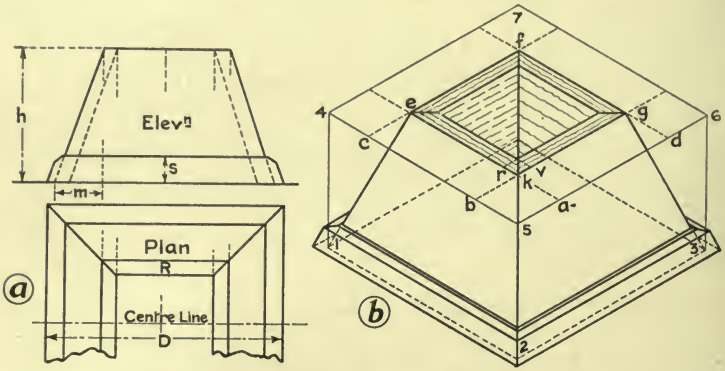


FIG. 316.—(*a*) Plan and elevation of ventilating shaft. (*b*) Isomeric projection of shaft shown in (*a*).

Frustum of Pyramid.—The **Ventilating Shaft** shown in plan and elevation at (*a*) Fig. 316, and in Isometric Projection at (*b*), is constructed in a manner similar to a *hopper*, *i.e.* it is a part of a *square-based pyramid* with the vertex of the pyramid omitted, thus making a *frustum*. Referring to the part plan (*a*), R represents the thickness of the wood from which the sides of the shaft are made, while on the elevation S is the width of a skirting board which is fitted around the lower edges of the shaft. The *height* of the shaft is shown at *h*, and the slope of the sides is shown at *m*. In making the Isometric Projection of such an object it is advisable to follow the rule already laid down and commence by making an Isometric Projection of an imaginary rectangular box to enclose the object as shown by fine lines at 1, 2, 3, 4, 5, 6, 7, in which the length and width, 2,3 and 2,1, are

made equal to the width D on plan minus the thickness of skirting board on each side, which is to be added after the main part of the Isometric of the shaft is completed. Referring to (*b*), set off along 5,6, 5,4, etc., distances 5*a*, 5*b*, 4*c*, 6*d*, etc., each equal to the distance *m* taken from the elevation. From points *a*, *b*, etc., draw lines parallel to 5,4, 5,6, etc., and intersecting at *k*, *e*, *f*, *g*. Then the lines *ke*, *ef*, *fg*, and *gk*, represent the top outer edge of the shaft. The distances *kv*, *kr*, are made equal to the thickness R on the plan, and from these points *v*, *r*, etc., lines are drawn parallel to 5,4, 5,6, etc., to show the thickness of the wood on the sides of the shaft as seen from above. Having completed the Isometric Projection of the main part of the shaft, the thickness of the skirting is measured from the elevation, and this thickness

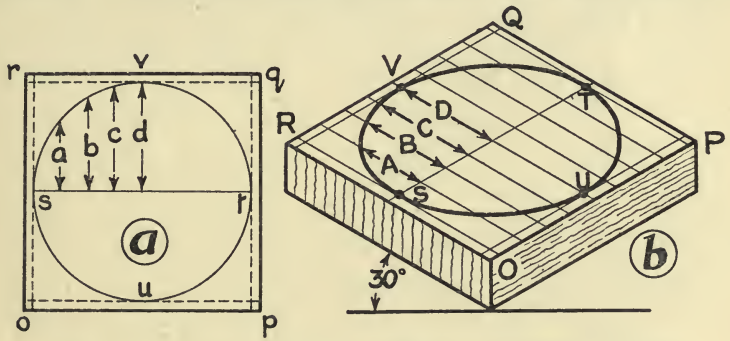


FIG. 317.—(*a*) Plan. (*b*) Isometric projection of circle on square slab.

and the width of skirting as shown at S are transferred to the Isometric Drawing, taking care to measure the distance S along a *vertical* axis of the Isometric Drawing and the thickness of the skirting along each of the arcs 2,3, 2,1.

Treatment of Curved Lines in Isometric Projection.—(*a*) Fig. 317 is the plan of a square stone slab *opqr*, on which a circle *svtu* is set out. *Make an Isometric Projection of the slab and circle.*

The square slab as shown in (*b*) is drawn as already indicated. The circle is reproduced as follows. Draw a diameter of the circle *st*, and having divided it into any convenient number of parts, set up *ordinates* as at *a*, *b*, *c*, *d*, etc. Referring to (*b*) draw the *margin lines* RO, RQ, QP, and OP, at the correct distance in from the outer edge of the square slab and from S and U. Midway between O and R, O and P, respectively, draw the *centre lines* ST, UV, parallel to the 30° axes to the left and right. Transfer from the plan to ST in (*b*) the distances already set out along

st in (*a*), and draw *ordinates* as shown at A, B, C, D *parallel* to OR. Measure the lengths of the various ordinates *a*, *b*, *c*, and *d* in plan and *transfer these lengths* to the respective ordinates A, B, C and D in (*b*). Draw a fair curve through the points thus determined to represent the Isometric Projection of this quadrant of the circle. The remaining quadrants are treated in precisely the same manner, making use in each case of the lengths of ordinates *a*, *b*, *c*, and *d* taken from the plan. Note that the *elliptical curve* SVTU, which is the Isometric Projection of the **circle** *svtu*, must be *tangential to the margin lines* at S, V, T, and R, just as the *circle* in plan is tangential to the margin lines on the slab at *s*, *v*, *t*, and *u*.

Isometric Projection of Conical Spire.—The diagram Fig. 318 is introduced as supplementary to that of the foregoing problem in order to show the method of treatment for a **cone**. Since the base of a cone is a *circle*, it is first necessary to make an isometrical drawing of the *base circle* of the conical spire as shown at 1,2,3,4. The method of setting out this part of the drawing is similar to that described in the foregoing problem, and no construction lines are shown in this case. Having completed the isometrical rendering of the circular base draw the lines 2,4, 1,3, representing two diameters of the base, and from the point 5, the intersection of these two diameters, set up a vertical line 5,6. On this line make the length 5,6 equal to the vertical height of the conical spire, and draw the lines 6,2, 6,3 carefully as *tangents to the ellipse* 1,2,3,4 as shown, thus completing the Isometric Projection of the whole cone.

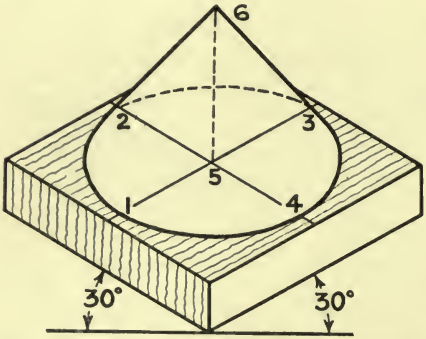


FIG. 318.—Isometric projection of conical spire.

Isometric Projection of Cylindrical Column.—The diagram, Fig. 319, shows how the construction outlined in the previous problem is varied when a **cylinder** has to be represented pictorially in Isometric Projection. The *ellipse* 1,2,3,4 is first set out, as before, to represent the *lower end* of the cylinder where it *rests upon the square slab*. Most readers of this book will be familiar with the idea of erecting a *scaffold* upon which to work while the top parts of a building are being constructed, and in any case such as this,

it is necessary to erect a *scaffold* in order to be able to complete the top portion of the drawing. Thus at the points *b*, *a*, etc., set up vertical lines *be*, *ad*, and *cf*, equal to the length of the cylinder, and draw the lines *de*, *ef*, *fg*, *gd*, to represent the top edges of a square block equal in diameter to the cylindrical column. From points, such as *h* on the margin line, erect perpendiculars *hk*, etc., meeting the top edges *ef*, etc., at points *k*, etc., and from these points draw construction lines across the top end of the square slab as shown at *km*, parallel to the axis *ed*. On these lines it is possible to find the position of points *n* and *p*, by drawing *vertical projectors* from the corresponding points *r*, etc., on the lower end of the cylinder. Having completed the *two ellipses*, it is necessary to draw *two straight lines*, *pr*, *st*, **tangential to the two ellipses** in order to complete the pictorial representation of the cylinder.

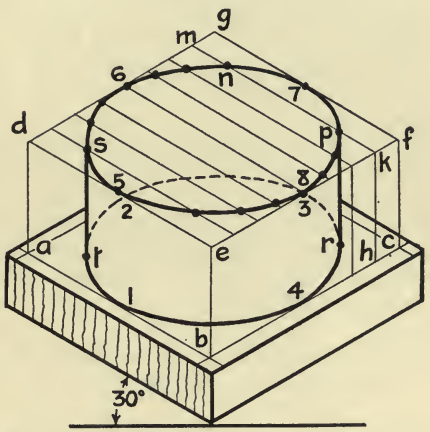


FIG. 319.—Isometric projection of cylindrical column.

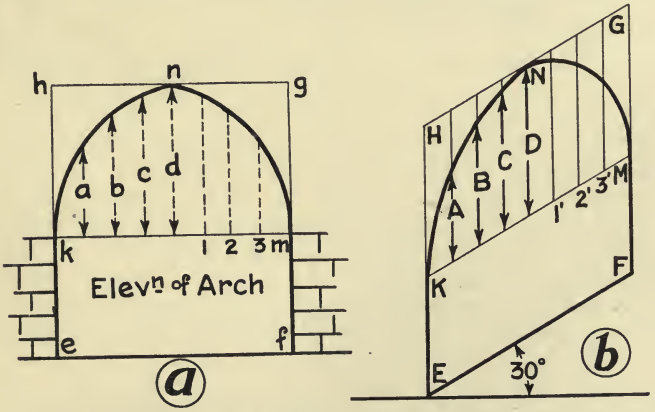


FIG. 320.—(*a*) Elevation of Gothic arch. (*b*) Isometric projection of Gothic arch.

Isometric Projection of Gothic Arch.—The front elevation of a **Gothic Arch** (*a*) is shown, Fig. 320, and the method of

arranging such a curved outline in Isometric Projection is shown clearly in (*b*). The span of the arch, *km* in (*a*), is divided into any convenient number of parts as shown at 1, 2, 3, etc., and these distances transferred to the Isometric Projection along the line KM at 1′, 2′, 3′, etc. The lengths of the respective *ordinates*, *a*, *b*, *c*, *d*, etc., on the elevation are then transferred to the corresponding ordinates in (*b*), as shown at A, B, C, and D, and a fair curve is drawn through the extremities of these ordinates. It should be carefully noted that if the two curves KN and NM are drawn to intersect at N (the centre point of the line HG), *these two curves are not exactly alike*, although they represent **two equal halves** of the outline of the arch; the curve KN being much *flatter* than the curve NM. This is a peculiarity of this type of pictorial projection with which the student should familiarize himself.

Construction of Isometric Scale.—It is not often that technical students are called upon to make use of an *Isometric Scale* in preparing a Pictorial Projection of this kind, but a word or two with reference to the matter may be of service. It has been already pointed out that in the projection of a cube suspended freely by one of its corners there is a *definite relation* between the **real lengths** of the edges of the cube and the **projected lengths** or *plans* of these edges upon the ground plane. The *ratio* of *Real Length* to *Projected Length* is $\sqrt{3} : \sqrt{2}$, and in order that an Isometric Drawing of an object may be correctly made, it is necessary, theoretically, to preserve this *ratio* as between *real lengths* and *projected lengths*. For instance, if the real length of an edge *ab* of a solid is 6 ins., the length of the line AB which, on the Isometrical Projection, represents the real length *ab* must be made shorter in the proportion of $\sqrt{3} : \sqrt{2}$. It is evident that to work out every case arithmetically would involve a considerable amount of labour; and this labour can be reduced to a minimum by constructing an **Isometric Scale**, as shown in Fig. 321. This is made as follows: Along a line *a*P set out **any unit measurement**, say 1 in., as at *b*, and at *b* erect a perpendicular *bc*, equal in length to *ab*, i.e. *one unit* long. Draw *ac*, the hypotenuse of the right-angled triangle *abc*. Then, by the property of right-angled triangles, the squares on *ab* and *bc* would each contain 1 sq. inch;

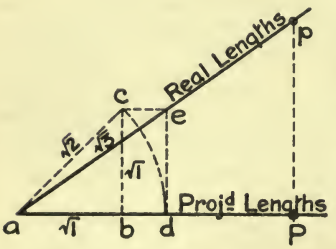


FIG. 321.—Isometric scale.

and the square on *ac* would contain 2 sq. inches. Thus *ab* and *bc* may be taken to represent $\sqrt{1}$, and *ac* to represent $\sqrt{2}$. With centre *a* and radius *ac* describe the arc *cd*. Then *ad*, which is equal to *ac*, is also $\sqrt{2}$. At *d* erect a perpendicular *de*, and make this *one unit long* by drawing the horizontal projector *ce*. Then since $ad = \sqrt{2}$, and $de = \sqrt{1}$, the *square on the hypotenuse ae* will be equal to the **sum of the squares on the two sides** *ad* and *de*, i.e. $ae = \sqrt{3}$.

To use the Isometric Scale, the lines *ae* and *ad* may be produced indefinitely. Assuming that the *real length* of a certain line on an object is equal to *ap*, this length is marked along the **Real Lengths** line from *a* to *p*, as shown. A projector, *p*P, is drawn at right angles to *a*P, in order to determine the position of the point P. Then the length *a*P on the **Projected Lengths** line is the correct length for making a line on an Isometric Drawing in order to represent correctly to scale a line whose

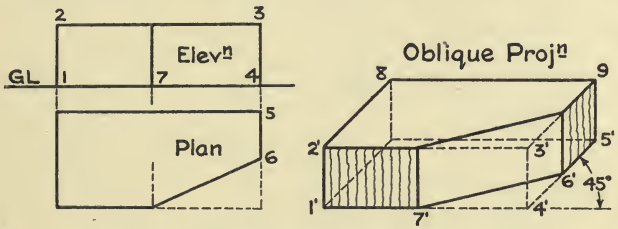


FIG. 322.—Oblique projection of brick "closer."

real length is *ap*. In practice, however, the Isometric Scale is dispensed with, and the lengths of lines on the Isometric Drawing are made *equal to their respective lengths on the object*, or equal to the scale length on the Orthographic Projection of the object.

Oblique Pictorial Projection.—This type of pictorial drawing is by far the easiest to understand and use, as the principles underlying it are very simple. These are illustrated in Fig. 322, where the plan, elevation, and oblique projection of a brick *closer* are shown. In making the oblique projection the *front elevation of the brick is reproduced at* 1′,2′,3′,4′, and from the points 1′, 2′, etc., projectors such as 4′5′ are drawn, at 45° to the horizontal line. The width of the brick is then marked along the 45° projectors as shown at 2′,8′, and 3′,9′; and the pictorial view of the back portion of the brick is completed by drawing horizontal lines 8,9, etc. The splayed part of the closer is obtained by making 1′7′ on the pictorial drawing equal to 1,7 on the elevation, and by making 5′,6′ on the 45° projector equal to the distance 5,6 on plan. By erecting perpendiculars at 6′ and 7′, and joining

points 6′, 7′, etc., the representation of the splayed portion of the closer is obtained.

Oblique Pictorial Projection of Dovetail Joint.—Fig. 323 shows the plan and elevation of a **Dovetail Halved Joint.** *Make an oblique projection of the two parts* A *and* B *of the joint to a larger scale, arranging the drawing in such a way as to avoid as far as possible the use of dotted lines to represent the unseen edges of the object.*

The piece A containing the *socket* is shown at A′, and that containing the *pin* has been arranged in a horizontal position at B′, and turned with the *face side* of the wood towards the back in order to show the shoulders and the inclined surfaces of the pin without the use of a large number of dotted lines. In A′ the front elevation A is reproduced, the distance C is made equal to *c*

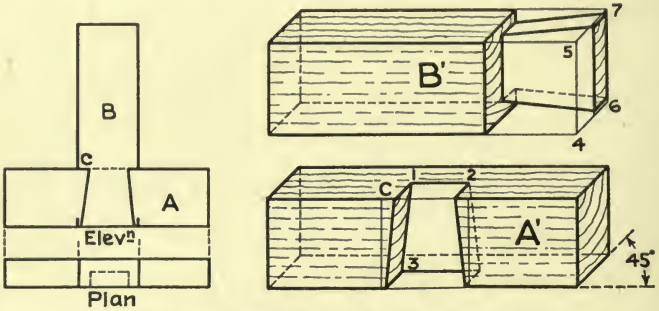


FIG. 323.—Oblique projection of dovetail halved joint.

on the front elevation in order to give the correct *splay* for the pin of the dovetail. The points 1 and 2 are obtained by means of 45° projectors from C, etc., making the distance C1 = one-half the thickness of the piece of wood A. All other lines, such as 1,2, 1,3, are parallel to lines already drawn, and may be obtained by using set-squares in the manner usually employed for drawing a series of parallel lines. The construction for drawing the pin-piece, B, is fairly obvious. It is an advantage in such cases to make an oblique projection of the *whole block from which the pin is to be cut*, as shown at 4, 5, 6, 7, and then proceed to insert lines on the drawing to represent *gauge lines* for half-thickness of wood, etc., as shown.

Diminished Oblique Projection.—Fig. 324 represents a-9 ins. **Brickwork pier** resting upon two courses of *footings* and a concrete foundation. The main object of introducing this sketch is to show the need for *diminishing the measurements* taken along

the 45° projectors in order to give a rendering which *approaches more nearly to a perspective drawing*. The *front edge* of the concrete base is made equal to the **full length** of this edge, but the length measured along *bc*, although representing a length equal to the front edge of the concrete base, is made **half the real length.** Similarly in setting out the margin lines for the first course of footings at *a*, *b*, and *c*, the distance along the *horizontal line* at *a* is made **full scale,** while the distance along the *oblique projectors* as at *b* and *c* are made to **half scale,** although the distances from the corner of the concrete to *b* and *c* are really the same in the original as that to *a*. This rule should be followed in all cases where a more correct rendering of the object in oblique projection is desired. In all such cases *measurements made along the horizontal axis are made full scale, and those along the oblique projectors are made* **one half** *their real length.*

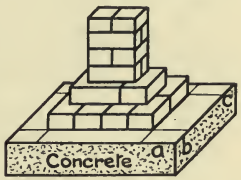


FIG. 324.—Oblique projection of brickwork pier.

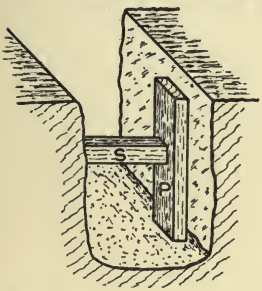


FIG. 325.—Oblique projection of timbering for excavation.

Freehand Sketches combined with Oblique Projection.—Fig. 325 is a rough oblique pictorial sketch of an **excavation** showing some of the *timbers* necessary for supporting the sides of the excavation : P being a *poling board* and S a *strut*. The student will find that for examination purposes

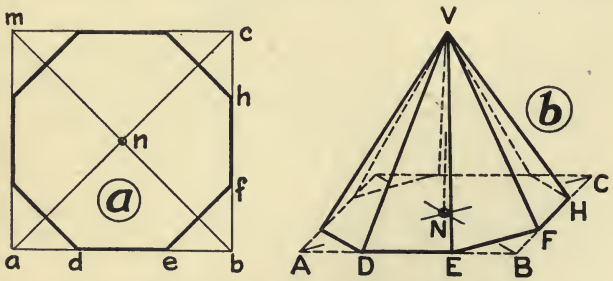


FIG. 326.—Oblique projection of of octagonal turret.

oblique pictorial drawing lends itself very well to this purpose, as sketches, partly *freehand* and partly *ruled*, may be executed

quickly and help to make details of construction clear to the examiner in a much more convincing way than orthographic drawings could possibly do.

(*a*) Fig. 326 is the plan of the base of an **octagonal turret.** The method of showing this turret in oblique pictorial projection is shown clearly in (*b*), and it is only necessary to draw attention to two points : (1) The length AB is made full scale, *i.e.* it is equal to *ab* on the plan, but the length BC is made *one half* the length of *bc* on plan. The same thing holds good with regard to the points D, E, F, etc., on the corners of the octagon, where the distances AD and BE are made **full scale,** *i.e.* equal to *ad* and *be* on the plan, while the lengths BF and CH are made **half scale,** *i.e.* one half the length of *bf* or *ch* on the plan. (2) The proper position for the vertex V is found by drawing lines such as AC to represent the diagonals of the base, and from the point of intersection N set up a *vertical* projector NV to the correct height of the turret. Join VD, VE, VF, VH, etc., to complete the pictorial drawing.

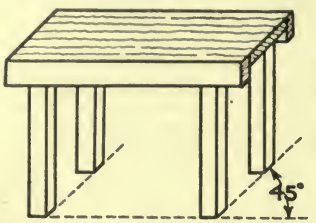


FIG. 327.—Oblique projection of work-bench.

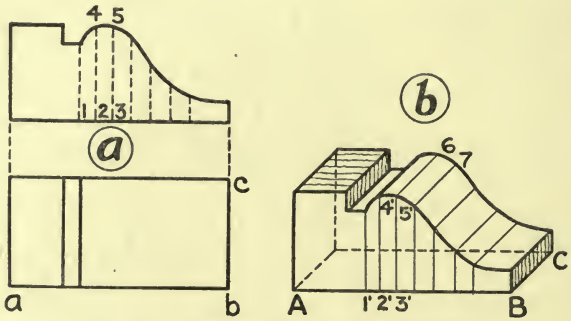


FIG. 328.—Oblique projection of stone corbel.

Fig. 327 shows a small table or workbench as it would appear in oblique pictorial projection, and does not call for any detailed explanation.

Curved Lines in Oblique Projection.—(*a*) Fig. 328 shows the plan and elevation of a **stone corbel.** The method of placing this in oblique projection is shown at (*b*), where it will be noted that the length AB, measured along the *horizontal axis,* is

made **full scale,** *i.e.* equal to *ab* in plan ; while the length BC on the *oblique projector* is made **half scale,** *i.e.* one half the length of *bc* on plan. The curved line is obtained by means of ordinates as shown, transferring the heights 2,4, 3,5 from the elevation to the respective ordinates in the oblique projection at 2′,4′, 3′,5′, etc., and drawing a curve through the points 4′, 5′, etc. Points, such as 6 and 7 on the curve of the back are obtained by means of 45° projectors from 4′ and 5′, making the distances 4′,6, 5′,7 in each case equal to the width BC of the corbel.

CHAPTER XXI

HORIZONTAL PROJECTION

Figured or Indexed Plans.—The theory of *Orthographic Projection* has already been considered at some length in Chapter XII (Solid Geometry), where it was shown that in that method of projection at least **two** Planes of Projection were necessary. In the type of projection with which this chapter deals only **one** Plane of Projection is employed, and the drawings so made are sometimes termed **figured plans**, though the term *plan* in this connection is somewhat misplaced. In Orthographic Projection the *height* of a point above the Horizontal Plane is indicated by the *distance apart* of the *elevation* of the point and the *Ground Line*. In Horizontal Projection **no Ground Line** is used, but, instead, the position of the point with reference to the plane of projection is denoted by an *index figure* placed by the side of the letter which marks the position of the point on the plane. Thus an *indexed plan* in this type of projection takes the place of plan and elevation in Orthographic Projection. The *index figure* which denotes the *height* of the point is usually placed at the *right-hand side* of the letter indicating its plan, and is written a *little lower* than the letter, thus A_{15}. If *no* sign is placed in front of the index figure, a positive sign (+) is understood, and this sign indicates that the number of units indicated by the index figure are units of height *above* the plane of projection. On the other hand, a minus sign (—) indicates that the position of the point is *below* the plane of projection. Thus the figured plan of a point *a* which appears as a_5 indicates that the actual point is immediately *over* its figured plan and 5 units **above** the plane of projection; while an indexed plan such as b_{-12} indicates that the actual point B is 12 units *below* the plane and exactly **underneath** its figured plan.

Fig. 329 shows the figured plan of a wire rope A_0B_{40}. The indexed plan A and B are 70 units apart. *Find* (a) *the real length of the rope, and* (b) *the inclination of the rope to the ground.*

At B set out B*b* at right angles to AB making it 40 units long to any convenient scale, and join A*b*. Then A*b* represents the **true length** of the rope and the *shaded angle* at (*a*) is the **inclination of the rope** to the ground.

Choice of a Suitable Unit.—N.B. The *unit* employed in indexed plans is often taken as ·1 in. ($\frac{1}{10}$ in.); but the figured plans may be indexed in *feet*, or *inches*, or in fact **any unit of linear measurement.**

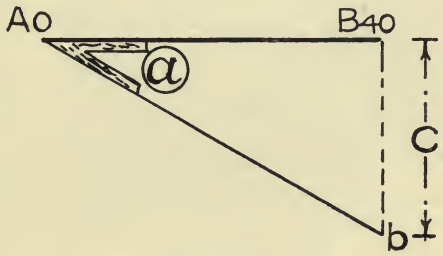


FIG. 329.—Figured plan of rope AB.

Gradients or Slopes.—In Fig. 330, 0,10 represents a barrow run 36 ft. long, the index figures indicating that the left-hand end of the barrow run rests upon the ground, *i.e.* level 0, while the other end of the run is 10 ft. above the ground. *What is the* **gradient** *of the barrow run?*

Draw a line *ca* parallel to 0,10, and at *a* set out *ab* at right angles to *ca*, making the distance *ab* = 10 units on the same scale as that to which 0,10 measures 36 units, and join *cb*. Then the shaded angle near *c* indicates the *gradient* or slope of the barrow run. In this connection the sketch, Fig. 331, may be interesting. **Indicator boards**, such as the one shown, are fixed at intervals alongside the railway tracks on the various lines, and indicate the character of the gradient in both directions, *i.e.* to right and left of the gradient post. The one illustrated in the diagram indicates that the track to the *left* is **level** while that to the *right* **slopes downwards**, the degree of slope being indicated by the *ratio* of the numbers 1 and 30, *i.e.* if a level line AB 30 ft. long is laid down the railway track will slope from the point A *on* the level line to the point B, 1 ft. *below* the level line, and at a **level** distance of 30 ft. away from A. Such a slope is classed as a *gradient of* 1 *in* 30. On main roads a gradient of 1 in 30 presents no difficulty to a cyclist, but a gradient of 1 in 20 is a difficult hill to climb, and a gradient of 1 in 10 is practically unrideable.

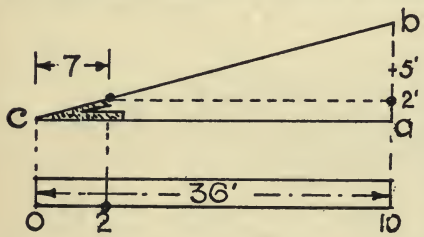


FIG. 330.—Indexed plan of plank for barrow run.

Referring again to Fig. 330, the point 2′ on *ab* is at a distance of 2 ft. (to scale) above *a*. Draw a level line from 2′ parallel to *ca*, and from the intersection of this line and the line *cb* draw a projector to determine the position of 2 in plan. Measure the distance 0,2. It measures 7 units, thus showing that the gradient in this case is 2 in 7 or 1 in 3·5.

Scales of Slope.—The position of an inclined plane with reference to a given horizontal plane may be conveniently represented by means of two parallel lines as shown at (*a*) and (*b*) in Fig. 332. The two parallel lines, one thick and one thin, indicate the plan of the *steepest path* up the plane in each case, while the short lines at 0, 5, 10, etc., are short *sample lengths* of the respective *levels*, which may be produced indefinitely in either direction to suit the convenience of the draughtsman.

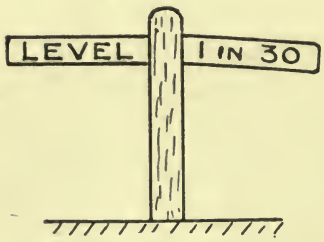
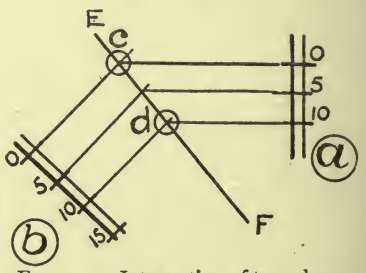


FIG. 331.—Indicator Board.



FIG, 332.—Intersection of two planes.

(*a*) and (*b*) Fig. 332 are the *scales of slope* representing two planes. *Determine the line of intersection of the two planes.*

Select *any two* corresponding levels on the scales of slope of the respective planes and produce them until they intersect. Thus the level lines at 0 in (*a*) and 0 in (*b*), if produced, meet at *c*, and the level lines at 10 in (*a*) and (*b*), if produced, meet at *d*. Then the point *c* which is *common* to the level 0 on *both* planes must be a *point on the intersection* of the two planes.

Similarly *d*, which is on the level 10 of *both planes*, is also a *point on the intersection*, and a straight line EF drawn through any two such points, *c* and *d*, represents the **plan of the intersection**, the index of *c* being 0, *i.e.* c_0, and that of *d* being 10, *i.e.* d_{10}.

Scales of Slope for Roof Surfaces.—In Fig. 333 A and B are two roof surfaces which intersect in a *hip*. The inclination of the roof surface A is shown at (*a*), and that of B is shown at (*b*). *Determine a scale of slope for each plane, and state which roof surface has the greater slope.*

Draw any two lines 1,2 and 5,6 on (*a*), and corresponding

lines 3,4, 7,8 on (*b*), the height 1,2 in (*a*) being equal to that of 3,4 in (*b*); while the height of 5,6 in (*a*) is equal to that of 7,8 in (*b*). Produce these lines in each case to meet on the hip at *d* and *e*, and draw a double line at E and D to indicate the scales of slope. The *intersection* of the lines 2,*d*, 6,*e*, and that of 4,*d*, 8,*e* with the double lines at D and E gives the scale of slope for the respective planes.

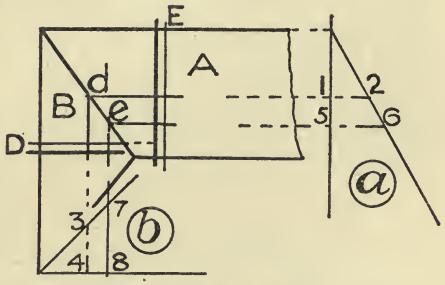


FIG. 333.—Roof surfaces, indicated by scales of slope.

The levels *d*,1 and *d*,3 are at the same height above the ground, as also are the levels *e*,5 and *e*,7. Hence, the fact that these *common levels occur closer together* on the scale of slope D than they do on that at E indicate that D is a **steeper** slope than E, or stated alternatively the *gradient* of D is **greater** than that of E.

Choice of Suitable Working Levels.—Fig. 334 is the indexed plan of a rod $A_{25}B_{38}$, the index figures of which represent the respective heights of the two ends of the rod above the ground. *Determine the height of the point* C *which is intermediate between* A *and* B.

In such a case there is no need to set out a projector 25 units long at *a* and 38 units long at *b*. Taking an *imaginary level* at a height of 25 units (*i.e.* the height of A) above the ground as a *working level*, the distance *bb'* is made equal to the **difference in level** of A and B, *i.e.* 38 − 25 = 13 **units**. Join *ab'*, and from C draw a projector *cc'*. This length, measured to the same scale as that used for *bb'*, is 8 units. Then, since the *working level* in this case has been chosen at a height of 25 units, the height of C above the ground is found by adding 8 units to the height of the working level line, *i.e.* 25 + 8 = **33 units**. The indexed plan of C should therefore appear as C_{33}.

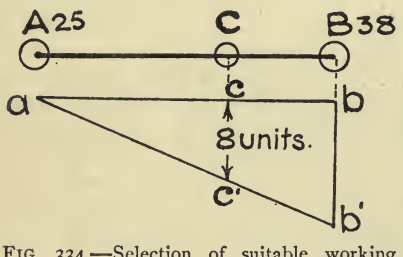


FIG. 334.—Selection of suitable working level.

Contour Lines.—In map drawing and survey work it is very convenient to have some method of representing on a *plane surface* (the map or plan) the *relative height* of various points shown on the map of the district or plan of the estate. This is done by means of **contour lines**, or *lines of contour*, which are the plans of a series of horizontal sections taken at equal distances from one another. Such a method is illustrated in Fig. 335, where the elliptical outlines in the lower half of the diagram are the *contour lines* of a hilly piece of country. These lines represent the *plans of imaginary horizontal sections* of the hill taken at levels 0, 50, 100, 150, etc., these index figures representing *feet*.

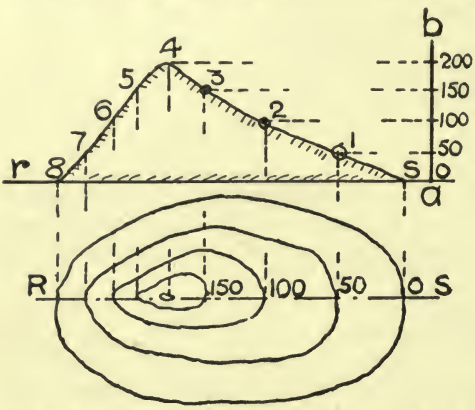


FIG. 335.—Contour lines of a hill and section of RS.

Determine the true shape of a vertical section of the hill on the line RS.

Set up a vertical line *ab* and draw a horizontal line *rs*. On *ab*, to any convenient scale, set out equal distances, 0,50, 50,100, etc., to represent heights. From the point at which the line of section RS intersects the contour line 0, at each end of the plan draw vertical projectors, to determine the position of 8 and *s* in the sectional view. From the points at which RS crosses the contour lines 50, 100, etc., draw vertical projectors, and from levels 50, 100, etc., on *ab* draw horizontal lines to meet these projectors at 1, 2, 3, 4 . . . 7. Through these points draw a fair curve 8,4,*s*, to represent the outline of the section. From the sectional view it is evident that the *gradient* at the *left*-hand side of the hill is greater than that at the *right*.

A little experience will help the observant student to discover by looking at a set of contour lines of a district, which parts have the steepest gradients and which the least steep ; *e.g.* the fact that the contour lines between R and 150 occur *closer together* than they are between 0 and 150 shows that R is the *steepest* side of the hill.

CHAPTER XXII

GRAPHIC STATICS

Statics is that branch of the **Science of Mechanics** which treats of the **Forces** which keep *bodies at rest or in equilibrium.* The reason for the inclusion of this branch of science in a text-book devoted to the study of **Geometry,** is that all the solutions of statical problems which immediately follow are solved **graphically,** *i.e.* by means of *lines.* Thus it follows that the solution of such problems is possible for any expert draughtsman even though his knowledge of mathematics may be very limited.

A *Force* may be defined as *that which moves, or tends to move, a body at rest; or which changes, or tends to change, the direction or rate of motion of a body already moving.* Thus the *weight* of an upper floor supported by a *column* is an example of a **force** acting upon the column. What is usually termed the *weight* of any object is in reality the result of an *attractive force, i.e.* **gravitation,** by which the Earth acts upon the particles of which the particular object is composed. A close study of this branch of the subject is of immense importance to the daughtsman and designer, since by its aid he is enabled to determine accurately the amount of force which is being exerted in any member of a simple or more complex structure, as, for instance, a roof truss or lattice girder, and is then able so to design the various members that there shall be sufficient material to give *adequate strength,* without having an *excess of material* such as would increase both the *cost* and the *weight* of any particular member to an undue extent.

Stress and Strain.—These terms are often used indiscriminately without any clear conception of their respective meanings. Referring again to the illustration already chosen, *i.e.* a *column* which supports an upper floor. The weight of the floor is a *weight* or **force** which, acting upon the material of which the column is made, *tends to change its shape and size.* This *force* is the **stress,** while the *change in shape* or size of the material in

the column is the **strain**. If the weight of the floor is, say, 10 tons, there is a *compression stress* of this magnitude acting along the length of the column: the result of this is that the fibres of the wood, or the particles of steel (according as the column is of timber or otherwise), may be *crushed*, and as a result the *length* of the column *shortened*. The *tendency to crush*, *i.e.* to alter its shape, is the **strain** in such a case.

As a converse example, consider the *tie rod* or *tie beam* in a *framed roof truss*. The tendency in such a case is for the feet of the principal rafters to *spread*, or move outwards, and thus to exert a **tensile** or **pulling stress** on the tie rod or beam. The result of this *stress* is that there is a tendency for the particles of steel in the tie rod to become *elongated*, and eventually to break away from each other, just as a chain would give way under such conditions if one of its links was faulty.

Parallelogram of Forces—*Def.: If two forces, acting at a point such as the joint between two members of a roof truss, be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented by the diagonal of the parallelogram which passes through the point.*

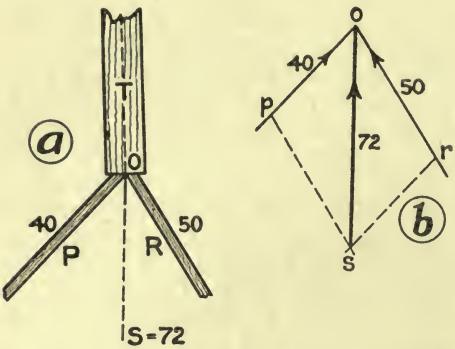


FIG. 336.—Parallelogram of forces (*b*) as applied to struts P and R in (*a*).

Components and Resultant. — In (*a*) Fig. 336 a plank T is being held in a vertical position by two struts, P and R. The youths who are holding up the struts P and R are exerting a force of 40 lbs. and 50 lbs. respectively. *Determine the position of a line*, OS, *indicating the direction along which a man must push in order to replace the two youths, and determine how much force he must exert along this line.* In this case the forces exerted by the youths are the **components**, and the man's force is the **resultant.**

For the purpose of solving such a problem, a **Parallelogram of Forces** is employed, as shown in (*b*), in which the lines *op* and *or* are drawn respectively *parallel* to the struts OP and OR. Choosing any suitable scale, *op* (one of the *components*) is made

to represent 40 lbs. and *or*, the other component, = 50 lbs. (N.B.—The unit chosen for such measurements is purely arbitrary, and depends upon the limits of the paper and the scales available, but it will usually be found that a scale of millimetres and centimetres is a very useful scale for such a purpose, as the results can be scaled off in such a way as to give a whole number and a decimal part.) Referring again to (*b*), draw *ps* parallel to *or* and *rs* parallel to *op*, thus *completing a parallelogram*, *opsr*. Draw the diagonal *os* and measure it, using the *same scale* as that already used for setting out the lengths *op* and *or*. The length of this diagonal represents a force, the *resultant*, the magnitude of which is 72 lbs. This is the *amount* of force which the man must exert, and the line OS in (*a*), which

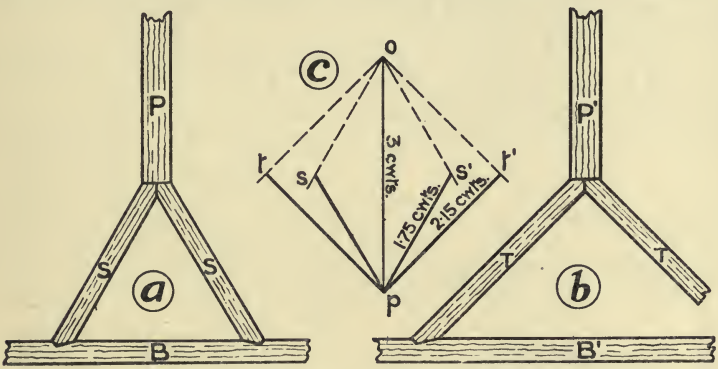


FIG. 337.—Resolution of forces (*c*) as applied to struts in (*a*) and (*b*).

is drawn parallel to *os* in (*b*), indicates the *line of action* along which the man's force must be exerted, while the arrow mark on *os*, pointing towards *o*, indicates the *sense*, or direction, towards which the force must act.

Resolution of Forces.—(*a*) Fig. 337 shows an upright wooden member P supported by two struts S, S, while (*b*) represents a similar member P′ supported by two struts T, T. The struts in (*a*) are more nearly upright than those in (*b*). *Assuming that the weight of P is the same in each case* (3 *cwts.*), *determine the relative amount of stress in the struts in* (*b*) *as compared with* (*a*).

This is the converse of the last problem. In that case the two *components* were known, and a **resultant** had to be determined. In this case the *resultant* is known, *i.e.* the weight of the piece P, and the two **components**, SS, or TT, have to be found.

In other words, a force of 3 cwts. acting along P must be **resolved** along *two lines of action*, *i.e.* the struts. This is a practical example of what is known as the **resolution of forces.**

For purposes of solution a *parallelogram of forces* is required, but the order of construction must be reversed. The weight of the piece P is a force of 3 cwts. acting in a *vertical* direction. Thus, set out *op* in (*c*) parallel to P, and to any suitable scale make *op* = 3 cwts. From *o* draw *os* and *os'* parallel to the struts SS in (*a*), and from *p* draw lines *ps*, *ps'*, to *complete the parallelogram osps'*. Then the **sides** *ps* and *ps'* of this parallelogram represent the required pair of **components** for the **resultant** *op*. Measuring these *components* to the same scale as that used for *op*, they show a magnitude of 1·75, which indicates that the pressure upon *each* of the struts S and S, as a result of the vertical weight (3 cwts.) of P, is 1·75 cwts.

Referring again to (*c*), and using the same resultant *op* = 3 cwts., draw *ot* and *ot'* parallel to the struts T, T in (*b*), and from *p* draw *pt* and *pt'* parallel respectively to *ot'* and *ot*, and thus complete the parallelogram. Then *pt* and *pt'* are the *components* for a **resultant** force of 3 cwts. when *resolved* along the *lines of action* TT. The lengths of *pt* and *pt'*, to scale, represent forces of 2·15 cwts., thus indicating that the struts in (*b*) are subjected to a *greater compressive stress* than those in (*a*), although the *weight* of P is the *same* in each case.

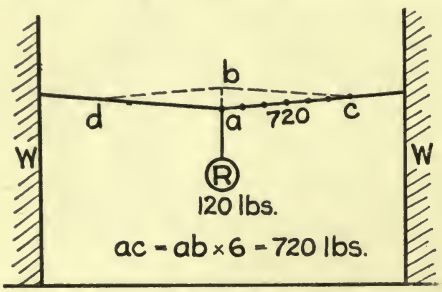


FIG. 338.—Parallelogram of forces applied to clothes line.

Fig. 338 is a very instructive case, which shows clearly that each of the components may, under certain conditions, be of very *much greater magnitude* than the **resultant.**

A **clothes line** is fixed between two walls W, and *sags* slightly at its centre, *a*. The line has been tested, and found to be capable of bearing a maximum tensile stress of 350 lbs. A youth, who weighs 120 lbs., is supporting his whole weight at R, and is surprised to find that the line breaks, although it is known to have stood a *tensile stress* of 350 lbs., which is much *greater* than his weight. *Why is this?*

Draw *ab* in line with the force R, and make *ab* to some *small* scale = 120 lbs., and complete the parallelogram by drawing *bc* and *bd* parallel to the two parts of the clothes line. Then *ac*

and *ad* are the *components* for a **resultant** the magnitude of which is represented by *ab*. Measuring *ac* and *ad* to the same scale as that used for *ab*, they are each found to represent a force of 720 lbs., which goes to prove that, although the weight of the youth is only 120 lbs., this force of 120 lbs., when *resolved* along two lines of action which *meet at a very obtuse angle* as at *dac*, may give as a result two *components*, each of which is **several times greater than the Resultant.**

Experimental Verification of Principle of Parallelogram of Forces.—Fig. 339 illustrates a simple and easily constructed piece of apparatus which may be used to illustrate this important principle. Two small pulleys *a* and *b* are screwed upon a piece of board as shown. Strings are passed over the pulleys and a weight of 6 lbs. is suspended from another string o*e*. In order to keep the angle *aob* between the two branches of the string constant, it is necessary to suspend weights of 5·4 lbs. and 4·8 lbs. at *c* and *d* respectively. While the strings and weights are in the position shown, draw a line o6 on the board in line with the string o*e*, and mark *six equal spaces* along this line. From the point 6 draw lines as shown, parallel to the strings *a*o and *b*o respectively, and thus complete the parallelogram, of which o6 is the diagonal through the point o at which the *resultant* of 6 lbs. acts. The *components* on o*b* and o*a* will be found to measure to scale, 4·8 units and 5·4 units respectively.

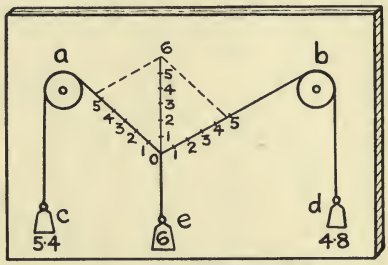


FIG. 339.—Model illustrating parallelogram of forces.

Triangle of Forces.—*Definition.—If three forces acting at a point are in equilibrium they can be represented in magnitude and direction by the three sides of a triangle taken in order.*

The diagram at (*a*) Fig. 340 shows the side elevation of a **platform** P which is supported at one end by the wall A, the other end being suspended from the wall by means of a **chain** C. A **weight** W of 3 tons hangs at the outer end of the platform as shown. *Find the pressure on the platform and the pull in the chain.*

Consider first the effect which the *weight* W has upon the *platform* and *chain*. It would have a tendency to crush the *platform* and **stretch** the *chain*. W is a force acting along a *vertical* line and in a *downward* direction as indicated by the

arrow mark. If P is to resist the *crushing* effect of the weight W it must *push outwards* from the wall, *i.e.* in an *opposite direction* to that in which the crushing force of W is felt. This outward *push* of the platform is the **reaction** of P and its *sense* is indicated by an arrow mark. Similarly if the *chain*, C, is to resist the tendency of the vertical force W to *elongate its links* or the particles of steel of which the links are made, it must *pull upwards* towards the wall as indicated by the arrow mark which is pointing upwards to show that the *sense* of the **reaction** in the chain is upwards, *i.e. away from* the platform. In order to determine the **compression stress** in the *platform* and the **tension stress** in the *chain* a triangle of forces (*b*) is drawn as follows. Draw a line *ab* parallel to the vertical force W, and to any convenient scale make *ab* 3 units to represent a magnitude of 3 tons. This is the only force (of the three in question) with regard to which all the data is complete; the other two are only *partly* known. In order to discover the missing data with reference to these two forces, draw lines *bc* parallel to P and *ac* parallel to C, starting from *a* and *b*, the two ends of the line representing the force which is *completely* specified. The intersection, *c*, of these two lines determines the respective lengths of *bc* and *ac*, which to the scale already chosen for *ab* represent magnitudes of 1·75 and 3·5 tons, thus showing that the *compression stress* in the **platform** is 1·75 tons and the *tension stress* in the **chain** is 3·5 tons. Note that the arrow marks on the respective sides of the triangle of forces (*b*) must follow around in a *circuit*, *i.e.* from *a* to *b*, *b* to *c*, and *c* to *a*. Positive and negative signs are sometimes used to indicate compression and tension stresses on diagrams of roof trusses, etc., which include a large number of members.

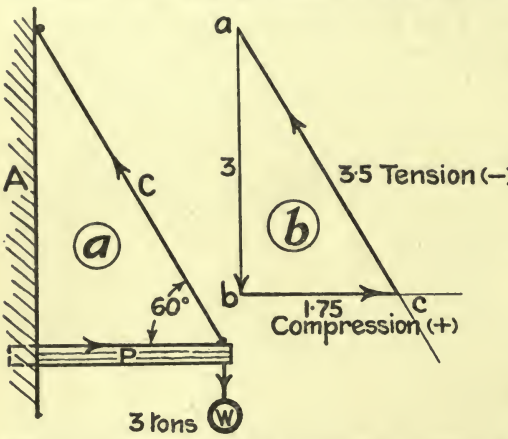


FIG. 340.—Triangle of forces as applied to a platform P and chair C.

Fig. 341 shows a variation of the foregoing example. A **jointed iron framework** AB, BC is fixed to a wall as shown,

and a **weight** of 7 cwts. is being hoisted up by means of a pulley at B. *What effect has this load upon the iron bars* AB *and* BC *respectively?*

As in the previous problem, the weight of 7 cwts. is a *force* acting in a *vertical direction.* In this case, however, the *inclined member* BC takes the place of the *platform* in the last case and is subjected to a *compression stress*, the *reaction* to which is indicated by the arrow mark on BC. AB, on the other hand, is subject to a *tensional stress*, and the reaction along AB is indicated. In order to determine the magnitude of the respective stresses a line 07 is drawn, parallel to the vertical force of 7 cwts. acting at B, and is made to represent 7 units on any suitable scale. The lines 7*n* and o*n* are then drawn parallel to BC and AB respectively, commencing to draw the line 7*n* at one end of the force 07 which is completely known, and finishing the line o*n* at o which is the other end of the force 0·7. Note that the arrow marks on the respective sides of the triangle of forces must **follow around circuitally** as they will do, if made to correspond with the arrow marks indicating the sense of the reactions in AB and BC respectively. In this case the **compression stress** in the member BC is 5·6 cwts., and the **tension stress** in AB = 3·9 cwts.

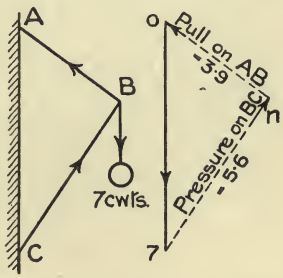


FIG. 341.—Triangle of forces applied to jointed iron framework.

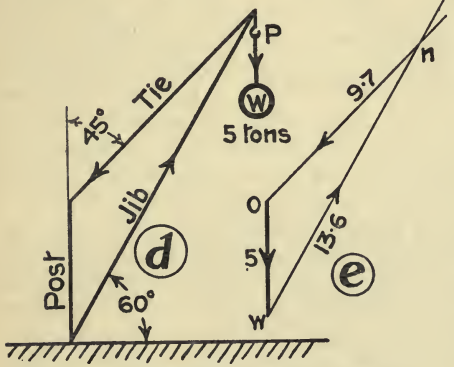


FIG. 342.—Triangle of forces (*e*) for crane (*d*).

Fig. 342 (*d*) illustrates a simple triangular framework—*e.g.* a **crane** in which there is no winding gear—the weight W of 5 tons being suspended as shown, or hoisted by means of ropes and tackle suspended from a hook at P. Determine the *compression stress* in the *Jib* and the *tension stress* in the *Tie.*

Referring to the triangle of forces (*e*) make o*w* = 5 units of any suitable length, and draw *wn* and o*n* parallel to the Jib and

Tie respectively. Measure *wn* and *on* to the same scale as that used for *ow*; *on* = 9·7 units and *wn* = 13·6 units, and these figures represent the magnitude, in tons, of the respective stresses in the Tie and Jib.

Fig. 343 (*a*) is a somewhat more complicated case along the same lines as the foregoing problem. A weight of 8 tons is suspended at C, but this weight is to be hoisted by means of a *crab* or *winch* at B, the rope passing over a pulley at A. *Find the stresses in the Jib and Tie.*

In order that the weight of 8 tons may be held in position it is necessary to exert an *equal amount of force, i.e.* 8 tons, along the rope AB, after it has passed over the pulley at A. The first step in the working of such a problem is therefore to determine by means of a *parallelogram of forces* (*b*) what is the **resultant** of the *two components* of 8 tons each acting along AB and AC. Draw *ab* and *ac* on (*b*) parallel to AB and AC respectively, and make each equal to 8 tons, to any convenient scale.

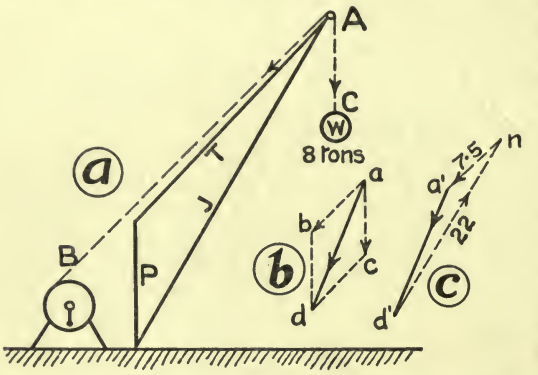


FIG. 343.—Triangle of forces (*c*) and parallelogram of forces (*b*) as applied to crane *a*.

Complete the parallelogram *abdc*. Then the *diagonal*, *ad*, of this parallelogram is the resultant of the two *components* of 8 tons each. Using this length as the first side of the *triangle of forces* (*c*) make *a'd'* = *ad* in (*b*) and complete the triangle of forces by drawing *d'n* parallel to the Jib (J) and *a'n* parallel to the Tie (T). The lengths of these **sides** of the triangle represent magnitudes of 22 tons and 7·5 tons respectively for the Jib and Tie. This case is interesting as showing how *slight* is the stress on the *Tie* as compared with that in the **Jib**. This is explained to some extent by the fact that the *rope* AB acts in a manner similar to a Tie in helping to support the weight at C.

(A), (B) and (C) illustrate three simple roof trusses, (A) and (B) having no Tie Beam. *Assuming that a weight of* 3 *tons acts vertically downwards at the ridge as shown, find the stresses in the various members of the truss.*

The *triangles of forces* are shown at (*a*), (*b*) and (*c*). In *each*

case the line 03 is made to represent 3 tons, and lines op, $3p$ in (*a*) are drawn parallel to the *principal rafters* of the roof truss. The sides $3p$ and op are equal in length, and, measured to the same scale as that used for 03, represent 3 tons each, which is the magnitude of the *compression stress* in each of the equally inclined rafters.

(*b*) is the *triangle of forces* for (B), in which $3r$ and or are drawn *parallel to the rafters* which are inclined at 60°, the *weight* at the ridge being 3 tons as before. In this case, however, the sides or, $3r$, measured to scale, represent a force of 1·75 tons, which is the magnitude of the *compression stress* in each of the rafters. From this it should be apparent that the *more nearly the slope approaches*

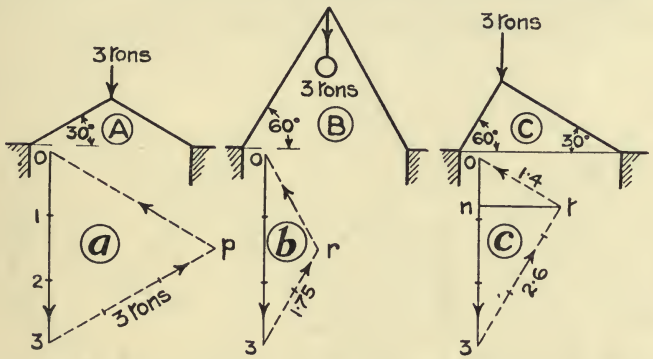


FIG. 344.—Triangle of forces (*a*), (*b*), and (*c*) for roof trusses (A), (B), and (C).

to a true vertical line the **smaller is the compression stress on the rafters,** even with the *same loading* in each case.

In C the two principal rafters are *not equally inclined*, and from what has been said with regard to B it should be obvious that we must expect the *stress* in the rafter inclined at 30° to be *much greater* than that in the one inclined at 60°. This is shown on the triangle of forces (*c*), where the side $3t = 2{\cdot}6$ tons, *i.e.* the *compression stress* in the rafter inclined at 30°; and the side $ot = 1{\cdot}4$ tons, *i.e.* the *stress* in the rafter inclined at 60°. The line nt on this triangle, which is drawn from the point t and parallel to the Tie Beam, measures the magnitude of the *Tensile Stress* in the *Tie Beam.*

(A) and (B), Fig. 345, represent a block of stone, the weight of which is 12 cwts., being hoisted by means of a rope or buckling chain which is passed under the block. *Assuming that the buckling chain has been tested, and will only stand a limited amount of stress, in which of the cases,* A *and* B, *is it most liable to fracture?*

(*a*) and (*b*) are the *triangle of forces* respectively for (A) and (B). From a study and comparison of these two triangles the student should be able to deduce the fact that the buckling chain in (A) is subjected to a *tensile stress* of 12 cwts., while in (B) it is subjected to a stress of *much less magnitude*, viz. 8·6 cwts.; hence it is *more liable to fracture* when arranged **tightly** as at A, than it is when **slung loosely** around the block as shown at B, the weight of the block in each of the cases (A) and (B) being identical.

Coplanar Forces.—In the various problems which immediately follow, the lines of action of the forces in each case are all in the *same plane*, and are thus classed as **Coplanar Forces.**

Polygon of Forces.—The principle involved in the *Polygon of Forces* may be stated as follows: *If two or more forces act at a*

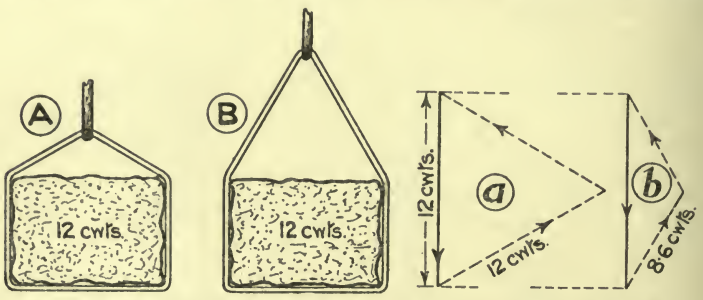


FIG. 345.—Triangles of forces (*a*) and (*b*) as applied to chain in A and B.

point, thus constituting a **System of Forces,** *the Resultant of the system may be found by constructing a* **Polygon,** the *sides* of which are *parallel to the respective forces in the given system;* the **resultant** of the system is represented in *magnitude* and *direction* by the **closing line** of this Polygon.

In Fig. 346 four ropes B, C, D, and E are attached to a **ring** A, and *forces* of 30, 50, 70, and 60 lbs. respectively are acting along the lines BA, CA, etc.

Determine by means of a Polygon of Forces (1) *Whether there is any tendency for the ring to move away from its initial position, and if so, in what direction.* (2) *The magnitude and line of action of a single force which can be introduced in such a way as to prevent the ring from moving.*

Concurrent Systems of Forces.—The four forces as shown act virtually at a *point*, the *centre of the ring.* Where the lines of action of the respective forces meet at a point as in this case, the forces constitute a **Concurrent System of Forces.** In cases

where **all** the lines of action of the forces *do not meet at a point*, the forces constitute a **Non-concurrent** System. Examples of the latter type will be found in the problems which follow this. Referring to the *Force Polygon*, Fig. 346, starting at the point numbered 1, and to any convenient scale, draw 1,2 *parallel to the force* of 50 lbs., lettered C in the system; draw 2,3 parallel to the force of 70 lbs. (D); 3,4 parallel to force E (60); and 4,5 parallel to force B (30). Draw the **closing line**, 1,5, of the polygon. This line, *measured to the same scale* as that used for the sides of the polygon represents a *magnitude* of 25 lbs. Its *sense* is always away from the point at which the Polygon was *started*, *i.e.* 1, and towards the point at which the Polygon was *completed*, *i.e.* 5.

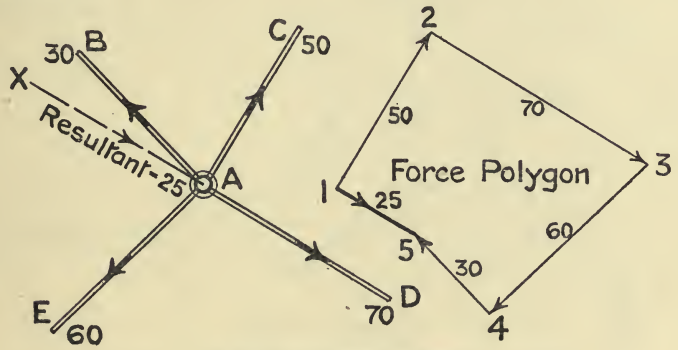


FIG. 346.—Polygon of forces.

Through the *centre point* of the ring draw a line AX *parallel to the closing line*, *i.e.* 1,5, on the Force Polygon and indicate its *sense* by means of an arrow mark as shown. Then AX is the **line of action** of the **Resultant** of the given system of forces, and this *Resultant* will have the *same effect* as the *whole system* if a force of 25 lbs. is exerted *along the line of action* AX.

In answer to part (1) of the question, the fact that the points 1 and 5 on the force polygon *do not coincide* proves that there *is* a tendency on the part of the system of forces to **move the ring** away from its initial position. In order to illustrate this point, assume that when making the force polygon the last side 4,5 when drawn and measured to scale, stood in such a position that the point 5 occurred *exactly at the same place* as the point 1. In such a case it would be impossible to draw the *closing line* of the polygon; or stated in another way, the *closing line has no length*. When such a case occurs it is proof that the **Resultant of the System is Zero.**

In answer to part (2) of the question, assuming that the respective forces of the system are operating, as already indicated, and it is required to add an *extra force* to the system, which shall be of such a *magnitude* and *direction* as to *prevent the ring from being moved*. In order to secure this end, a force of 25 lbs., *i.e.* **of the same magnitude as the Resultant**, must be introduced into the system. It must act along the line AX, or along this line produced beyond A ; and its *sense* would be indicated by an arrow mark placed in the *reverse position* to that indicated on the diagram. A force such as this, which exactly *balances* or **equilibrates** the remainder of the system and so keeps the whole system *at rest*, is spoken of as the **Equilibrant** of the system.

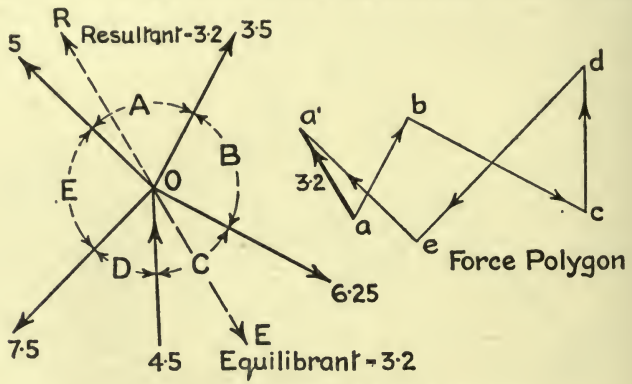


FIG. 347.—Polygon of forces for a concurrent system.

Bow's Notation.—In the more complicated examples of this type great confusion arises unless some systematic method of lettering the respective forces is adopted. In **Bow's Notation**, or *method of lettering*, instead of placing a letter near to or on the line of action of each force as in the previous problem, the **angular spaces** between the forces are lettered. This method, as will be seen, eliminates any danger of mistakes arising as a result of faulty lettering.

In Fig. 347 five forces act at a point O, thus constituting a *concurrent* system. The *angular spaces* between the forces are lettered as shown at A, B, C, D, and E. Then the force of 3·5 lbs.—the line of action of which separates the angular spaces A and B—is described as the force AB. Similarly that acting between the angular spaces B and C is described as the force BC, and so on. Before commencing to draw the Force Polygon, note that the force CD is unlike each of the other forces in that its *sense* is towards the point O, while the sense of each of the other

forces is *away from* O, as indicated by the arrow mark in each case.

Commencing the force polygon at *a*, draw *ab* parallel to the force AB and make it to scale = 3·5. In the same way make *bc* parallel to BC and = 6·25; *cd* parallel to CD and = 4·5, noting particularly that the *sense* is in a northerly direction. Following the sense marks on the remaining forces DE and EA, note that the side *de* of the force polygon crosses the side *bc*, and the side *ea'* crosses *ab*. Note also that the same letter, *a*, occurs *twice* in the force polygon, the point at which the polygon is commenced being lettered *a* and the finishing point *a'*. If this method of lettering the force polygon is adhered to systematically in all cases, there can be no misapprehension as to the *sense* of the Resultant, since this will always be indicated on the *closing line* of the polygon by an arrow mark pointing from *a*—the *starting point* towards *a'*—the finishing point. The closing line in this case measures 3·2, thus indicating that the *magnitude* of the Resultant is 3·2 lbs. On the diagram of the system of forces, draw a line EOR to represent the line of action of the Resultant. If a force of 3·2 lbs. is exerted along this line of action OR, and having its sense outwards from O towards R, this force—the **Resultant**—would have the *same effect* in moving the body, O, away from its initial position as the *whole system* of forces would have. On the other hand, if a force of **Equal magnitude**, *i.e.* 3·2 lbs., was exerted along the line of action OE, and having its *sense* outwards from O and towards E, as indicated, this force would serve as an **Equilibrant**, *i.e.* it would exactly *balance* or *equilibrate* the whole system and *prevent the body at* O *from being moved away from its initial position.*

Non-concurrent Systems of Forces. Funicular Polygon.—Fig. 348 illustrates a system in which the lines of action of the respective forces **do not all meet at the same point.** *Determine the Resultant of the system.*

Letter the angular spaces as before by *Bow's notation*, and draw the sides of the *force polygon* parallel to the respective forces. Draw the **closing line** *aa'* of the polygon. This measures 4·1 lbs., and its *sense* is from *a* towards *a'*, but the *data is incomplete*, as only the *sense* and *magnitude* of the Resultant are known. In the simpler *concurrent* system of forces the *line of action* of the Resultant was known also, since it must act *through the common point of intersection of all the forces of the system.* In a **non-concurrent** system, there is *no* common point of intersection for the lines of action of the respective forces, and in order to determine the line of action of

the Resultant, an additional construction is necessary, which is known as a **Funicular Polygon** (*funis—a thread*).

Select any point O within the force polygon as a *pole*. The position of this point is arbitrary, and experience will show what is the best position for it in any particular case. From O draw the *rays oa, ob . . . . od'* to the respective points on the force polygon. Commencing at *any point* in the angular space A of the system draw a *thread n*1, parallel to the *ray oa* in the force polygon. Similarly, draw the *thread* 1,2 from the point of intersection of *n*1 and the force AB (*i.e.* the point 1) parallel to the *ray ob* on the force polygon, and continue the construction in a similar manner by drawing the *thread* 2,3 parallel to *oc*;

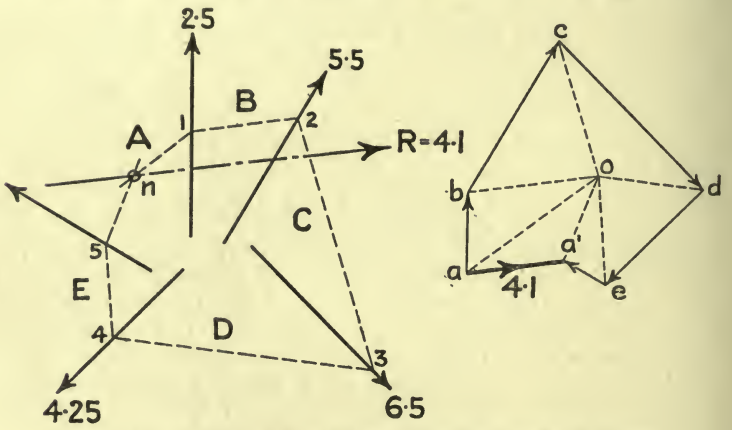


FIG. 348.—Polygon of forces for non-concurrent system of forces.

3,4 parallel to *od*; 4,5 parallel to *oe*; and 5*n* parallel to *oa'*. Thus in the angular space A *two* threads, *n*1, and 5*n*, have been drawn. The *first thread*, *n*1, intersects the *last thread*, 5*n*, at the point *n*. This is a point **on the line of action of the Resultant.** Through *n* draw the line *n*R parallel to the closing line, *aa'*, of the force polygon. This is the *line of action* of the Resultant of the system, its *sense* being identical with that of *aa'* of the force polygon, *i.e.* from *n* towards R, and its *magnitude*, as found from the length of the closing line *aa'* of the force polygon, is 4·1 lbs.

Application of the Principle of the Polygon of Forces to Loaded Beams.—Fig. 349 shows a **beam** *supported* at each end (the supports being 38 ft. apart) and carrying a *concentrated load* of 4 tons at a point 26 ft. away from the left-hand end of the beam. *Determine the* **reaction** *at each of the supports* 1 *and* 2.

Draw a **load line** *ab parallel to the line of action* of the vertical force AB, *i.e.* 4 tons, acting between the spaces A and B. Select any pole, O, and draw the *rays oa, ob.* Commencing at any point, 3, on the line of reaction at the left-hand support draw a *thread* 3,4 parallel to the *ray oa.* Note that this *thread* passes across the space A, between the line of reaction 1,3 and the line of action of the force 6,4. From 4 draw 4,5 parallel to *ob,* and from 5, at which this *thread* intersects the line of reaction 2,5, draw the closing line 5,3 of the *funicular polygon.* Starting from *o* on the *force polygon* draw a ray *on* parallel to 5,3, the closing line of the funicular polygon. Then the part *an* of the load line, *measured to the same scale* as that used for *ab,* represents the amount of pressure at the *left* support, viz. 1 ton 6 cwts., while the length *nb* on the load line represents the amount of pressure

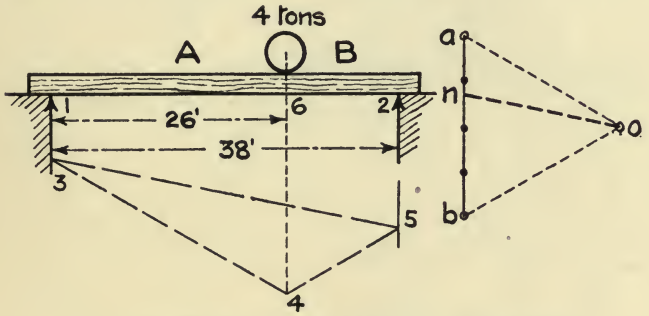


FIG. 349.—Polygon of forces as applied to loaded beam.

at the *right* support, viz. 2 tons 14 cwts. The supports at 1 and 2, therefore, must *push up* in order to withstand the *crushing* effect of the weight carried, and this *push up* or **Reaction** must be 1 ton 6 cwts. **vertically** along 3,1, and 2 tons 14 cwts. vertically along 5,2.

Fig. 350 shows a **beam** supported at both ends and carrying loads of 3 tons, 4 tons, and 2 tons, as shown. *Determine* (1) *the reaction at each support, and* (2) *the line of action and the magnitude of a single force* (i.e. *a concentrated load*) *which would have the same effect as the given loading.*

Draw a vertical line *ad* to serve as a *load line,* and draw vertical lines to represent the lines of action of the respective forces AB, BC, and CD as shown at 2, 3, and 4. On the *load line* make *ab* (to scale) = 3 tons; *bc* = 4 tons; and *cd* = 2 tons. Select a *pole o,* and draw rays *oa, ob, oc,* and *od.* From any point 1 on the *line of reaction* at the left support, and in the space A, draw 1,2 parallel to *oa*; similarly in the space B draw 2,3

parallel to *ob*; in C draw 3,4 parallel to *oc*; and in D draw 4,5 parallel to *od*. Note that if this method is carefully followed, the *same* letter *occurs twice* in each case: *e.g.* the correct *thread* for the space **D** is that parallel to *o*d, and so on. Draw the closing line 1,5 of the *funicular polygon*, and on the *force polygon* draw *on* parallel to 1,5. Then **a***n*, the *intercept* on the load line, measures the magnitude of the reaction at the support nearest **A**, *i.e.* the *left* side, and **d***n* measures the magnitude of the reaction at the support nearest **D**, *i.e.* the *right* side.

Produce the *first thread* (1,2) and the *last thread* (4,5) of the funicular polygon to meet at 6. This point is *on the line of action* of the **Resultant** of the system, and R6 is the line of action of the Resultant, its **point of application** being at S.

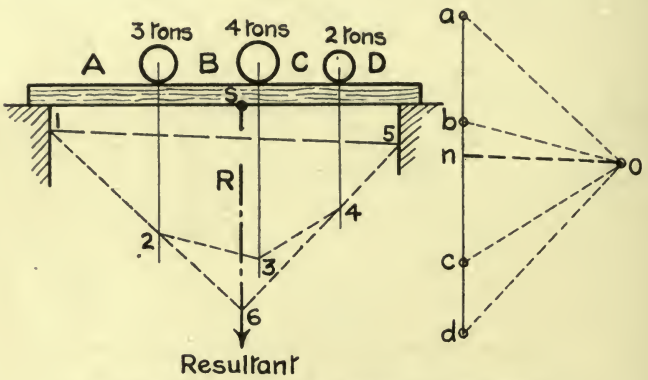


FIG. 350.—Polygon of forces applied to loaded beam.

Fig. 351 represents the **revolving platform** of a crane, the platform being supported at *one point* D. The *platform* carries a **crane**, the weight of which is 3 tons, the *maximum load* to be raised by the crane being 8 tons. The *winding gear* of the crane weighs 1½ tons, and the *engine* E weighs 5 tons. *Find the minimum weight which must be added to the chain box* (*marked?*) *in order to balance or equilibrate the remaining forces which act upon the platform.*

Draw the *lines of action* of the respective forces as shown. The line of action of the chain box, engine, and winding gear are represented by drawing vertical lines through the centre of each. The weight of the crane, 3 tons, may be taken as being a vertical force of this magnitude, acting at *c*, the **centroid** or *centre of gravity* of the framework of the crane. The position of *c* is determined by bisecting the angles between the members of the

framework of the crane. The weight of 8 tons to be lifted by the crane is a vertical force of this magnitude and its line of action is represented by the line RP. Letter the spaces between the forces as in the preceding problems; thus, reading from right to left of the platform, PR = 8 tons, RS = 3 tons, ST = 1½ tons, TV = 5 tons, leaving one force VW partly known, *i.e.* not known except as regards its line of action. Draw a load line *pw* and set out to scale, *pr* = PR, *rs* = RS, *st* = ST, *tv* = TV. Since the magnitude of the force VW is not known it is not possible at this stage to mark the position of *w* on the load line. Select any pole, *o*, and draw the *rays op*, *or* . . . *ov*, leaving *ow* to be filled

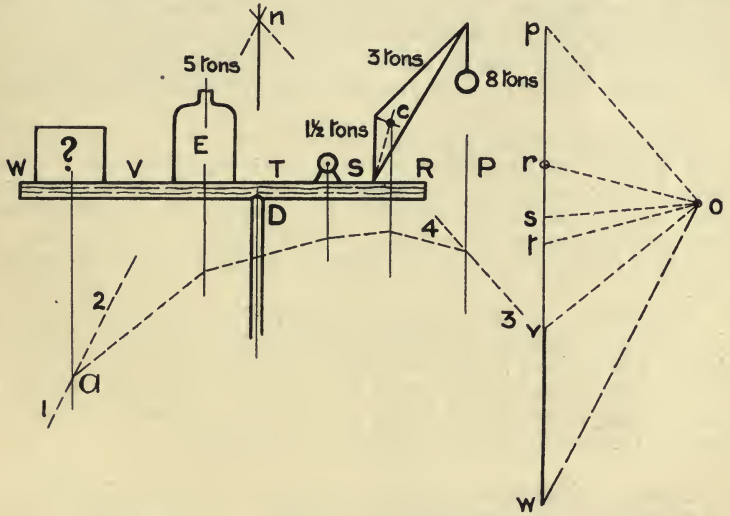


FIG. 351.—Polygon of forces for revolving platform of crane.

in later when the position of *w* on the load line is determined. Commencing at the right-hand side of the 8 ton force PR, draw a *thread* 3,4 parallel to the ray *op*, and continue with the remaining *threads* of the funicular polygon until the *thread* in the space V crosses the line of action of the force VW at *a*. The thread 1,2 is filled in as follows. In the previous problem it was noted that the *intersection of the first and last threads* was a **point in the line of action of the Resultant** of the system. Thus, if the positions of first and last threads of the funicular polygon are known, the line of action of the resultant can be determined by *producing the first and last threads until they meet.* In this case, the given data is different, viz. the *first thread* 3,4 is known, the *line of action* of the Resultant is known, since it must pass through the point D

on which the platform rotates. Produce the *first thread* 3,4 to meet the line of action of the Resultant at *n*. Then the *last* thread 1,2 must also pass through this point, *n*, if produced, and must also pass through the point *a* at which the thread in the space V crosses the line of action of the unknown force VW. Hence, if the line *n*2 is drawn and produced towards the point 1, the position of this thread is as shown at 1,2. Starting from *o* on the force polygon draw the ray *ow*, parallel to the thread 1,2, the position of which has now been located. Then *vw* on the load line represents the magnitude of the force VW, *i.e.* the weight which must be added to the chain box in order to **equilibrate** the system. *vw* measured to scale = 11 tons 4 cwts. It is evident, however, that, as this weight, if placed in the chain box, is only just enough to *balance* the system, *extra weight must be added* to this amount (11 tons 4 cwts.) in order to afford a **margin of safety**.

EXAMPLE 1.—Work the given case, taking the following data in place of that given: weight of crane $4\frac{1}{2}$ tons; maximum weight to be lifted 8 tons; weight of winding-gear 2 tons; weight of engine 4·75 tons. Determine the amount of loading necessary in the chain box, the position of the point of support being as shown (D), and the position of each of the weights on the platform being the same as in the given case.

EXAMPLE 2.—Taking the data with regard to loading exactly as given in the diagram, find by how much the weight necessary in the chain box would vary: (*a*) if the point of support, D, was placed further to the right, *i.e.* nearer the weight to be lifted by the crane; (*b*) if the point of support was placed further to the left, *i.e.* nearer the point (the chain box) at which the extra weight is to be added.

Stress Diagrams for Roof Trusses.—Fig. 352 shows a method of applying the foregoing principles to the determination of the stresses in the respective members of a **roof truss** (*a*) made up of *principal rafters*, *king post* or *king rod*, *struts*, and *tie beam* or rod. The loading of the roof due to slates, boards, etc., is taken as being **concentrated loads** of 3 tons *at each support*, and 7 tons at the *ridge* and at the junction of the upper ends of the *struts* and *principal rafters*, as indicated on the diagram. *Find the stress in each member of the truss.*

Commence by lettering the forces in such a way that each member of the truss has a letter on each side of it. Thus the strut at the left-hand side is denoted by the letters A and B placed on either side of it, while that at the right-hand side is lettered C and D. Similarly PA denotes the lower end of the principal rafter at the left-hand side, and RB denotes the upper end of the same member.

Stresses at Joint between Principal Rafter and Tie Beam. —The complete stress diagram is shown at (*b*), but in order to make the various steps in its construction clearer, separate units of the complete diagram are shown at (*c*), (*d*), (*e*) and (*f*). Referring to (*b*) the load line *ov* is obtained by making *op* = OP, *i.e.* 3 tons, to any convenient scale ; and to the same scale, make *pr* = PR, *i.e.* 7 tons ; *rs* = RS, *i.e.* 7 tons ; *st* = ST, *i.e.* 7 tons ; and *tv* = TV, *i.e.* 3 tons. The whole loading of the truss is thus represented by the total length of load line W, and since the

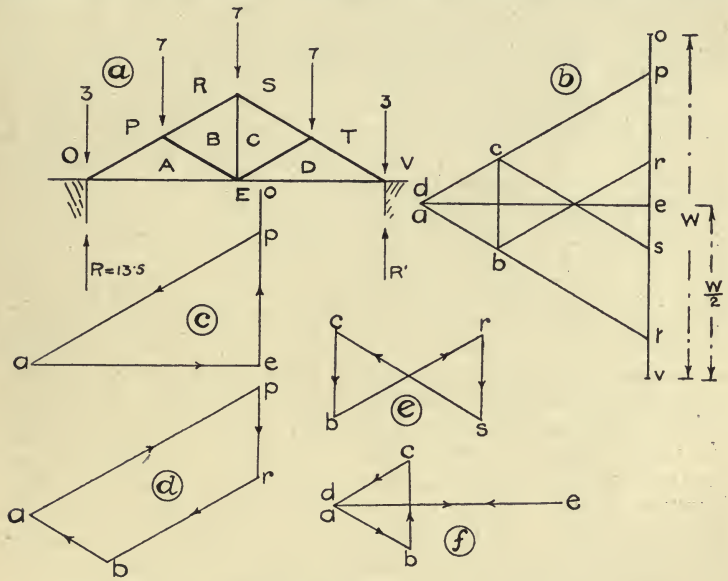


FIG. 352.—Stress diagrams (*b*) to (*f*) for roof truss (*a*).

loading is *symmetrical* on each side of the king post, the weight carried at each support is represented by the length $\frac{W}{2}$ = *oe* or *ve* on the load line. Thus the **reactions** at the supports, R and R′, will each be 13·5 tons.

The unit diagram (*c*) is the stress diagram for the joint at the foot of the principal rafter at the left-hand side of the truss. In this diagram *op* and *pe* are equal in length to *op* and *pe* in (*b*), *i.e.* *op* = 3 tons, and the total length *oe* = 13·5 tons, this being the *reaction* R at the left support. *Four* forces act at this point, viz. OP, downwards, which is known *completely ;* PA, the **principal rafter**, which is only *partly* known ; AE, the **tie rod**, with regard

to which the data is also incomplete; and the **upward force** OE, *i.e.* the **reaction** at the left support, which is *completely* known. The object of the stress diagram (*c*) is to determine the hitherto *unknown* particulars with regard to the principal rafter and the tie rod. The arrow marks on the diagram indicate the *sense* of the respective forces; thus *ep*, which represents the reaction, acts upwards; PA acts *towards* the joint, thus indicating that this member is in **compression**, while AE acts *away from* the joint, showing that the tie rod is in **tension**. Thus the lines *pa* and *ae* in (*c*) drawn respectively parallel to PA and AE in (*a*) *measured to the same scale* as that used for *op* and *pe*, represent the magnitude of the stress in the principal rafter (*pa*) and the tie rod (*ae*).

Stresses at Joint between Strut and Principal Rafter.—At this joint *four* forces act, viz. PR, PA, RB, and AB. Of these four forces only *two* are *completely* known, *i.e.* the stress in PA, which has already been determined in the unit diagram (*c*), and the weight PR of 7 tons. The object of the diagram (*d*) is to discover the *missing data* with regard to the members RB and AB. In (*d*) *pr* is made equal to *pr* in (*b*), *i.e.* to represent 7 tons. Draw *pa* and *rb* parallel to the principal rafter, making *pa* = to *pa* in (*c*), and from the end *a* of *ab* (the *known* quantity) draw *ab* parallel to the strut AB, and intersecting *br* at *b*, and in this way determine the lengths *ab* and *br*, which, *measured to the same scale as pr*, represent the **magnitude** of the stress in the strut (*ab*) and the top end of the principal rafter (*rb*). The *sense* marks in (*d*), which follow round in a *circuit*, indicate the **nature** of the stresses in AB and BR. Thus the *sense* mark on *pr* points **downwards**, as representing the force PR of 7 tons, acting in a vertical direction. The remaining arrow marks on the diagram must *complete the circuit*, that on *br* pointing towards *b*; that on *ab* towards *a*; and on *ap*, towards *p*. Thus it is noted that the stress in the upper end of the principal rafter AB is a **compression** stress, as indicated by the arrow mark on *br*, which points in a downward direction, while that in the strut AB is also a compression stress. It should be specially noted that *pa* and *br* in (*d*) are not equal in length, thus showing that the compression stress in the *lower end* of the principal rafter at PA, is **greater** than that in the *upper* end of the same member, *i.e.* at RB.

Stresses at Ridge.—At this joint there are *four* forces, viz. RS (7 tons) and RB, both of which are *completely* known. The two remaining forces SC and BC are only *partly* known, and by means of the diagram (*e*) the missing data is determined. In this diagram *rs* is made equal to *rs* on the load line in (*b*), *i.e.* 7 tons, and from *r* draw *rb* parallel to RB and equal to *rb* in (*d*).

From *b* draw *bc* parallel to BC, and from *s* draw *sc* parallel to SC and intersecting *bc* at *c*. Then *bc* and *sc* represent the magnitude of the stresses in the king post BC and the principal rafter SC, and since the loading is symmetrical, *rs* and *cb* are equal in length.

Stresses at Foot of Ring Post.—At this joint *five* forces act, each of which has been completely determined in one or other of the unit diagrams (*c*), (*d*), or (*e*). For instance, the stress in the king post BC, the strut AB, and the tie-rod AE, have been determined, and since the loading of the truss is symmetrical about the centre line of the king post, the stress in the strut CD is equal to that in AB ; the stress in the tie-rod DE is equal to that in AE, and so on. In the diagram (*f*) *ae* is equal to *ae* in (*c*) and (*b*), and *ab* is equal to *ab* in (*d*), and the remaining lines of the force diagram are completed by drawing *cd* parallel to CD, and *bc* parallel to BC on the roof truss.

Roof Truss unsymmetrically Loaded.—In cases where the loading of the roof truss on the right- and left-hand sides of the king post is not equal, the load line must be drawn as in (*b*) Fig. 352, and the reaction at each support determined by means of a funicular polygon as already explained in the case of loaded beams.

Centres of Gravity. Centroids of Plane Figures.—The **centre of gravity** or **centroid** of a piece of sheet material is a point on the surface of the material such that if the sheet is suspended from this point *it will balance in any position.*

The *centroid* of a **square** or **rectangular** piece of sheet material (A and B, Fig. 353) *is the intersection of the diagonals* of the figure, as indicated at *a* and *b* respectively.

The *centroid* of a **triangular** sheet D is obtained by drawing the *medians* of the triangle, *i.e.* by joining the respective corners of the triangle to the centre point of the opposite side. The intersection of the medians at *d* being the point required. The *centroid* of a quadrilateral such as (C) is obtained as follows. Draw the diagonals 1,2, 3,4, and on 3,4 make 4,6 equal to 3,5. Join 2,6 and 1,6, and thus complete the triangle 1,2,6. Draw the medians 2,8 and 6,7 of this triangle. The intersection of the medians at 9, *i.e.* the *centroid* of the triangle 1,2,6, is also the centroid of the quadrilateral 1,3,2,4.

The same method is applied to a slightly different figure in (F), where 2,6 is made equal to 1,5, the centroid of the triangle 3,4,6, *i.e. f*, being also the centroid of the whole figure 1,3,2,4.

It should be noted, in dealing with any figure such as (E), that the *centroid* of the whole sheet will occur on the centre line

3,4, since the figure is symmetrical about 3,4, and must occur at a point *e*, *nearer* 4 than 3, *i.e. below* the horizontal centre line 1,2, since there is more material in the part of the sheet *below* 1,2 than in the part *above*.

Stresses in Masonry Dams.—(*a*) Fig. 354 shows a section of a **masonry dam,** such as is used for holding back the water in a *reservoir*. The object of a *stress diagram* in such a case is to determine whether the *weight of the masonry* at the respective levels of the dam is sufficiently heavy to withstand the **pressure of the water.** PA is the *inner* face of the dam, and this face usually has a *batter* or slope, as indicated by the space between the line PA and the dotted vertical line PD. The strength of

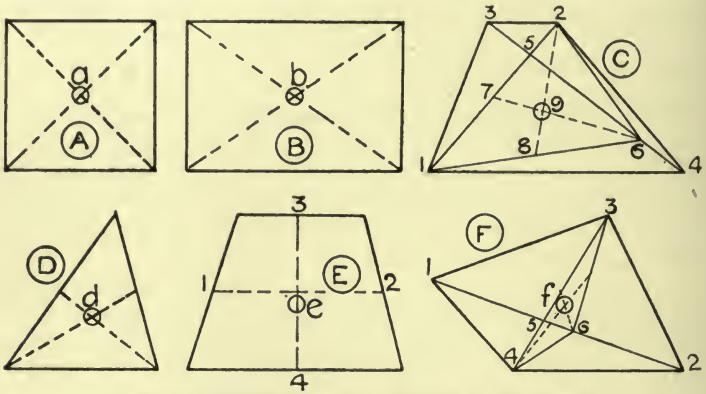


FIG. 353.—Centroids or centres of gravity of plane figures.

the masonry at the top of the dam, PR, may be comparatively small, where the pressure of the water is *least*, but must be increased very greatly at AB, where the pressure is *greatest*.

Two separate cases arise for solution, 1*st* to determine the stresses *when the reservoir is* **empty,** and 2*nd* when the reservoir is **full.**

1*st case.*—When the reservoir is empty the only stresses to be considered are those due to the *weight of the masonry* in the dam. Referring to (*a*) Fig. 354. Draw *horizontal section lines* AB, JK, FH, and UT, as shown. The *weight* of the *block of masonry* PRTU is taken as a **vertical force** 1,2 acting through the **centroid** *n* of this part of the cross-section. In order to determine what the magnitude of this force is, it is necessary to find the *area of the cross-section* PRTU in, say, square feet. Thus, assuming that PR = 7 ft., PU = 15 ft., and UT = 10 ft.: the area of the cross-section PRTU may be approximately determined

by multiplying the *length* PU by the *width of the section* taken at the level of *n*, the *centroid* of the section, *e.g.* 15 × 9 = 135 sq. ft. Consider a piece of masonry having a section PRTU and *one foot long;* such a block would contain **135 cub. ft.** of stone. Taking 150 lbs. as the *weight per cubic foot* of an average specimen of *sandstone*, the weight of a piece of masonry 1 ft. long and of a section PRTU would be 135 × 150 = 20,250 lbs. or approximately **9 tons,** and this must be considered as a *vertical force* 1,2 acting through *n*. The areas of the respective cross-sections UTHF, FHKJ, and JKBA, and the

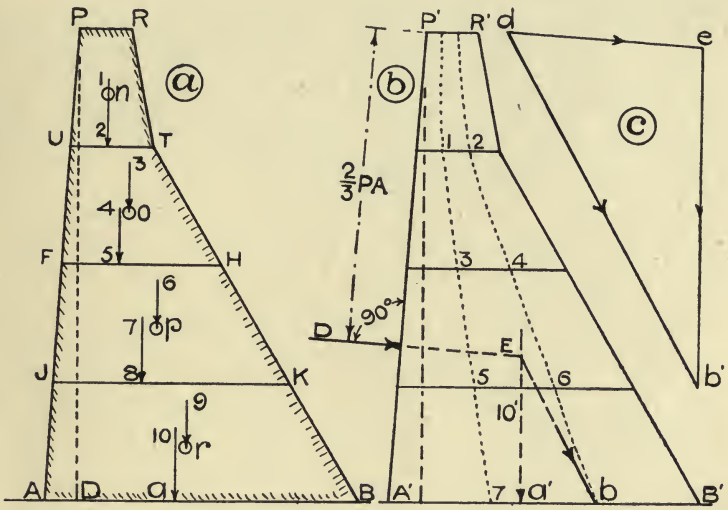


FIG. 354.—Stress diagram for masonry dam.

weight of one foot run of masonry of each section must be determined as already explained in connection with PRTU. The weight of the block UTHF is a *vertical force* 3*o*, **acting through the centroid** *o*. Find the resultant of these two forces. Its *line of action* is shown at 4,5, which, obviously, must occur *nearer* the line of action of 3*o* than that of 1,2. The weight of the next block of masonry FHKJ is a vertical force acting through the centroid *p* at 6*p*. The resultant of this force and the force 4,5 is shown at 7,8, which is nearer the line of action of 6*p* than that of 4,5. Similarly the weight of the block JKBA is a vertical force acting at 9*r*, through the centroid *r*, and the resultant of this force and that last found (*i.e.* 7,8) is shown at 10*a*.

Centres of Pressure.—The points 2, 5, 8 and *a* on the

respective horizonal section lines UT, FH, etc., are the **centres of pressure** or *centre of stress* for these sections.

Line of Resistance.—Having obtained the *centres of pressure* for the respective sections as shown at 2, 5, 8 and *a*, draw a fair curve through these points. This curve is the **line of resistance** for a dam of the given cross-section, and the position of this line with regard to other lines on the cross-section enables the designer to form an opinion as to the relative stability of the dam. The *line of resistance* is omitted from (*a*) in order to avoid confusion.

2nd case.—When the reservoir is **full**, the *weight of the stone* or other material has to be considered just as when the reservoir was *empty*, but in addition the *weight of the water pressure* against the inner face of the dam must also be considered. For practical purposes this pressure may be taken as being a force D in (*b*), acting at **right angles to the inner face** of the dam at a point **two-thirds of the slant height** of the inner face, *i.e.* two-thirds of P′A′ measured from P′. Taking this distance as 40 ft. the *intensity of the pressure of the water* per square foot is equal to 40 multiplied by the *weight of a cubic foot of water*, *i.e.* approximately 62 lbs. = 62 × 40 = 2480 lbs.

Referring to (*b*), the line 10′*a*′ is the final resultant at the *centre of pressure* as already determined in (*a*). Draw a *triangle of forces* (*c*) making *de* parallel to the direction of the pressure of water at D and, to any suitable scale, representing the magnitude of the resultant water pressure at D, *i.e.* 2480 lbs.; say 1·1 tons. Continue the triangle by drawing *eb*′ parallel to 10*a*, in (*a*) or (*b*). Draw the closing line *db*′ of the triangle. This represents the *magnitude* and *direction* of the **resultant pressure** of the *weight of the masonry* and the intensity of the *pressure of the water* against the inner face of the dam. In the diagram (*b*) draw a line E*b* parallel to the *closing line db*′ on the force triangle (*c*). The line of action of E*b* intersects the horizontal section line A′B′ at *b*, which is the **centre of stress** for the level A′B′, when the **reservoir is full.**

The *centre of stress* for each of the sections JK, FH, and UT may be determined in a similar manner, by using the lines 7,8, 4,5, and 1,2, respectively, in (*a*) in a manner similar to that employed in dealing with 10*a*. A fair curve drawn through the centres of stress of the respective horizontal sections JK, etc., is the **line of resistance** for a dam of the given section when the reservoir is **full.**

Referring again to (*b*). Each of the horizontal section lines is divided into *three equal parts* as 1,2, 3,4, 5,6 and 7*b*. The

centre space so found, *i.e.* 1,2, 3,4, etc., is called a **middle third**, and if fair curves are drawn through these points as shown, *i.e.* 1,3,5,7, and 2,4,6,*b*, the *central space* between these curves is the *middle third* of a dam of the given cross section. If the dam is properly designed, the *lines of resistance* should occur **within this middle third**, whether the reservoir is empty or full.

Moments of Forces.—The *moment* of a force about a point may be defined as *the measure of the tendency of the force to produce* **rotation** *about the given point.*

A cyclist pressing his foot down upon the pedal of his machine is a simple example of the principle of moments, or the tendency to produce rotation.

Referring to Fig. 355, suppose a bar *ad* is free to move around upon an axle *a*. A force of 60 lbs. is applied to the bar at a point B, 9 ins. from the centre of the axle. In such a case the *tendency to produce rotation* is dependent upon two factors: (1) the **amount** of force expended; (2) the **distance** away from the axle at which the force is applied. The two factors may be denoted by using the terms (1) Power, (2) Arm; the *power* being a **force** of a given magnitude, and the *arm* being a given **length** or distance. Thus in dealing with the *moment* of a force, two terms of different denominations have to be considered. Usually, in arithmetical work, it is not possible to multiply together *two terms of different denominations* such as *pounds* and *acres*, *yards* and *shillings*, and so on. In this case, however, **both terms** must be considered, and in speaking of the result, *i.e.* the *moment*, a joint term is used, which is made up of *two denominations*, (1) a *linear* measurement, and (2) a *weight* or force, thus *foot-lbs.*, *inch-tons*, etc. Referring again to Fig. 355, a *force* of 60 lbs. applied at B on an *arm* 9 ins. long would have a **moment** or *tendency to produce rotation* equal to $60 \times 9 = 540$ inch-lbs. or $\frac{3}{4} \times 60 = 45$ ft.-lbs. If, however, the same force (60 lbs.) is applied at D the moment would be $60 \times 12 = 720$ *inch-lbs.* or 60 *ft.-lbs.* Thus the turning tendency may be increased either (*a*) by employing the same *arm* and *increasing the magnitude of the force*, or (*b*) by using the **same force** on a **longer arm**.

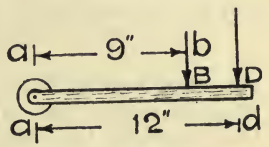


FIG. 355.—Moment of a force D about a point (*a*).

Boring Tools and Moment Principle.—The three familiar **boring tools** shown in Fig. 356, illustrate the principle of the *moment* of a force about a point or *centre*. In using each of the three, the *brace*, the *gimlet*, and the *bradawl*, the strength of

the operator's arm, wrist, or fingers is employed as the *power*, but the *arm*, or distance at which the power operates from the centre, varies in each case. In using the **bradawl**, a *very small amount* of resistance has to be overcome, since the point of the bradawl is comparatively small, and is only used to clear a passage through the wood sufficiently large for a nail by cutting and opening the fibres. In this case the operator's fingers exert a *power* on an *arm* equal to *f*, that is the *distance from the central axis* of the bradawl to the outer circumference of the wooden handle. In using the **gimlet**, a *greater turning tendency is required* in order to overcome the *greater resistance* offered in boring out, say, a $\frac{1}{4}$ in. hole. In this case the former *power*, *i.e.* the strength of the operator's fingers and wrist, may be the *same* as that used for the bradawl, but the length of *arm* is increased and is now represented by the distance *de*. The **brace** is used when *larger* holes are to be bored, and consequently **much greater resistance** has to be overcome, as the cutter of the centre bit or twist bit removes the waste wood from the boring.

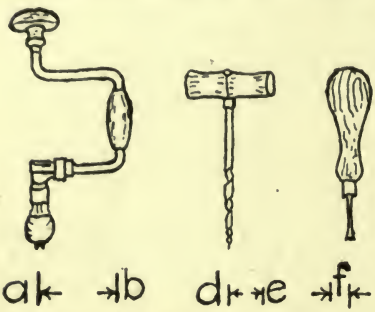


FIG. 356.—Moment principle as applied to boring tools.

In order to overcome the greater resistance involved, the *power*, *i.e.* the strength of the operator's hand and fore-arm, must be brought to bear at a greater distance away from the central axis of the boring tool, and for this reason the brace is made with a *crank* such that the length of *arm* on which the force works is represented by the distance *ab*.

Graphic Determination of Moment.—It has already been demonstrated that the *turning tendency* or *moment* of a force may be calculated, since it is the **product** of a *length* into a *force*. It is often more convenient, however, to determine the magnitude of the *moment* of a force **graphically**, *i.e.* by means of lines. The method of doing so is shown in Fig. 357. A force AB of 16 lbs. operates at a distance away from a point P. *Determine the magnitude of the moment of* AB *about* P, *i.e. the tendency which the force* AB *has of producing rotation about* P.

On a separate diagram draw a line *ab*, parallel to AB, and make *ab* = 16 to any convenient scale. Select any point *o* as a pole, and join *oa* and *ob*. Draw *oh* at right angles to *ab*. The

line *oh* is the **moment base** for the diagram, and, while it may be made of *any length* it makes the work of calculation much easier if the *moment base* is made equal in length to some definite *linear unit*, such as 1 in., or some *multiple* of this unit. Through P draw a line 3,4, parallel to AB, and from any point O on AB draw O3 parallel to *oa*, and O4 parallel to *ob* on the force triangle. From O draw OH at right angles to AB and 3,4. Then, from the nature of the construction the two triangles 3O4 and *aob* are **similar** *in all respects*. Hence the following ratio holds good: *ab* : 3,4 :: *oh* : OH. From this it follows that *ab* × OH = 3,4 × *oh*; and since *ab* represents the *magnitude* of the force AB, and OH is the *arm* on which the force AB is operating, the *moment* of AB about P is the **product** of 3,4 and *oh*. In the given case, the *moment base oh* is made *to scale* to represent 1 ft. To the *same linear scale* OH is measured and found to be 2·2 ft. Measure the length 3,4 and find the *product* of 3,4 × *oh*, which is 35. The two units employed in this case are the *foot* as a unit of **length** and the *pound* as a unit of **weight**. Hence the moment of AB about P is expressed as 35 ft.-lbs.

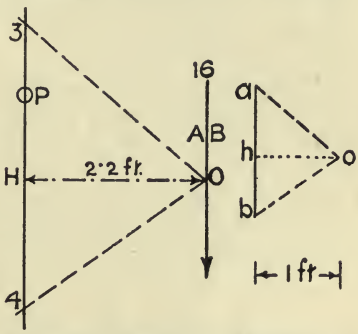


FIG. 357.—Graphical construction for determining the moment of a force.

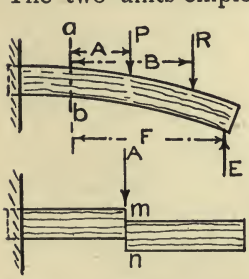


FIG. 358.—Bending moment and shearing stress in a cantilever.

Bending Moment and Shearing Stress Diagrams.—Fig. 358 shows a **cantilever** or beam having one end built into a wall while the other end of the beam is unsupported. If this cantilever has to support weights such as P and R, these weights have a tendency to **bend** the beam, and the *measure* of this tendency to produce bending is spoken of as the **bending moment**. The *bending moment* at any section of the beam, such as that indicated at *ab*, is the *algebraical sum* of the moments of all the forces such as P and R on one side of the section *ab*. In taking the **algebraical** sum of several quantities or *terms*, both **positive** and **negative** terms must be added. In this case there are *two positive* terms and *one negative* term. The positive terms are (1) the **product** of the *force* P and the

distance A ; (2) the **product** of the *force* R and the *distance* B ; while the negative term or quantity is the **product** of the *force* E, which is *assumed* to be acting vertically upwards, and the *distance* F. In computing the *algebraical sum* of these positive and negative quantities, therefore, the calculation would appear as follows : bending moment at the section *ab* is $= (P \times A) + (R \times B) - (E \times F)$.

In addition to this tendency to produce bending, any load such as A applied to a cantilever (see lower diagram, Fig. 358) has a tendency to **shear** or cut the material of which the beam is made in a transverse direction, as indicated at *mn*. This tendency to produce *shearing* is termed the **shearing stress,** and the shearing stress at any section such as *mn* is the resultant of the transverse forces acting at one side of the section.

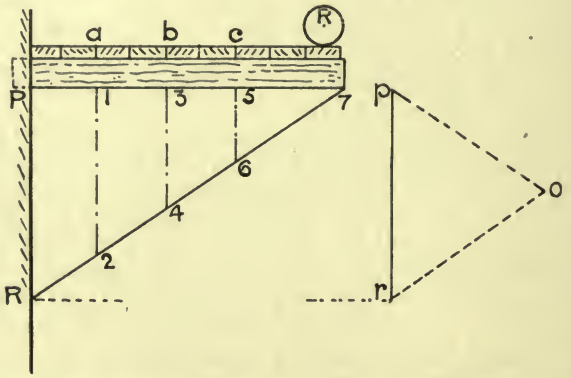


FIG. 359.—Bending moment diagram for cantilever with concentrated load R.

Fig. 359 shows a **cantilever** which projects 3 ft. 6 ins. beyond a wall into which one end is built, and a **concentrated load** R of 2½ tons is supported at the outer extremity of the cantilever. *Determine the magnitude of the maximum bending moment and also the bending moment at vertical sections of the cantilever at a, b,* and *c.*

The load R is *concentrated* at one point, and the bending moment diagram differs from that which would be necessary if the load was *distributed* along the whole length of the cantilever.

The *greatest tendency* to produce bending will be at P, the *point of support,* and as R is 3 ft. 6 ins. away from the wall, the maximum bending moment is the product of the weight, R (2½ tons), and the distance between R and the wall (3 ft. 6 ins.).

Thus $2{\cdot}5 \times 3{\cdot}5 = 8{\cdot}75$ foot-tons. Make PR to represent 8·75 to any convenient scale, and join R7. Then the ordinates 1,2, 3,4, 5,6 of the triangle P7R, if measured to the same scale as that used for PR, will represent the magnitude of the bending moment at the respective sections *a*, b, and *c* of the cantilever.

Fig. 360 shows a **cantilever** which projects 4 ft. from a wall and carries a **distributed load** of 12 tons. *Determine the maximum bending moment and also the bending moment at the section* PR *of the cantilever.*

The cantilever in this case is shown *tapering* from a **wide end** *nearest the point of support* to a **narrower end** at the *point furthest away from the point of support.* From what has already been demonstrated in the foregoing problem it is evident that the *greatest strength* of the beam must be arranged **at or near its point of support**, while at the *outer* end, where the bending moment is practically nil, the strength of material may be reduced to a minimum with safety.

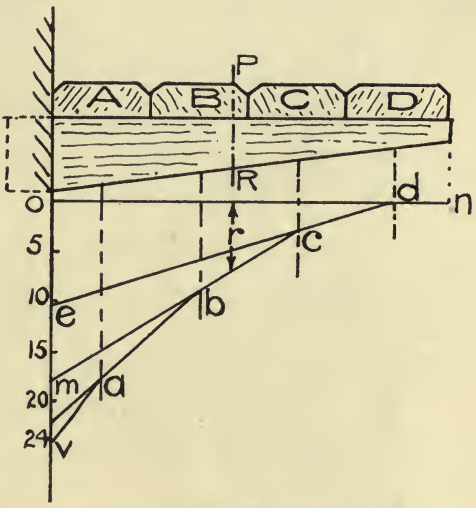


FIG. 360.—Bending moment diagram for cantilever with distributed load ABCD.

In order to represent diagrammatically a *distributed* load of 12 tons, 4 heavy baulks of timber A, B, C, and D are shown A *distributed* load of 12 tons is equivalent to a *concentrated* load of 12 tons acting at the **centre of the length** of the cantilever, *i.e.* at a point 2 ft. away from the point of support. Hence the maximum bending moment is $12 \times 2 = 24$ foot-tons. The distributed load is assumed to have been divided into four loads, the lines of action of which pass through the centre of the width of each of the baulks A, B, C, and D, as shown by the dotted lines at *a*, *b*, *c*, and *d*. Draw a *horizontal line on* equal in length to that of the cantilever, and to any suitable scale set out 0,24, to represent the *maximum bending moment.* Consider the bending moment at the centre of D, *i.e.*

along the line D*d*. The weight of D is one-fourth of the total distributed load, *i.e.* $\frac{1}{4}^2 = 3$ tons; and this force acts at a distance of 3 ft. 6 ins. from the point of support. Hence the B.M. at this section is $3 \times 3{\cdot}5 = 10{\cdot}5$ ft. tons. Mark a point *e* on the vertical line 0,25 such that *oe* represents 10·5 units *on the same scale* as that used for setting out 0,25, and from *d* draw *de*, thus *completing the triangle ode*. The weight of C, 3 tons, acts at a distance of 2 ft. 6 ins. from the point of support, and the B.M. at the C*c* is therefore $3 \times 2{\cdot}5 = 7{\cdot}5$ *ft.-tons*, which is indicated by a length *em* of 7·5 *units*, on the line 0,24. From the point *c*, at which the lines *de* and C*c* intersect, draw the line *cm*, and thus *complete the triangle cem*. In the same way the B.M. for B is $3 \times 1{\cdot}5 = 4{\cdot}5$ ft.-tons, and that for A is $3 \times 0{\cdot}5 = 1{\cdot}5$ ft.-tons, and these magnitudes are represented at V, etc., on the line 0,24, and the lines *va*, etc., drawn as shown. Having obtained a series of points such as *v*, *a*, *b*, *c*, and *d*, draw a **parabolic curve** to pass through the series of points. The bending moment at any section of the cantilever is then represented by an *ordinate* drawn at right angles to *on* to meet the *parabolic curve*. Thus the B.M. at the section PR is represented by an ordinate *r* which is *interpolated* or *fitted in* between the reference line *on* and the parabolic curve. This curve follows the straight lines *va*, *ab*, *bc*, and *cd* so closely that it is *omitted* from the diagram in order to avoid confusion.

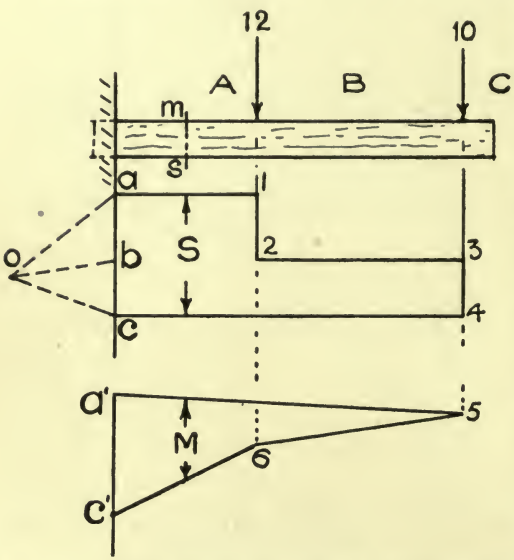


FIG. 361.—Bending moment and shearing stress diagrams for cantilever.

Fig. 361 shows a **cantilever** which supports two **concentrated loads** of 12 cwts. and 10 cwts. as shown. The **shearing stress** and **bending moment** diagrams for such a beam are shown. Letter the spaces between the forces as indicated at A, B, and C,

and make *ab* and *bc* = 12 units and 10 units respectively to any suitable scale. Select a pole *o* and join *oa*, *ob*, and *oc*. Then the *stepped* diagram *a*,1,2,3,4,*c* is the **shearing stress** diagram for the given beam, on which the shearing stress at any vertical section of the beam, such as *ms*, is represented by the ordinate S which fits between the sides *a*1 and *c*4 of the stepped diagram at a point immediately underneath *ms*. From this it should be obvious that the Shearing Stress at any section of the beam immediately underneath B is much smaller than that at S.

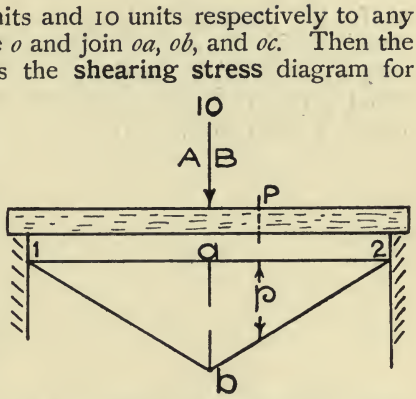


FIG. 362.—Bending moment diagram for beam supported at both ends and carrying a concentrated load.

The Bending Moment diagram is obtained as follows: make *a'c'* = *ac* and draw *c*6 parallel to *oa* and in the space A between the point of support and the first force of 12 cwts.; and from 6 draw 6,5 parallel to *ob* and in the space B between the lines of action of the forces AB and BC. Complete the *funicular polygon* by joining 5,*a'*. Then the B.M. at any section *ms* of the beam is represented by the ordinate M on the diagram.

Beams supported at Both Ends.—Fig. 362 shows a beam *supported* (but **not fixed**) at both ends; and carrying a **concentrated** load AB of 10 tons. The *maximum bending moment* in such a case is immediately *underneath the weight* as shown at *ab*, while, if the triangle 1,2,*b* is completed in such a manner that the line *ab*, to scale, represents the maximum B.M., the *ordinate p* represents the B.M. at any section of the beam, such as that indicated at P.

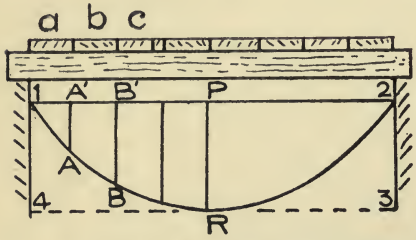


FIG. 363.—Bending moment diagram for beam supported at both ends and carrying a distributed load *a*, *b*, *c*, etc.

Fig. 363 shows a **beam supported at both ends** and carrying a **distributed** load equal to that of AB in Fig. 362. Here, as in the last case, the *maximum* B.M. is *at the centre*, PR, but in order to determine the B.M. at any other section of the beam, such as *abc*, etc., it is necessary to draw a *parabolic curve*

to fit the rectangle 1,2,3,4 (see p. 142). Having drawn this *parabola* with an axis PR, the B.M. at any section of the beam may be obtained by *interpolating* an ordinate such as A'A, B'B, etc., in order to find the B.M. at the joint between the timbers *a* and *b*, *b* and *c* respectively.

EXERCISES

(*Answers are given on p.* 403)

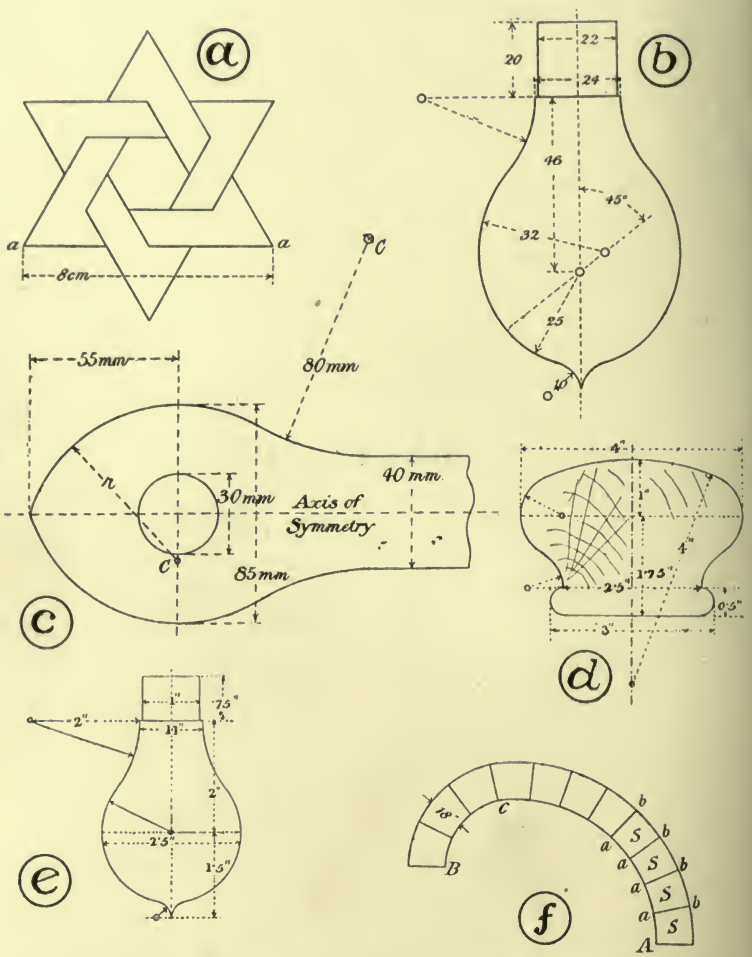


FIG. 364.—Illustrating Exercises designed to test Accuracy and Neatness.

Preliminary Exercises designed to test Accuracy and Neatness. (See Fig. 364.)

1. The given diagram (*a*) is a pattern made up of interlacing equilateral triangles. Taking the length of the side *aa* of a triangle as 8 centimetres, copy the diagram, making the remaining measurements proportionate to *aa*. (*Bd. of Educ.*)

2. Copy the given diagram (*b*), which represents an electric filament bulb, the units of measurement of which are given in millimetres. (*Bd. of Educ.*)

3. End of eye-bar (*c*). Set out the figure to the given dimensions. Show clearly the constructions for locating the centres C, C, and measure the radius *r*. Determine and mark the junctions of the arcs and lines. (*Bd. of Educ.*)

4. The section of a hand-rail is shown in (*d*). Draw this figure to the given dimensions. (*L. C. C. Scholarship.*)

5. Draw, to the given dimensions, the electric lamp (*e*). (*L. C. C. Scholarship.*)

6. The diagram (*f*) shows a rampant arch ring, 18 inches deep, made up of eleven voussoirs or arch stones S, S, S, ... having equal lengths of arc *aa*, *aa*, ... along the soffit A*c*B, the joints *ab*, *ab*, ... being perpendicular to the curve of the soffit. The soffit is made up of two circular arcs A*c*, *c*B, each of 90 degrees, and of 6 ft. and 3 ft. radius. Draw the figure to a scale of $\frac{1}{2}$ in. to 1 ft. (*Bd. of Educ.*)

Exercises on Chapter I. Scales.

7. In a scale drawing of a machine, a certain length measuring $10\frac{1}{8}$ ins. is known to represent 3 ft. 2 ins. Construct a scale to use with the drawing, reading feet and inches, and make it long enough to measure 4 ft. (*Matric.* 1907.)

8. Draw a scale of 25 ins. to 1 mile, reading up to 500 yds. No unit smaller than 10 yards need be shown.
Draw a line representing 370 yds. (*Bd. of Educ.*)

9. Determine $\frac{1}{96}$ of 2 ins. by means of a diagonal scale.

10. If 2 chains be represented by 4 ins. construct the same scale to show poles and yards.

11. A line 17·5 yds. long is represented on a certain drawing by 3·5 ins. Construct the scale.

12. Draw a diagonal scale of 5 cm. = 1 yd. to read feet and inches.

Exercises on Chapter II. Angles and Angle Measurement.

(*A Table of Trigonometrical Ratios will be found on Page 25.*)

1. By the aid of your protractor, and without using the tables, find the value of

$$\sin 23^\circ + \tan 23^\circ.$$

Now take out the true values of the sine and tangent from the tables, add them, and calculate the percentage error in your first answer. (*Bd. of Educ.*)

2. Construct a right-angled triangle ABC to the following data, C being the right angle :—

Base BC = 10 ins., base angle ABC = 13°.

Measure the height CA. Obtain tan 13° from the tables. Calculate the height CA. (*Bd. of Educ.*)

3. Take from the tables the chord and tangent of 22°. Construct an angle of 22° by using the value of the chord, and a second angle of 22° by using the tangent. Verify the results by measuring the two angles with your protractor, writing down, to the first decimal, what each angle measures. (*Bd. of Educ.*)

4. In any triangle, if A, B, C be the angles, and a, b, c the opposite sides, then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

Calculate cos C when $a = 6$ cm., $b = 5$ cm., $c = 4$ cm.

Now verify the formula (and your answer) by construction, measurement, and, if you like, the use of the tables. (*Bd. of Educ.*)

5. In the equation $\frac{a}{\sin A} = \frac{b}{\sin B}$, you are given

$$A = 41^\circ, \quad B = 72^\circ, \quad a = 7{\cdot}6 \text{ cm.}$$

Determine b by graphical construction, and verify your answer by numerical calculation, using the tables. (*Bd of Educ.*)

6. In working this question employ a decimal scale of $\frac{1}{2}$ in. to 1 unit.

Draw a circular arc, radius 10 units, centre O. Mark a chord AB of this arc, 3·47 units long, and draw the radii OA, OB. Measure the angle AOB in degrees.

From B draw a perpendicular BM on OA, and at A draw a tangent to meet OB produced in N. Measure carefully BM and AN (on the above unit scale), and calculate the sine and the tangent of the angle AOB.

Give the correct values of the angle, the sine and the tangent, taken directly from the examination tables supplied. (*Bd of Educ.*)

7. The polar or vector co-ordinates of two points A and B are (3 ins., 30°) and (4·2 ins., 100°). That is, if O is the pole, and OX the line of reference,

$$OA = 3 \text{ ins.}, \quad XOA = 30^\circ$$
$$OB = 4{\cdot}2 \text{ ins.}, \quad XOB = 100^\circ$$

Draw OX and plot the points A and B. Measure AB and the perpendicular OC from O on AB, and calculate the area of triangle OAB. Verify your answer by calculating the value of $\frac{1}{2}$ OA . OB sin AOB. (*Bd. of Educ.*)

8. Taking the height of the cliff (Fig. 24, p. 22) and the height of the pole as given, *i.e.* 95 ft. and 30 ft. respectively, and the angles of depression as 35° and 50°, find (*a*) the lengths of the guy ropes and (*b*) the width of the river.

Exercises on Chapter III. Construction of Triangles.

1. Construct a triangle whose perimeter shall be 7 ins.; the base AB = 2 ins., and the angle BAC = 50°.

2. Construct the triangle whose sides are 10 ft. 6 ins.; 14 ft. 0 ins.; and 16 ft. 3 ins., when $\frac{1}{8}$ in. represents one foot.

3. Draw a triangle in which the sides are in the ratio 3 : 4 : 7, the longest side to be 3 ins. long.

4. The medians of a triangle are 1·7 ins., 2·3 ins., and 2·0 ins. long respectively. Construct the triangle and measure and write down (*a*) the length of its sides, (*b*) the magnitude of its angles measured in degrees. (See Fig. 33.)

5. A piece of wire 12 ins. long is bent at two points in such a way as to make a triangular template, the angle at the vertex of the triangle being 52°, and its altitude 3·4 ins. Construct the triangle and write down the lengths of its sides and the magnitude of its angles. (See Fig. 30.)

6. In a survey the following measurements were made, lengths being in chains and angles being reckoned anti-clockwise from the east.

$$AB = 32_{16}°,\ BC = 51·3_{142}°,\ CD = 67_{41·4}°.$$

Plot the points A, B, C, D to the scale of $\frac{1}{2}$ in. to 10 chs. Measure from your drawing the length and direction of AD. Also give the easterly and northerly components of AD. (*Bd. of Educ.*)

7. A surveyor is making a map on which he wishes to locate two inaccessible objects H and K situated towards the north. He lays off a base line DE, 20 chs. or $\frac{1}{4}$ m. long, going due east. When stationed at D, he measures the angles EDK, KDH, by means of a sextant, and finds them to be 51° and 55°. When stationed at E the angles DEH and HEK measure 48° and 62°.

Plot the points D, E, H, K to a scale of 8 ins. to the mile. Measure the distance and direction of HK. (*Bd. of Educ.*)

8. In the plan of an estate, drawn to a scale of 1 in. to 100 yds., three trees A, B, C are marked, and their relative positions measure, in yards,

$$AB = 232_{79·6}°,\ BC = 358_{6·8}°,$$

angles being measured anti-clockwise from the east.

In order to locate two places P and Q on the estate, lying on the north side of BC, the angles subtended by the trees at P and Q are measured with a sextant and are as follows:—

$$APB = 36·2°,\ BPC = 22·3°,\ APC = 58·5°.$$
$$BQA = 18·1°,\ BQC = 78·8°,\ AQC = 60·7°.$$

Plot the five points to scale, and measure the length and direction of PQ. (*Bd. of Educ.*)

9. A person, 6 ft. high, walking along a road with the sun directly behind him, observes that at a place where the road is level his shadow measures $7\frac{1}{2}$ ft., and at a place where the road has an upward gradient his shadow measures $5\frac{1}{4}$ ft. What is the angle which the sun's rays make with the horizontal? What is the inclination of the road? (*Bd. of Educ.*)

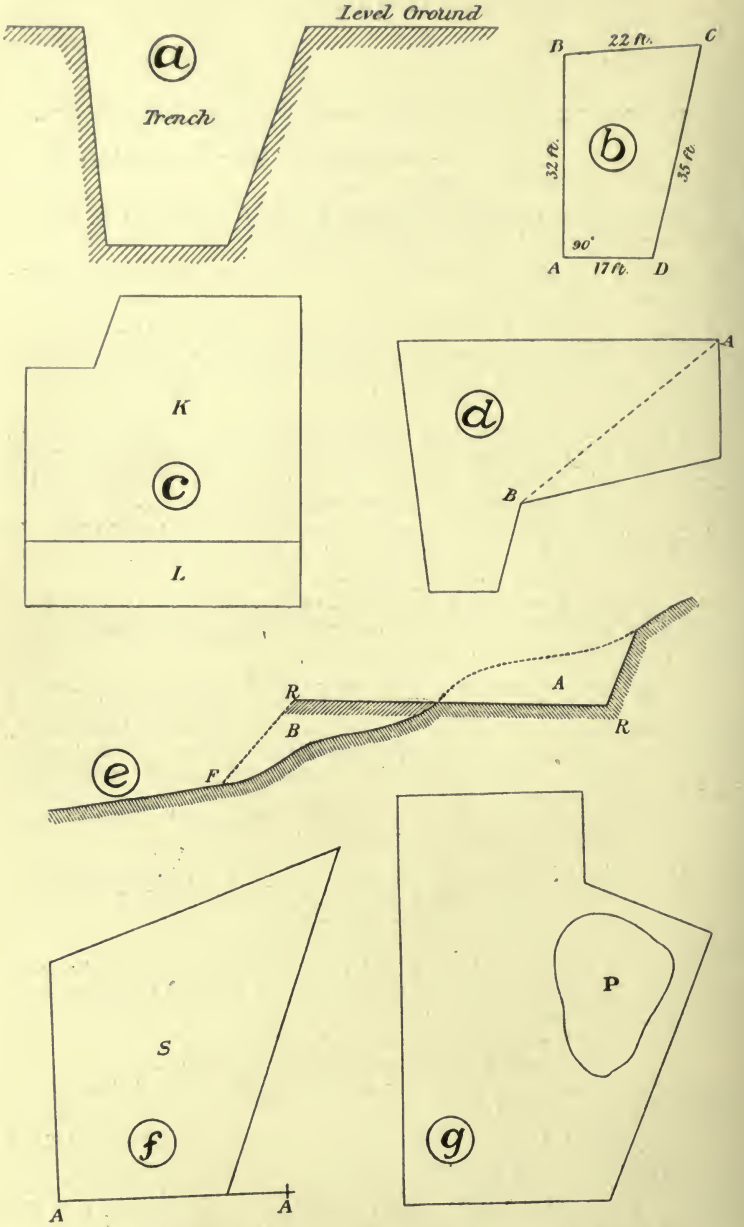


FIG. 365.—Illustrating Exercises on Plane Rectilinear Figures.

Exercises on Chapter IV. Plane Rectilinear Figures.

(See Fig. 365.)

1. Construct a rectangle, one side 2 ins. long, having its area half that of an equilateral triangle of $2\frac{1}{2}$ ins. side. (See page 47.)

2. Draw a circle of 3 ins. diameter and in it describe carefully a regular pentagon. Without calculation, draw an isosceles triangle on a 2-in. base which shall have the same area as the pentagon. (See Fig. 70.)

3. A trench is dug of the shape and size given in (*a*), the scale for the diagram, after enlarging it to twice the given size, being 1 cm. to 1 ft. By measuring the diagram to scale find the cross-sectional area of the trench in square feet.

The material removed expands 10 per cent. in bulk, and is filled alongside the trench in an embankment of triangular section with the sides sloping at 45°. Draw this embankment to scale. State its height. (*Bd. of Educ.*)

4. The diagram (*b*) gives the dimensions of a plot of level land. Draw this figure to a scale of $\frac{1}{8}$ in. to 1 ft. Measure the lengths of the two diagonals in feet and determine the area of the plot in square yards. (*Bd. of Educ.*)

5. The plan of a hall is given at (*c*). Copy the given plan twice the size shown. L is the platform, and K the body of the hall. Working to a scale of $\frac{1}{2}$ in. to 10 ft. find the area of K in square feet. Calculate the sitting accommodation of K, 30 per cent. of its area being occupied by passages, and allowing one person to every 4 square feet of the remainder. (*Bd. of Educ.*)

6. You are given the plan of a building site (*d*). Copy this plan twice the given size. Working to a scale of 1 in. to 10 yds. draw a triangle as shown at AB and determine the area of the space occupied by the site. (*Bd. of Educ.*)

7. A road is formed along the side of a hill as shown in cross-section at RR (*e*), by cutting away material A, and depositing it as an embankment B. The scale of the figure being 1 cm. to 2 yds. determine :—

(*a*) The number of cubic yards of material removed at A per yard length of road.

(*b*) The point F, reached by the foot of the embankment, assuming the material removed at A to extend $\frac{1}{12}$ in bulk.

N.B.—The point F on the diagram is purposely misplaced, and is not to be copied. (*Bd. of Educ.*)

8. The building plot S in (*f*) is to be drawn twice the given size. Taking the scale of the drawing, after enlargement, as $\frac{1}{8}$ in. to 10 yds., find the area of the plot in square yards.

A rectangular plot of land of the same area has the frontage (or side) AA. Find the depth (that is the other side) of the plot in feet, and draw the rectangle to scale. (*Bd. of Educ.*)

9. The figure is the plan of a field in which there is a pond P ; scale $\frac{1}{8}$ in. to 10 yds. Find the total area of the field (including the pond), and the area of the pond in square yards. (*Bd. of Educ.*)

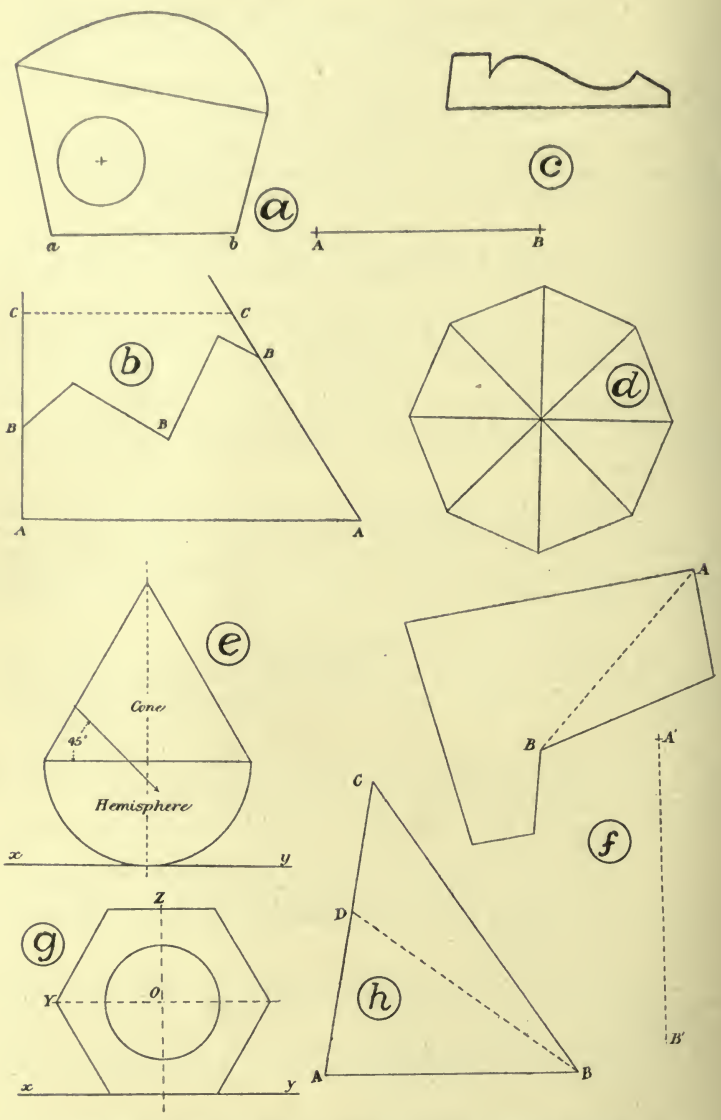


FIG. 366.—Illustrating Exercises on Similar Rectilinear Figures.

Exercises on Chapter V. Similar Rectilinear Figures.

(See Fig. 366.)

1. On AB in (*a*) as base draw a figure similar to the given figure on *ab*. What is the ratio of AB to *ab*? What is the ratio of the areas of the two figures after deducting the area of the circular patch in each case?

2. The plan AABBB of a piece of ground (*b*) is to be drawn double the size shown. Find the area of the plot. Scale 1 cm. to 2 yds.

It is required to replace the crooked fence BBB by a straight fence such as CC, parallel to AA, and so placed as to enclose an additional 50 sq. yds. of ground. Show the position of CC.

3. The given cross-section of a moulding (*c*) is one-fourth of the actual size of the moulding. Enlarge the cross-section so that the *width* of the moulding is five times that shown. What ratio does the width of the moulding on the *enlarged* drawing bear to the width shown in (*c*)?

4. Enlarge the given figure (*d*) (which is the plan of an octagonal lantern light) so that one side of the octagon on the enlarged drawing may represent a length of 10 ft. to a scale of 1 in. = 1 ft.

5. The drawing (*e*) is the elevation of a cone and hemisphere. Enlarge the drawing so that the *area* of the triangle which represents the cone may be *twice* that of the *original* triangle.

6. Enlarge the given figure (*f*) and alter its position so that the line AB of the enlarged figure shall be situated at A′B′.

7. Make an enlarged drawing of a nut similar to that shown in (*g*), and equal in area to the *combined* areas fo the given nut and another nut in which the half diagonal OY is 1 in. long.

8. ACB in (*h*) represents the plan of a triangular plot of ground. Enlarge this plan in such a way that the length of AB is increased to that shown at DB, and so that the base DB of the new plan occupies the position of DB on the given drawing.

9. Make a pentagon of 1 in. side and enlarge it so that its area may be equal to the combined areas of this pentagon and another of $\frac{3}{4}$ in. side.

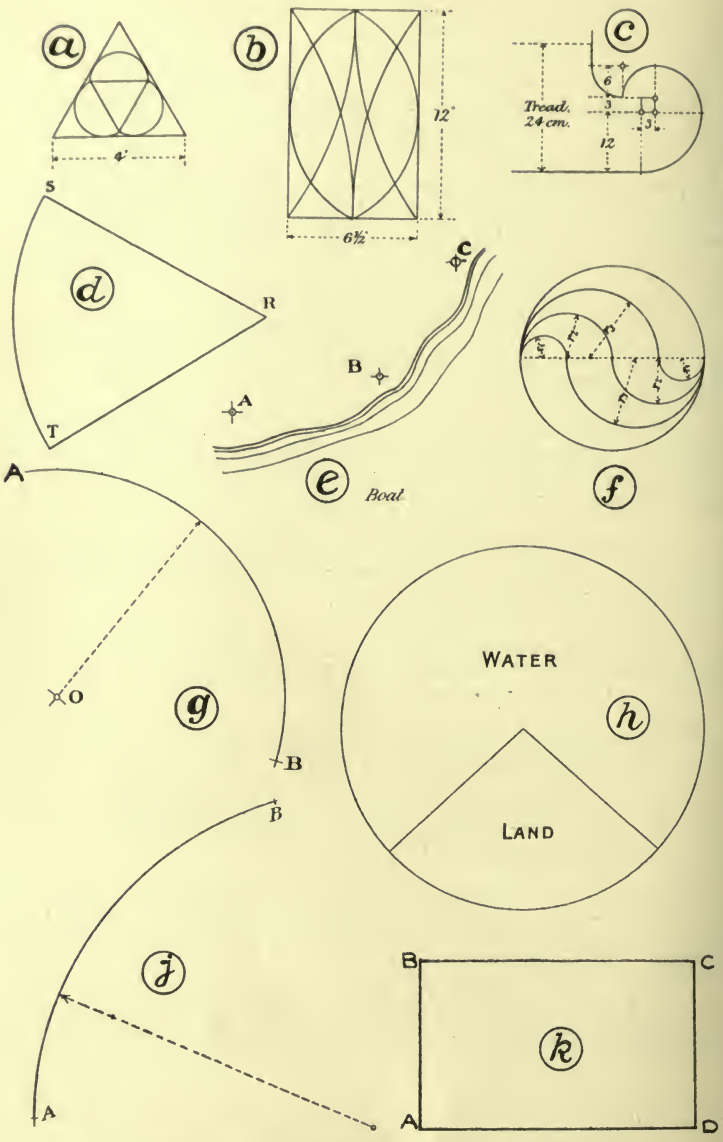


FIG. 367.—Illustrating Exercises on Circles, etc.

Exercises on Chapter VI. Circles, Etc. (See Fig. 367.)

1. Draw an equilateral triangle of 4 ins. side (*a*), and in it describe three semicircles as in the diagram. Your method of construction must be clearly shown. (*Bd. of Educ.*)

2. Geometrical pattern for panel (*b*). Set out the figure to the given dimensions, $\frac{1}{4}$ linear size. Show carefully the constructions for determining the centres of the circular arcs. (*Bd. of Educ.*)

3. The curve (*c*) representing the curtail of a stone step is constructed of circular arcs, each of 90°. Set out the figure to the given dimensions, which are in centimetres. Scale $\frac{1}{4}$. (*Bd. of Educ.*)

4. In the given sector of a circle RST in (*d*) inscribe a circle to touch the arc ST and the straight lines RT and RS. (*Bd. of Educ.*)

5. A, B, C in (*e*) are three station points on land. A person in a boat P, in order to locate his position, measures with a sextant the angles subtended by pairs of points and finds them to be

$$\text{APB} = 39°, \quad \text{BPC} = 11°, \quad \text{APC} = 50°.$$

Plot the position of P. Tracing paper may be used if desired. Measure PA and PC, the scale of the figure being 1 cm. to 100 yds. (*Bd. of Educ.*)

6. Divide a circle of $1\frac{1}{2}$ ins. radius (*f*) similarly to the given diagram, the radii of the smaller circles r_1, r_2, and r_3, being $\frac{3}{8}$ in., $\frac{3}{4}$ in., and $1\frac{1}{8}$ in. respectively. (*Bd. of Educ.*)

7. The given circle (*h*) is divided into two sectors, the areas of which represent the relative areas of land and water on the earth's surface. If the area of land is 52 millions of square miles, what is the area of the water?

Draw a concentric circle, the area of which shall represent the area of land, the given circle representing, as before, the total surface of land and water.

8. Find the length of the given circular arc AB in (*g*) by the following approximate method: Bisect the arc AB in C, and the arc AC in D. Draw the tangent at A to meet the radius OD produced at E. Join EB. Then the arc AB = AE + EB nearly.

Use tracing paper and a pricker to find the length of the arc more accurately and say whether AE + EB is too long or too short, and by how much.

9. The following construction is sometimes used for finding the approximate length of a circular arc: Let AB (*j*) be the arc. Draw the chord AB and produce AB to C, making BC = half AB. Draw the tangent at B and with centre C radius CA, cut this tangent in D. Then BD = arc BA nearly. Apply this construction to the given arc. Measure BD in centimetres. Also, by means of tracing paper and a pricker, or otherwise, find the exact length of the arc. Is BD too long, or too short, and by what per cent.?

10. In the rectangle ABCD (*k*), which should be re-drawn so that AB = $1\frac{1}{2}$ ins. and BC = $2\frac{1}{2}$ ins., find two points (P and Q) on the line BC such that the angles APD, AQD shall be each 70°. (*Bd. of Educ.*) (*Hint: Construct segment of* 70° *on* AD *as chord.*)

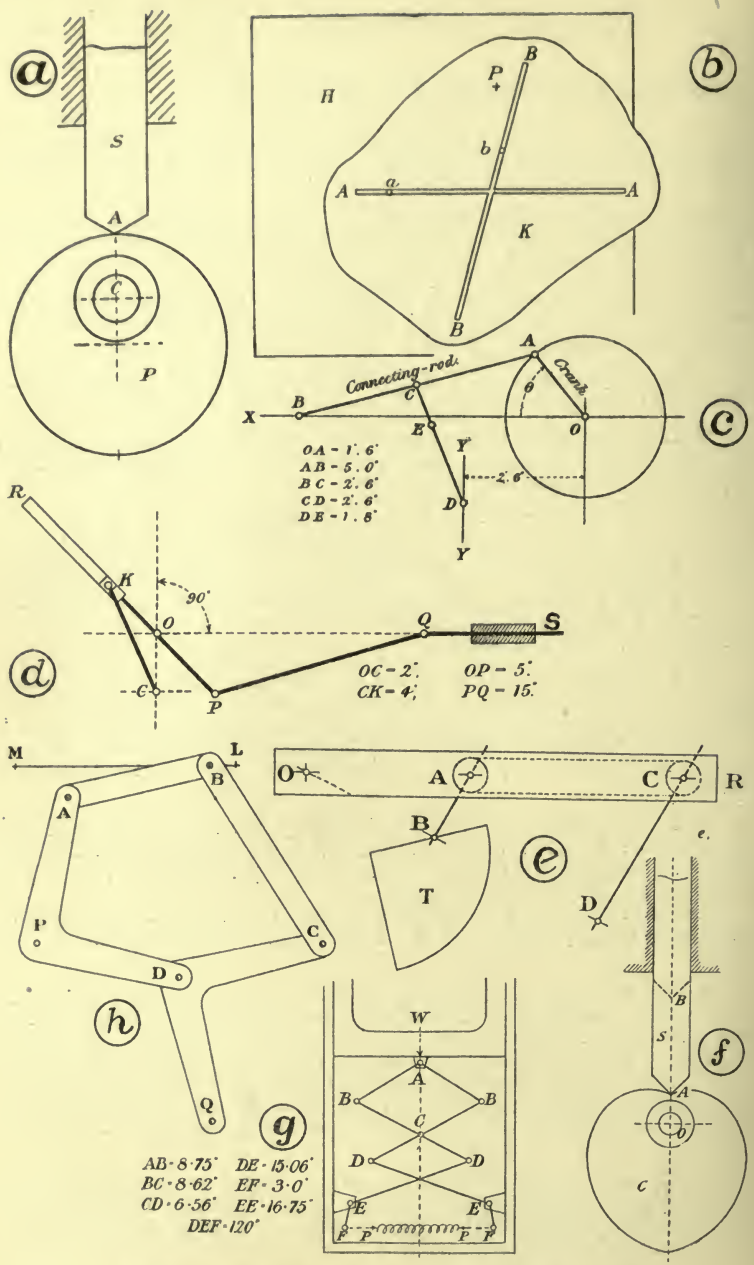


FIG. 368.—Illustrating Exercises on Loci and Spiral Curves.

Exercises on Chapters VII. and VIII. Loci and Spiral Curves. (See Fig. 368.)

1. The circular plate P in (*a*) turns about the axis C, set eccentrically, thus forming a cam. The slider S has its lower end A resting on the cam, and thus receives an up and down motion. Mark successive positions of A corresponding to angular positions of the cam of 30°, 60°, 90°, 120°, 150°, and 180° from the given position. (*Bd. of Educ.*)

2. A piece of sheet material K in (*b*) has a motion determined by the condition that the two grooves AA, BB cut in it slide over two pins *a*, *b*, secured to a fixed plate H. Determine the path traced on H by the point P carried by K. (*Bd. of Educ.*)

3. In the mechanism shown in (*c*) one end of the link CD is pinned to the connecting-rod and the other end moves in the vertical slide YY.

Working to the dimensions given on the diagram, and to a scale of 1 in. to 1 ft., plot carefully the eight positions of the point E in CD corresponding to the eight angular positions θ of the crank of 0°, 45°, 90°, ... 315° (*Bd. of Educ.*)

4. In the mechanism shown in (*d*), the slide S which oscillates with a "quick return" motion, is actuated as follows: CK is a crank turning uniformly about C, its end K moving in the slot of the link ROP which rotates about O at varying speed. The slide S is connected to the link by the rod PQ.

Set out a line diagram of the mechanism to the given figured dimensions, scale 1 cm. = 1 in., for the position in which the crank angle OCK = 32°. (*Bd. of Educ.*)

5. OR in (*e*) is a bar free to turn about O. AB, CD are parallel arms fixed to two equal pulleys pinned to the bar and connected by a band, so that when one arm is turned the other turns with it, and the two thus remain parallel. The point B in the arm AB is moved round the figure T. The arm CD carries a pencil at D ; find the curve traced by D.

You will see that the region commanded by B is bounded by two concentric circles. Draw these circles. (*Bd. of Educ.*)

6. A sliding piece S in (*f*) has its end A resting on a cam C which rotates uniformly about O. During half a turn of the cam the slider is lifted from A to B at uniform speed, and during the next half turn it falls from B. to A in like manner. The shape of the cam is shown *incorrectly*, and you are required to set out the *true* shape.

Take OA = $\frac{3}{4}$ in., AB = 1$\frac{1}{2}$ ins., and *do not copy the diagram.* (*Bd. of Educ.*)

7. The figure (*g*) shows an arrangement for balancing the weight of the window of a railway carriage by means of the pull of a spring PP, acting through linkwork ABCDEF as shown. Set out the linkwork to scale, $\frac{1}{4}$ size, working to the dimensions given on the diagram, and making the angle BAB = 110°.

N.B.—The window and door frame need not be drawn. (*Bd. of Educ.*)

8. Four pieces of sheet metal (*h*) are hinged together at A, B, C, D, these points forming the corners of a jointed parallelogram. One of the pieces is pinned or hinged to the drawing board at P, and the hinge-point B is moved in a straight line from L to M. Find the locus of Q, that is the path that would be traced by a pencil moving with Q.

9. Copy the versed sinic spiral shown in Fig. 154 twice the given size.

10. Draw two versed sinic volutes, similar to that shown in (*a*) Fig. 155, twice the given size, and arrange them to form a pair as shown at the top of the column in (*b*), Fig. 155.

Exercises on Chapter IX Plane Curves.

ELLIPSE.

1. Draw the elevation of an elliptical arch as shown in Fig. 159, span 10 ft., rise 3 ft., and show the necessary framework (made of $2\frac{1}{4}$-in. battens) for "striking" a plaster mould on the intrados of the arch. (Scale $\frac{1}{2}$ in. = 1 ft.)

2. Using the method of construction shown in Fig. 160, draw an ellipse 9 ft. × 5 ft. Assuming that the space within the ellipse is to be excavated to a depth of 3 ft., how many cubic feet of earth will have to be removed? (Scale $\frac{1}{2}$ in. = 1 ft.)

3. Draw the front elevation of an elliptical masonry arch, span 15 ft., rise 5 ft. 6 ins. Scale $\frac{1}{2}$ in. = 1 ft. Show a keystone 14 ins. wide at its lower (or intrados) end, and nine voussoirs on each side of the keystone. The height of the keystone and voussoirs is left to your judgment. The top ends of the voussoirs are to be finished as shown in (*e*), Fig. 192.

4. Using the method shown in (*a*), Fig. 163, draw an ellipse to fit a rectangular framework 5 ft. × 3 ft. Scale $\frac{3}{4}$ in. = 1 ft.

5. Draw two circles of 4 ins. and $2\frac{1}{2}$ ins. diameter respectively. These are the major and minor auxiliary circles of an ellipse (*b*), Fig. 163. Draw the ellipse.

6. Copy the problems illustrated in Figs. 166 and 167 double the given size.

7. An arch for a bridge spanning a river 90 ft. wide is to be made by drawing a parallel to an ellipse, as shown in Fig. 168. The crown of the arch is to be 25 ft. above the normal level of the river. Draw the outline of the arch. Scale 1 in. = 10 ft.

PARABOLA.

8. Draw a parabola as shown in (*a*) Fig. 174, making the distance apart of the vertex V and the focus F = $\frac{3}{4}$ in.

9. Using part of the parabola drawn in the preceding question, draw normals to the curve at intervals of $\frac{3}{4}$ in. employing the method illustrated in Fig. 175.

10. Draw the elevation of an arch made up of two parabolic arcs as indicated in Fig. 178, but employing the method of construction shown in Fig. 179, making the span AB = 20 ft. and the rise CD = 9 ft. Scale $\frac{1}{4}$ in. = 1 ft.

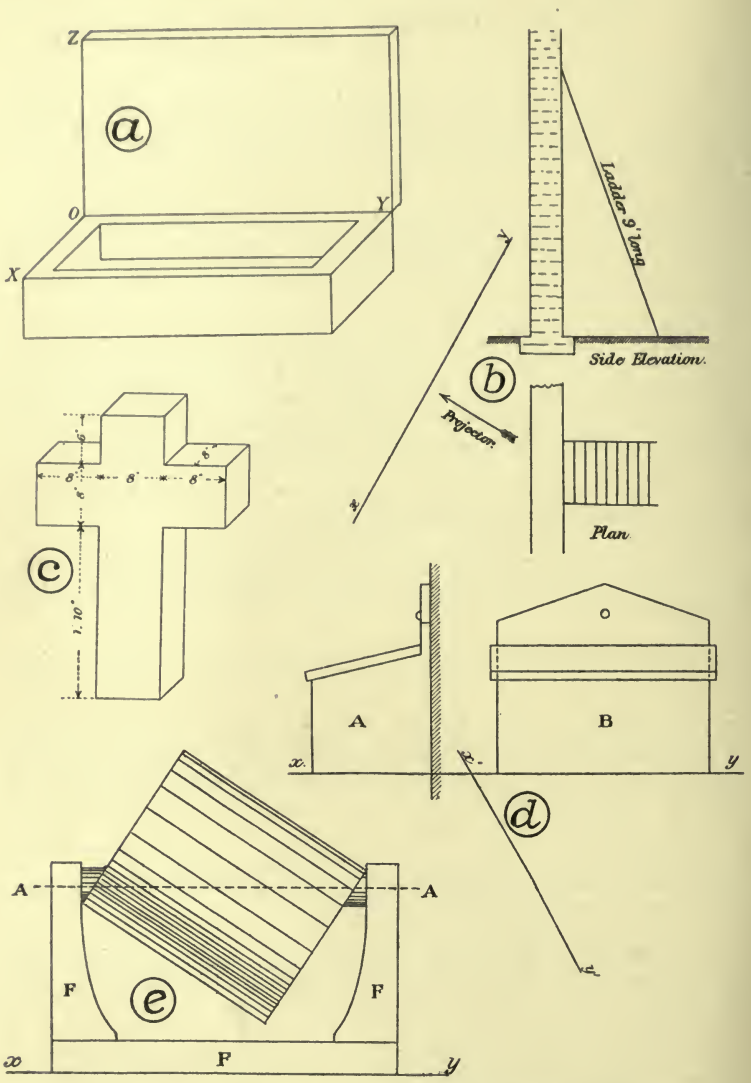


FIG. 369.—Illustrating Exercises on Orthographic Projection.

N.B.—Figs. (*a*), (*b*), (*d*) and (*e*) should be enlarged to three times the original size.

Exercises on Chapter XII. Orthographic Projection

(See Fig. 369.)

1. The figure (*a*) shows the size and shape of a rectangular box with the lid open at right angles along the hinge OY, dimensions parallel to OY and OZ being set off full size, and parallel to OX half size.

(*a*) Draw an end elevation of the box with the lid open at 60°, that is an elevation on a vertical plane parallel to ZOX.

(*b*) From the elevation project a plan.

(*c*) From the plan project a new elevation on a vertical plane which makes 45° with the first vertical plane.

N.B.—The pictorial view is not to be copied, but dimensions are to be measured from it. (*Bd. of Educ.*)

2. The projections are given of a ladder (*b*) leaning against a wall. Draw an elevation of the ladder on the given ground line (*xy*). Scale $\frac{1}{2}$ in. = 1 ft.

3. The dimensioned pictorial view (*c*) gives particulars of a stone cross. Draw the plan of the cross. Scale 1 in. = 1 ft.

Project an elevation on a VP making 30° with the front vertical face.

4. Two elevations (*d*) of a kitchen salt box are given. Copy the elevation A and from it project a plan. From the plan project an elevation on $x'y'$.

N.B.—The view B need not be drawn.

5. The drawing (*e*) shows a concrete mixing machine which consists of a cylinder mounted obliquely on an axis AA and a framework F. Draw a plan of the cylinder, omitting all details of the framework.

6. The base of a right pyramid is an equilateral triangle of 2 ins. side. The height of the pyramid is $2\frac{1}{2}$ ins. Draw the plan of the pyramid when it lies with one triangular face (not the base) on the ground. (*Bd. of Educ.*)

7. The height of a square pyramid is 3 ins., the base is a square of 2 ins. side. Draw its plan and elevation when the axis is vertical; also draw a plan when one of the sloping edges of the solid is (*a*) vertical, (*b*) horizontal. (*L. C. C. Scholarships.*)

8. A hopper (Fig. 238) is 3 ft. 6 ins. square at its wide end, 2 ft. wide at its narrow end, and the distance between the ends is 2 ft. 6 ins. Determine the correct shape for a face template for marking the outer face of one of the sides of the hopper.

9. Taking the length of the plan AB of the strut shown in Fig. 239 as being 10 ft. and its inclination to the ground as 35°, find the true length of the strut.

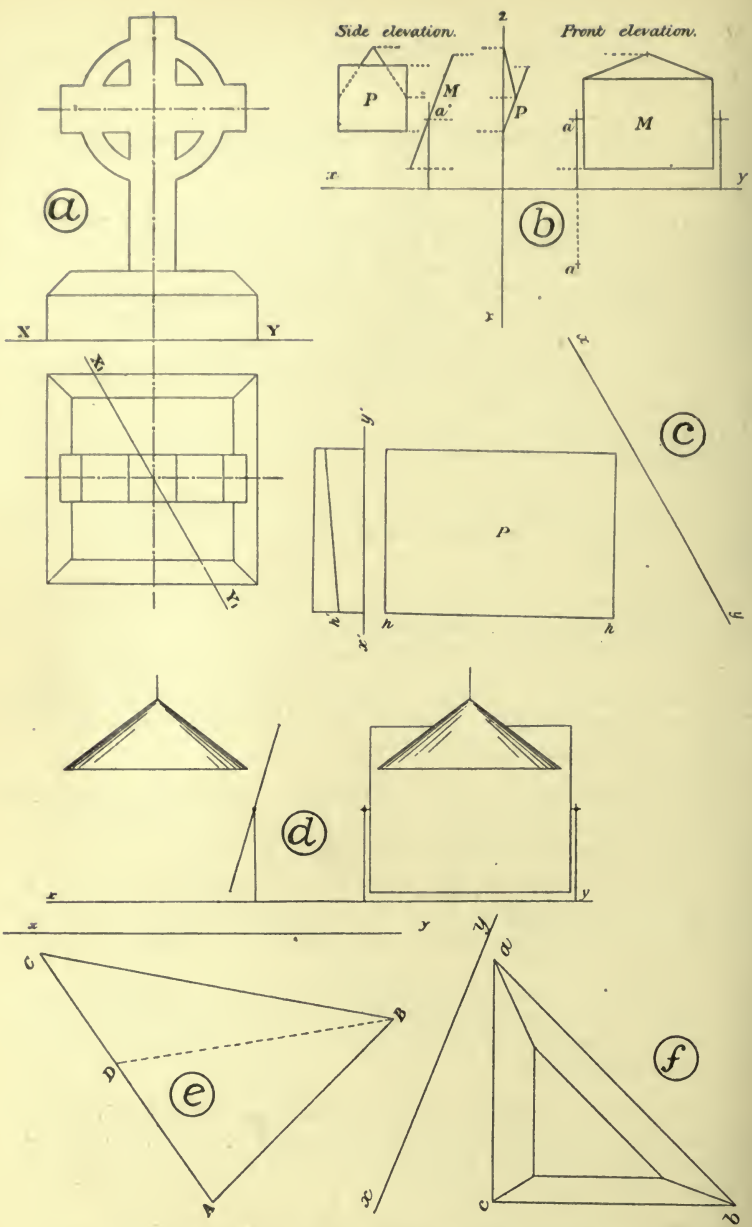


FIG. 370.—Illustrating Exercises on Orthographic Projection—*continued.*

N.B.—The diagrams shown above (with the exception of (*f*)) should be enlarged to three times the original size.

Orthographic Projection. (See Fig. 370.)

1. Draw the plan and elevation of the given cross and base (*a*); and a new elevation on a ground line parallel to XY. (*Bd. of Educ.*)

2. Two elevations (*b*) are given of a mirror M, tilted backwards, and a picture frame P hanging from the wall. From the front elevation project a plan, one point, *a*, of which is drawn.

Determine the plan of the image of the picture frame in the mirror. Note.—If K is any point in space, and K′ its image, then the plane of the mirror bisects KK′ at right angles. (*Bd. of Educ.*)

3. Writing desk (*c*). The plan and elevation are shown with the desk closed. Draw the plan when the lid is opened by turning about the hinge HH through an angle of 135°. From your plan project an elevation on *xy*. (*Bd. of Educ.*)

4. The figure (*d*) represents a hanging lamp shade and a tilted mirror. Draw the two elevations of the image of the conical shade in the mirror.

N.B.—If P is any point in space, and P′ its image, then the plane of the mirror bisects PP′ at right angles. (*Bd. of Educ.*)

5. The triangle ABC in (*e*) represents a thin metal plate resting on the horizontal plane. Determine the plan of the portion ABD after it has been bent upwards about BD through an angle of 60°. Draw an elevation of the bent plate on *xy*. (*Bd. of Educ.*)

6. The plan is given (*f*) of a thin 60° set square resting on its short edge BC, $\frac{1}{4}$ full size.

(*a*) Determine the height of the corner A and index its plan *a* in inches.
(*b*) Draw the elevation of the set square on *xy*.
(*c*) Determine and measure the angle which the edge AB makes with the vertical plane. (*Bd. of Educ.*)

7. (No diagram.) A 45° set square rests on the HP with one corner in XY, and with its hypotenuse, which is 3 ins. long, inclined at 35° to the XY. It is turned about its hypotenuse as hinge through 50°. Represent the set square full size in plan and elevation. (*London Matric.*)

8. Draw the plan, elevation, and new elevation of the clock face shown in Fig. 218 twice the size shown.

9. Draw the plan and elevation of the archway shown in Fig. 219 three times the size shown, and make a new elevation of the archway as it would appear from the S.E., *i.e.* draw the new ground line G′L′ at 45° to the original ground line GL.

10. Determine the true shape of the opening in a roof which slopes at 30°, and which is penetrated by an octagonal shaft as shown in Fig. 234. Side of octagon $\frac{3}{4}$ in.

11. Find the true shape of the hole which must be cut in a roof surface (as shown in Fig. 235) to allow a cylindrical pipe, 14 ins. diameter, to pass through. Scale 3 ins. = 1 ft.

12. A line is stretched tightly from one corner of the ceiling to the opposite corner of the floor in a room 14 ft. long, 12 ft. wide, and 10 ft. high, as shown in Fig. 240. Find the length of the cord.

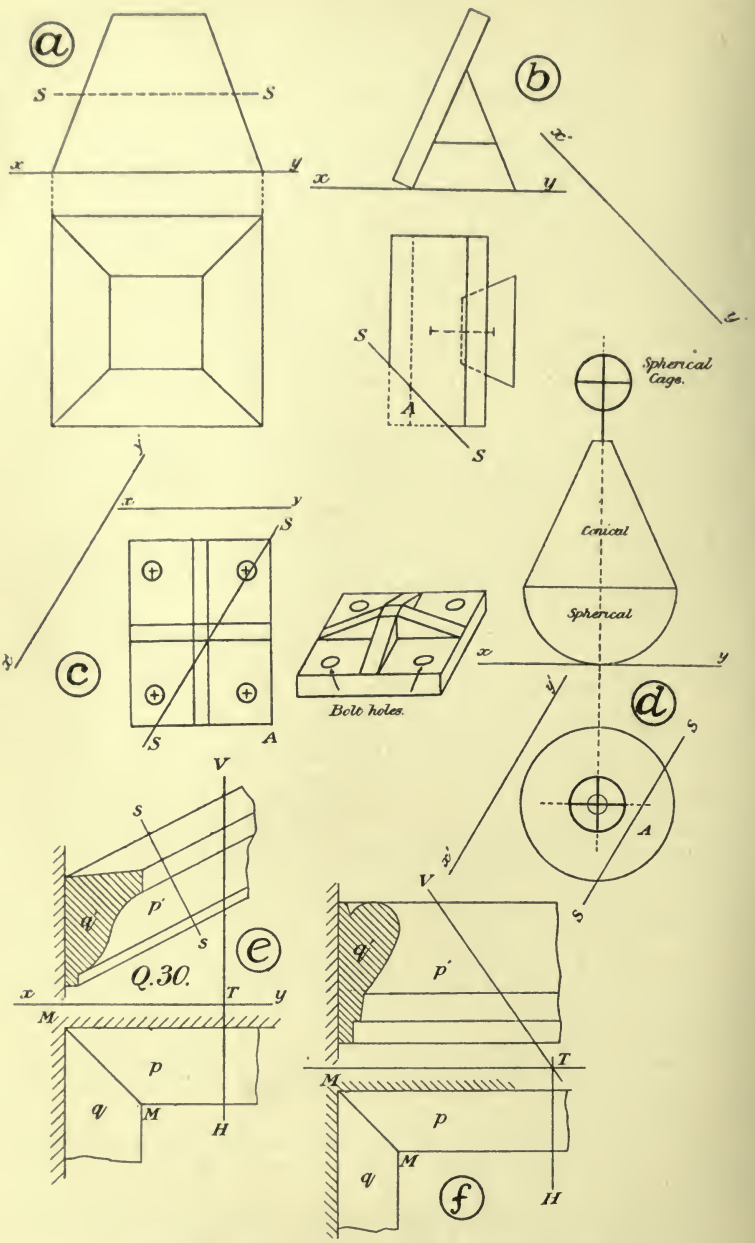


FIG. 371.—Illustrating Exercises on Sections of Solids.

Exercises on Chapters XIII and XIV. Sections of Solids. (See Fig. 371.)

1. You are given (*a*) the plan and elevation of a frustum of a square pyramid. Set out the true shape of one of its sloping sides (see Fig. 238). Draw the plan of the mid-section SS and find its area a, and the areas a_1 and a_2 of the ends of the solid. Measure the height and calculate the volume of the frustum by using the prismoidal formula :—

$$\text{volume} = \text{height} \times \frac{a_1 + 4a + a_2}{6}$$

2. A picture frame (*b*) is shown in plan and elevation. Draw an elevation on $x'y'$, the corner A of the frame being supposed cut off by the V plane SS and removed.

3. A cast-iron cover or lid (*c*) is shown in plan and in pictorial projection. From the plan project an elevation on xy, the lower part of the cover resting on the horizontal plane, and vertical dimensions being taken from the pictorial view, in which vertical lines appear full size.

SS is a vertical section plane. Draw a sectional elevation on $x'y'$ (parallel to SS), the front portion SAS of the cover being supposed removed.

4. Bell buoy (*d*). Draw a sectional elevation of the buoy on $x'y'$ taken parallel to the vertical section plane SS, the portion A of the buoy in front of SS being supposed removed.

5. P and Q in (*f*) are two portions of a horizontal picture rail which meet in the corner of a room and are mitred together at MM. The form of the cross-section is shown at q'. Find the shape of the common section at the joint MM. Also draw the *plan* of the section of P by the plane VTH.

6. A moulding running horizontally is shown (*e*) in plan and cross-sectional elevation at q, q'. An inclined or raking moulding is given at p, p'. The two mouldings are required to mitre correctly together at a vertical section MM. Determine the form of the cross-section (by the plane SS) of the raking moulding P. Find also the shape of the section of the latter by the vertical plane VTH.

7. Draw the plan and elevation to a scale of $\frac{1}{3}$ full size of a frustum of a cone. Diameter of larger end 10 ins., diameter of small end 6 ins., length 12 ins., and calculate the volume of the frustum by using the prismoidal formula :—

$$\text{volume} = \text{height} + \frac{a_1 + 4a + a_2}{6}$$

(N.B.—The diagrams shown in Fig. 371 should be enlarged to three times the original size.)

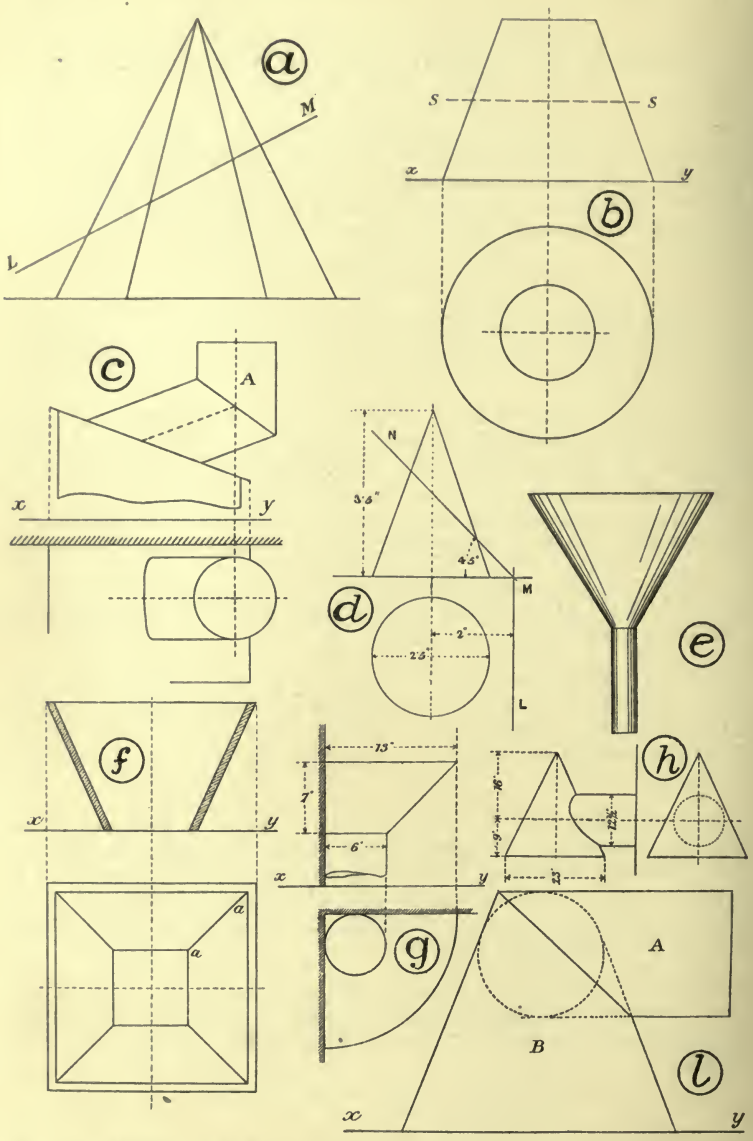


FIG. 372.—Illustrating Exercises on Developments and Interpenetration of Solids.

N.B.—The above diagrams (with the exception of the dimensioned sketches) should be enlarged to three times the original size.

Exercises on Chapters XV. and XVI. Developments and Interpenetration of Solids. (See Fig. 372.)

1. A hexagonal pyramid (*a*) is cut by a plane whose vertical trace is L'M' and whose horizontal trace is perpendicular to the VP. Draw the true shape (*i.e.* the development) of a piece of sheet material for making the lower part of the pyramid, *i.e.* the frustum below L'M'.

2. A frustum of a cone is shown (*b*) in plan and elevation. Find the true shape of a piece of sheet material for making the lower part of the frustum, *i.e.* between *xy* and the horizontal section plane SS.

3. The diagram (*c*) shows the roof of a small shed, through which passes a cylindrical pipe. Draw the plan of the hole which must be cut through the roof so as just to allow the pipe to pass. Set out the shape of a piece of sheet metal to cover this roof, showing the hole. Develop the surface of the vertical branch A of the pipe.

4. If the lower part of the cone, shown in (*d*), including the cut surface, is to be made from thin sheet copper, find the shape to which the sheet must be cut.

5. The hopper shown in the diagram (*f*) is required to be lined on its inner surface with sheet metal bent out of one piece. Find the shape of this piece, the joint being along AA.

6. The elevation is given of an elbow pipe (*l*) made in sheet metal. A is cylindrical, and B conical.

(*a*) Draw the plan of the pipe showing the elliptical join.

(*b*) Draw a development of the surfaces showing the shapes to which the two sheets must be cut before being bent and joined.

7. Find the true shape of a piece of sheet material for making the inclined branch B of the cylindrical pipe shown in (*c*).

8. A funnel (*e*) is to be made up from two pieces of sheet iron or tin. The finished model is to be three times as large as that shown. Determine the correct shape of each of the two pieces of material, *i.e.* one piece for making the upper (or conical) part, and the other for making the lower (or cylindrical) part of the funnel.

9. A hopper (*g*) for a cylindrical pipe in the corner of a room is shown to be made of sheet metal. It will be seen that its upper end is a quadrant and its lower end a circle.

Working to the figured dimensions and to a scale of quarter size, draw the template for marking out the plate—that is, develop the surface of the hopper. Only one of the two symmetrical halves of the template need be drawn. (*Bd. of Educ.*)

10. Cylindrical pipe with conical cowl (*h*). To a scale of 1 in. to 10 ins. and working to the given dimensions, draw the plan and elevation as shown, and add a side elevation as viewed in the direction indicated by the arrows. Determine by construction and show in both elevations the curve of intersection of the surfaces. (*Bd. of Educ.*)

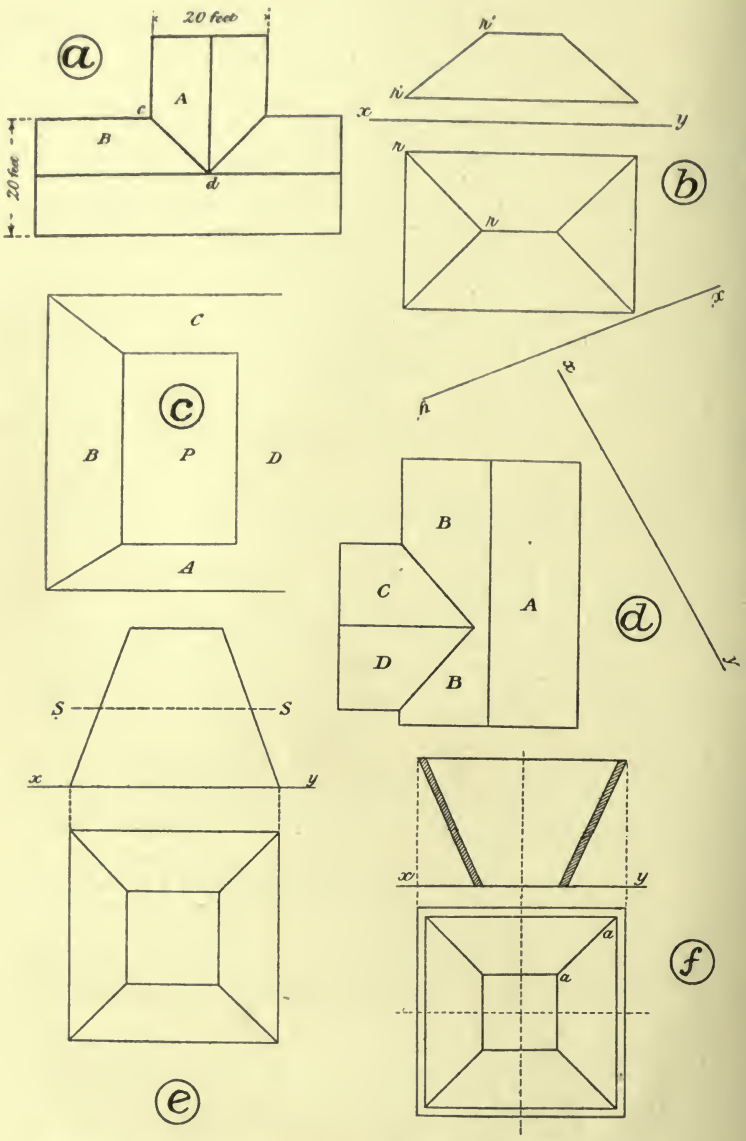


FIG. 373.—Illustrating Exercises on Roof Surfaces, etc., and Oblique Cuts in Carpentry.

Exercises on Chapters XVII. and XVIII. Roof Surfaces, etc., and Oblique Cuts in Carpentry, etc. (See Fig. 373.)

1. (*No diagram.*) The roof of a house is rectangular in plan, and two adjacent surfaces are inclined to the horizontal at 30° and 40° respectively. Find the inclination to the horizontal of the "hip," or line in which the two surfaces meet.

What is the dihedral angle between the surfaces? (*Bd. of Educ.*)

2. (*No Diagram.*) The roof of a house is rectangular in plan, and two adjacent surfaces are each inclined at 32° to the horizontal. What is the inclination to the horizontal of the "hip" or line where the two upper surfaces meet? (*Bd. of Educ.*)

3. The diagram (*a*) is the dimensioned roof plan of a house. The roof planes are both inclined to the horizontal at 33°. Find the length and inclination of the valley rafter CD, that is, of the line whose plan is *cd*. Employ a scale of $\frac{1}{4}$ in. to 1 ft. (*Bd. of Educ.*)

4. The plan and elevation of a "hip" roof are given in (*b*). Copy the given drawing twice the size shown, and draw an elevation on $x'y'$. Working to a scale of $\frac{1}{12}$ in. to 1 ft., find the true length and inclination of the hip rafter RR. (*Bd. of Educ.*)

5. You are given the plan of a roof with gable ends (*d*), linear scale $\frac{1}{4}$ in. to 10 ft. Copy the given diagram twice the sizes shown. The roof surfaces A and B being inclined to the horizontal at 40°, what is the inclination of the surface C? Draw an elevation of the roof on *xy*. Set out the true shape, and measure the area of the surface BB. Find the dihedral angle between the roof surfaces B and C. (*Bd. of Educ.*)

6. The rectangle P in (*c*) shows a horizontal plot of ground raised above the surrounding level, and having the embankments A, B, C, and D on its four sides. The embankment A has a slope of 40° to the horizontal. Find the *slopes* of the embankments B and C. The embankment D has a slope of 23°; show its *boundary*.

7. The diagram (*e*) shows plan and elevation of a ventilating shaft. Copy the drawings three times the original size, and find the dihedral angle between any two sides of the shaft. (*Bd. of Educ.*)

(Hint: see Fig. 287.)

8. (*f*) shows a hopper in plan and sectional elevation.

Copy the drawings three times the original size, and determine the correct bevel for fitting the two sides of the hopper which meet at *aa*.

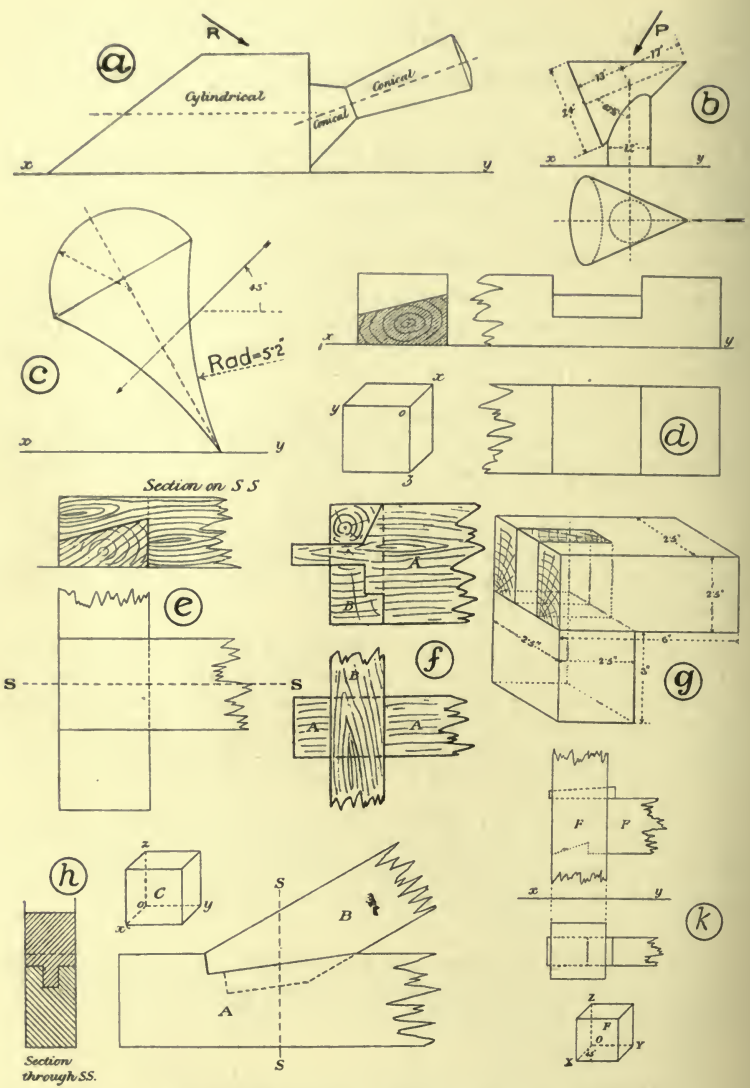


FIG. 374.—Illustrating Exercises on Shadows and Pictorial Projection.

Exercises on Chapters XIX. and XX. Shadows and Pictorial Projection. (See Fig. 374.)

(The Diagrams should be copied three times the given size.)

1. The diagram (*a*) represents a scoop resting upon the HP. Determine the shadow of the scoop as cast on the HP, and the shadow of the cylindrical portion of the scoop as cast upon the conical parts. The rays of light are parallel to R in elevation, and parallel to *xy* in plan. (*Bd. of Educ.*)

2. (*b*) represents a conical metal cowl supported upon a cylindrical pipe. Draw the plan and elevation as shown, working to the given measurements and using a scale of $\frac{1}{4}$ full size. Draw the shadow of the cowl as cast upon the HP, the rays of light being parallel to P in elevation and parallel to *xy* in plan. (*Bd. of Educ.*)

3. (*c*) is the elevation of a solid of revolution, transverse sections of which taken at right angles to the axis (dotted) being circles. Determine the shadow of the solid as cast upon the HP, the rays of light in elevation being inclined at 45° to the *xy* and in plan parallel to *xy*.

4. (*d*) represents a piece of quartering in which a sinking has been made to receive another timber at right angles. The bottom of the sinking is inclined to the horizontal as indicated in the sectional view. Make an oblique pictorial view of the piece of wood in such a way that measurements taken along the horizontal axis are drawn full size, while those on the oblique projectors are drawn half full size as indicated in the pictorial view of the cube. (*Bd. of Educ.*)

5. (*e*) shows a timber joint similar to that in (*d*). Make an isometric projection of the piece which runs in the direction indicated by SS.

6. (*f*) shows plan and sectional view of a tusk tenon joint between two joists A and B. Make an isometric projection of the piece B and an oblique pictorial projection of the piece A.

7. (*g*) shows an oblique pictorial projection of an open mortise and tenon or angle bridle joint. Working to the given measurements make an isometric projection of the two pieces of wood showing the joint taken apart.

8. (*h*) represents the joint between a principal rafter B and tie-beam A. The dotted lines on the elevation and the lines on the sectional view indicate the character of the joint. (*a*) Make an isometric drawing of the part of the tie-beam, showing the mortise and sinking made to receive the principal rafter. (*b*) Make an oblique pictorial projection of the principal rafter in the manner indicated in the cube, where measurements along the axis *oy* and *ox* are full scale and those along the oblique axis *ox'* are half scale.

9. (*k*) shows an upright wooden member mortised to receive a horizontal rail F, the mortise being made equal to the whole thickness of F, and slightly wider than F, a wedge being afterwards inserted in the mortise to bring F down into position on the dovetail.

(*a*) Make an isometric projection of the upright member or style, making the necessary allowance for wedge room.

(*b*) Make an oblique pictorial projection of the rail F, working to the following measurements and employing a scale of $\frac{1}{2}$ full size.

Width of F = $3\frac{1}{2}$ ins.; thickness $1\frac{3}{4}$ ins.
Post $3\frac{1}{2}$ ins. × $3\frac{1}{2}$ ins. Other measurements to same scale.

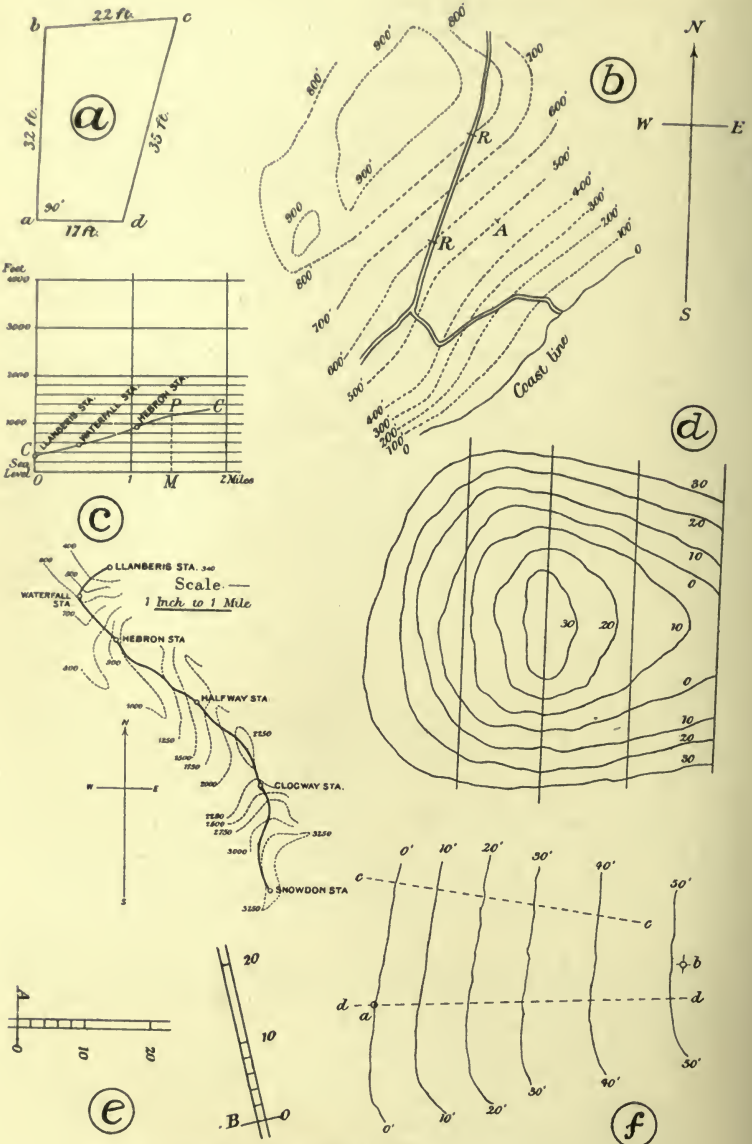


FIG. 375.—Illustrating Exercises on Horizontal Projection.

Exercises on Chapter XXI. Horizontal Projection.

(See Fig. 375.)

1. Draw the quadrilateral *abcd* in (*a*) to the given dimensions; scale $\frac{1}{8}$ in. to 1 ft. This is the plan of a piece of ground ABCD which has a slope of one vertical in six horizontal, the lines of steepest ascent being parallel to the diagonal AC. Find the area of the plan, and also the area of the plot in square yards.

What are the vertical heights of B, C, and D above the level of A. (*Bd. of Educ.*)

2. A portion of a contoured map of a district is given in (*b*), showing a road going over the side of a hill. The scale of the map is $\frac{1}{2}$ in. to 1 mile, and the contour lines are at heights in feet above the sea-level as marked. Only so much of the diagram need be copied as is necessary to answer the following questions:—

(*a*) Measure to scale the length of the section of the road RR.

(*b*) What is the vertical rise in this length? And at what angle is this portion of the road inclined to the horizontal?

(*c*) What would be the direction and slope of a path going straight up the hill and starting from A? (*Bd. of Educ.*)

3. You are given (*c*) a map of a portion of Snowdon to a scale of $\frac{1}{2}$ in. to 1 mile, contoured in feet above sea-level, and showing the line of the railway from Llanberis station to the summit of the mountain. By means of tracing paper and a pricker, or otherwise, measure the length of the railway. If the average slope of the line is 1 ft. rise in x feet of length, what is x? Where is the railway steepest, and what is its inclination to the horizontal at this place?

A curve CC is begun, showing the height MP above sea-level of any point P on the railway, distant OM miles from Llanberis station as measured along the railway line on the map. Complete this curve. (*Bd. of Educ.*)

4. The given plan (*d*), contoured at vertical intervals of 10 ft., defines the shape of the surface of a piece of ground; horizontal scale $\frac{1}{2}$ in. to 100 ft.

Draw vertical sections through the four given lines; scale for heights $\frac{1}{4}$ in. to 10 ft. (*Bd. of Educ.*)

5. A and B in (*e*) are the scales of slope of two planes. Draw the plan of a sphere of $\frac{1}{2}$ in. radius, resting on the horizontal plane and touching the two planes, both of which pass over the sphere. Show the points of contact with the planes, and write down their figured heights. Unit = 0·1 in. (*Bd. of Educ.*)

6. The form of the surface of a piece of ground is shown by a plan (*f*) contoured at vertical intervals of 10 ft., the linear scale being $\frac{1}{4}$ in. to 10 ft. Plot a zigzag path, connecting the two given points A and B, the several lengths having a constant slope of one vertical in two horizontal, and the corners of the path lying on the lines *cc*, *dd*. What is the total length in feet of this zigzag path? (*Bd. of Educ.*)

7. (*No diagram.*) A line AB, 10 cm. long, has its end A in the horizontal plane, and AB is inclined at 35° to the latter. Determine the length of its plan, *ab*, and the height of B.

Represent AB by a figured plan. Draw an elevation on a vertical plane which contains B and is 3 cm. from A. (*Bd. of Educ.*)

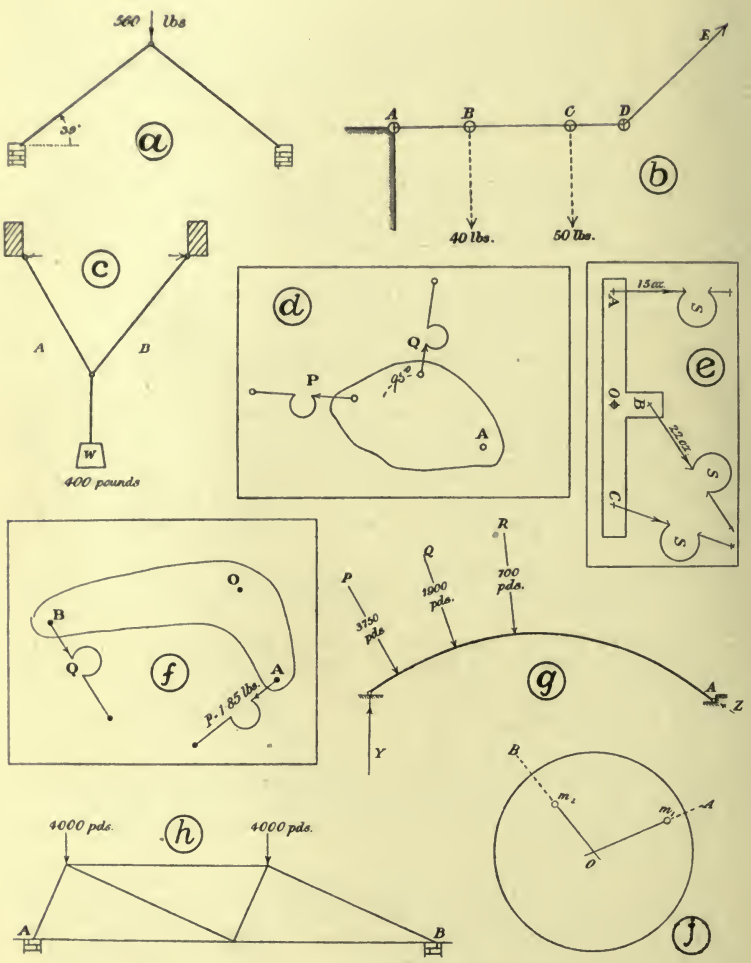


FIG. 376.—Illustrating Exercises on Graphic Statics.

Exercises on Chapter XXII. Graphic Statics.

(See Fig. 376.)

1. The diagram (*a*) shows a simple roof truss resting on walls and loaded at the apex with 560 lbs., the inclination of the rafters being 39°.

(*a*) Find and measure the thrust in each rafter.

(*b*) Find and measure the horizontal thrust on the walls. (Force scale: 1 in. to 100 lbs.) (*Bd. of Educ.*)

2. ABCD in (*b*) is a horizontal rigid bar, hinged at A, loaded at B with 40 lbs., at C with 50 lbs., and retained in place by a cord DE (passing over a pulley) attached to a pin at D. Copy the drawing three times the given size, and find the stress in the cord DE. Employ a funicular polygon, using for the scale of loads $\frac{1}{4}$ in. to represent 10 lbs. (*Bd. of Educ.*)

3. A load of 400 lbs. in (*c*) is carried from overhead beams in the manner shown, where A and B are two bars pinned to the beams and to each other. Find the pull in each bar. Find also the horizontal force pulling the beams towards each other. (*Bd. of Educ.*)

4. A piece of card (*d*) is pinned temporarily to a drawing board, and forces P and Q are applied to the card, as shown. Copy the diagram three times the original size. It is desired to balance these two forces by a force of 30 ozs. acting at A, so that the card shall remain at rest on removal of the pins. What must be the line of this third force, and what the original magnitudes of P and Q? (*Bd. of Educ.*)

(*Hint.—The line of the required force must pass through the intersection of* P *and* Q.)

5. A cardboard lever (*f*) is pinned or hinged to a board at the point O. Forces P and Q are applied to the lever as shown. The linear scale of the drawing is $\frac{1}{8}$ full size, and the magnitude of P = 1·85 lbs. What is the moment of P about O? Find the magnitude of Q if the lever is balanced. Show a force which, being applied to the lever near O, shall relieve the pin from all pressure and allow of its being removed without disturbing the equilibrium. (*Bd. of Educ.*)

6. In a laboratory experiment ABC in (*e*) is a cardboard lever, pinned to a board at O and free to turn about this point. Forces which can be measured by the springs S are applied to the lever at A, B, and C in the directions indicated. If the forces A and B measure 15 ozs. and 22 ozs., what should the force at C measure when the card is in equilibrium?

What is the force on the pin at O? (*Bd. of Educ.*)

(*The lever* ABC *should be enlarged to twice the size given, keeping the angle between the forces at* B *and* C *as shown in the diagram.*)

7. Find the resultant of the three given forces P, Q, and R in (*g*), and measure its magnitude. These three forces are balanced by two forces, Y and Z, the former acting along the given vertical line, and the latter through the given point A. Copy the diagram twice the size shown, and determine the magnitude of Y and the magnitude and line of Z.

Employ a scale of 1 in. to 1000 lbs. (*Bd. of Educ.*)

8. A disc rotating about O in (*j*) has two masses, m_1 and m_2, fixed to it as shown, which exert centrifugal forces on the bearing O of 5 lbs. and 8 lbs. respectively along the radii OA and OB. Find the resultant of the centrifugal forces. Measure its magnitude and the angle which its line makes with OA. (*Bd. of Educ.*)

9. The given roof truss (*h*) is loaded as shown. Find and measure the vertical supporting forces at A and B. Draw the figure twice the size shown, and determine the thrust in the horizontal upper member of the frame. (*Bd. of Educ.*)

ANSWERS TO EXERCISES

(*Pages* 373 *et seq.*)

Exercises on Chapter II. Page 374.

(1) Actual error, say, 1·5 per cent. ; percentage error 1·8 per cent.
(2) CA (measured) 23 ins. ; calculated 23·09 ins.
(4) Cos C = $\frac{3}{4}$ or ·75. (C = 41·5° ; hence, cos C lies between ·7547 and ·7431.)
(5) *b* = 11·0 (measured) ; 11·01 (calculated).
(6) AOB = 20° ; BM = 3·4 ; AN = 3·6 ; sin AOB = ·342 ; tan AOB = ·364.
(7) AB = 4·2 ins. ; OC = 2·8.
Area = 11·76 sq. ins. (11·84 sq. ins. by calculation).
(8) (*a*) 152·6 ft. ; 164·4 ft. (*b*) 105 ft.

Exercises on Chapter III. Page 375.

(4) 1·93 ins. ; 2·36 ins. ; 2·64 ins.
(5) 3·4 ins. ; 3·5 ins. ; 5·1 ins.
(6) $96\frac{6}{6}$° ; 40 chns. ; 88 chns.
(7) 46·3 chns. ; 14° N. of E.
(8) 200 yds. ; 19° S. of E.
(9) 38·5° ; 24°.

Exercises on Chapter IV. Page 377.

(3) 28·35 cub. ft.
(4) 36 ft. and 41 ft. ; 71·18 sq. yds.
(5) K = 2889 sq. ft. ; 506 people.
(6) 637·5 sq. ft.
(7) 5·9 cub. yds. ; FR = 4·8 cm.
(8) 2841·5 sq. yds. ; 56·8 yds.
(9) 18,298 sq. yds. ; P = 2756 sq. yds.

Exercises on Chapter V. Page 379.

(1) 6 : 5 ; 137 : 89.
(2) 131 sq. yds.

Exercises on Chapters XVII and XVIII. Page 395.

(1) 26°; 118°.
(2) 24°.
(3) 15 ft. 9 ins.; 24·5°.
(4) 16 ft. 6 ins.; 30°.
(5) 36·5°; 1047 sq. ft.; 115°.
(6) B = 28·5°; C = 35°; width of D in plan = 1·4 cms.
(7) 98°.
(8) Bevel set to 51° (dihedral angle = 102°).

Exercises on Chapter XXI. Page 399.

(1) Area of plan = 648 sq. ft.; area of plot = 664 sq. ft.
B = 4 ft. 6 ins.; C = 7 ft.; D = 1 ft. 6 ins.
(2) (*a*) 2200 yds.; (*b*) 200 ft.; 2°; (*c*) 44° W. of N. at slope of 6°.
(3) $5\frac{1}{2}$ m.; $x = 9\frac{5}{8}$; between levels 2500 and 2750; 30°.
(6) 128 ft.
(7) 8·2 cm. and 5·7 cm.
(8) AB = 40·7 ft.; AC = 31·3 ft.

Exercises on Chapter XXII. Page 401.

(1) Rafters = 445 lbs.; horizontal thrust = 345 lbs.
(2) $57\frac{1}{2}$ lbs.
(3) A = 300 lbs.; B = 200 lbs.; horizontal force = 150 lbs.
(4) New force makes 113° with Q; P = 28 ozs.; Q = 13·5 ozs.
(5) M = 8·6 lb.-ins.; Q = ·95 lb.; force = 2·17 lbs.
(6) 24·7 ozs.; 58 ozs.
(7) Resultant = 6300 lbs.; Y = 4475 lbs.; Z = 2885 lbs.
(8) 8·25 lbs. at 30° with OA.
(9) A = 5400 lbs.; B = 2800 lbs.; horizontal thrust = 3500 lbs.
(10) 8 cwts.
(11) 325 lbs.
(12) B = 80 lbs.; C = 100 lbs.

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